Ph 221C Assignment #3 Solutions

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Electric Dipole Moment

Forbidden terms

- $\bar{u}'\gamma^5 u(p+p')^\mu$
  We can express this term in terms of the last one using the Gordon identity

$$\bar{u}(p')\left[(p+p')^\mu - 2iS^{\mu\nu}(p-p')_\nu\right]\gamma_5 u(p) = 0$$

- $\bar{u}'\gamma^5 u(p-p')^\mu$
  The Ward identity $k^\mu \mathcal{M}_\mu = 0$ for QED implies

$$0 = iq_\mu V^\mu(p,p') = q^2\bar{u}'\gamma^5 u.$$

  Note that the Ward identity holds for an off-shell photon.

- $\bar{u}'\gamma^5\gamma^\mu u$
  Again by the Ward identity

$$\begin{aligned} 0 &= (p-p')^\mu \bar{u}'\gamma^5\gamma^\mu u \\ &= \bar{u}'\gamma^5\left(\not{p} - \not{p}'\right)u \\ &= 2m\bar{u}'\gamma^5 u \end{aligned}$$

- $\bar{u}'\gamma^5 S^{\mu\nu} u q_\nu$
  Pauli's theorem states that any 4 × 4 matrix can be written as a linear combination of the 16 matrices,

$$1, \gamma^\mu, \gamma^{\mu\nu}, \gamma^5\gamma^\mu, \gamma^5.$$

  Since both $\gamma^5 S^{\mu\nu}$ and $\epsilon^{\mu\nu\sigma\rho}S_{\sigma\rho}$ transform as tensors under the Lorentz group, they must be proportional by Pauli's theorem. We can fix this constant of proportionality by evaluating a single component S^{01} or using Srednicki (94.27),

$$S^{\mu\nu} i\gamma^5 = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}S_{\rho\sigma}$$

Electric Dipole Interaction

Perhaps the cleanest solution is to first simplify the new interaction term in position space and then convert to momentum space.

$$\mathcal{L}_{\text{int}} = \alpha\bar{\Psi}S^{\mu\nu}\Psi\epsilon_{\mu\nu\rho\sigma}F^{\sigma\rho}$$

Introducing an external electric field $F^{0i} = E^i$ and vanishing magnetic field $F^{ij} = 0$ we have

$$\begin{aligned}\mathcal{L}_{\text{int}} &= 2\alpha\bar{\Psi}S^{kl}\Psi\epsilon_{kl0i}E^i \\ &= \alpha\bar{\Psi}\epsilon^{klm}\sigma_m\Psi\epsilon_{kl0i}E^i \\ &= -2\alpha\bar{\Psi}\sigma^i\Psi E^i \\ &= -d_e\vec{S}\cdot\vec{E}\end{aligned}$$

with $\vec{S} = \bar{\Psi}\frac{1}{2}\sigma\Psi$ and $d_e = 4\alpha$. To convert to momentum space recall that the interaction term

$$\mathcal{L}_{\text{int}} = g_e\bar{\Psi}\gamma^\mu\Psi \propto -g_e\vec{S}\cdot\vec{B}$$

so we can convert to momentum space by analogy with the magnetic dipole term.

Direct Approach

More explicitly (following Peskin 6.2) we can consider the expansion of the Dirac spinors in the non-relativisitc limit. First consider the vector potential

$$A^{\text{cl}}_\mu(x) = (\phi(x), 0)$$

which is

$$\tilde{A}^{\text{cl}}_\mu(q) = (2\pi\delta(q^0)\tilde{\phi}(q), 0)$$

in momentum space.

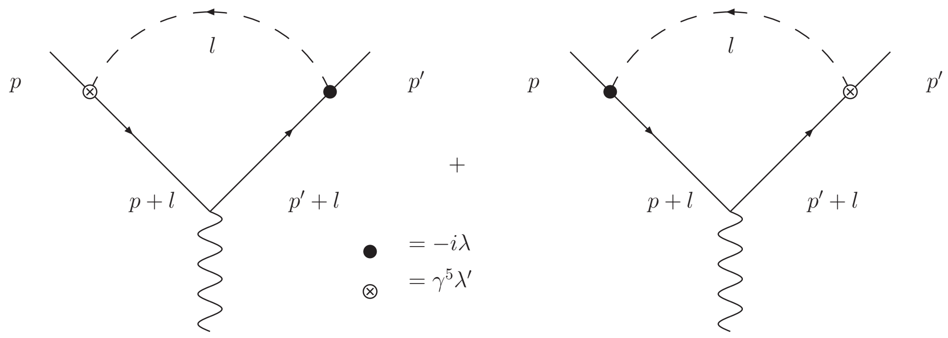
$$i\mathcal{M} = ie\bar{u}(p')V^\mu(p', p)u(p)\tilde{A}^{\text{cl}}_\mu(q)$$

CP Violation

(a)

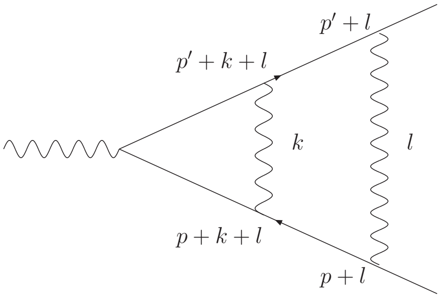
(b)

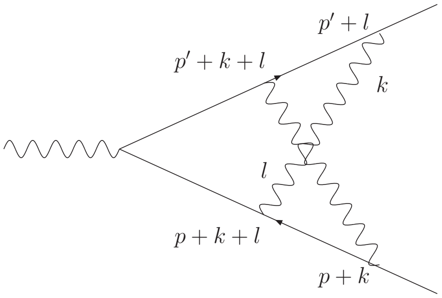
(c)



(d)

IR Divergences from soft virtual photons





The amplitude for the first diagram is

$$i\mathcal{M}_{(1)} = \int \frac{d^4k d^4l}{(2\pi)^8} \bar{u}(p')(ie\gamma^\mu) S(\not{p}' + \not{l})(ie\gamma^\nu) S(\not{p}' + \not{l} + \not{k})(ie\gamma^\rho) \\ \times \; S(\not{p} + \not{k} + \not{l})(ie\gamma_\nu) S(\not{p} + \not{l})(ie\gamma_\mu) u(p) \frac{-i}{k^2 + i\epsilon} \frac{-i}{l^2 + i\epsilon} \tag{0.1}$$

where S is the fermion propagator

$$S(\not{k}) = \frac{\not{k} - m}{k^2 + m^2}$$

For soft photons $k, l \to 0$ and

$$S(\not{p} + \not{l}) \approx \frac{i(\not{p} - m)}{2p \cdot l}$$

Using this to simplify the amplitude,

$$i\mathcal{M}_{(1)} = -e^4 \int \frac{d^4k d^4l}{(2\pi)^8} \bar{u}(p')\gamma^\mu(\not{p}' - m)\gamma^\nu(\not{p}' - m)(ie\gamma^\rho)(\not{p} - m)\gamma_\nu(\not{p} - m)\gamma_\mu u(p) \\ \times \; \frac{1}{k^2 l^2} \frac{1}{2(p' \cdot l)2(p' \cdot (l + k))2(p \cdot l)2(p \cdot (l + k))} \tag{0.2}$$

Simplifying the numerator using the Dirac equation

$$\left(\not{p} + m\right) u(p) = 0$$

and gamma matrix algebra we have

$$i\mathcal{M}_{(1)} = -e^4 \int \frac{d^4k d^4l}{(2\pi)^8} \frac{1}{k^2 l^2} \frac{(p' \cdot p)^2}{(p' \cdot l)(p \cdot l)(p \cdot (l + k))(p' \cdot (l + k))} \bar{u}(p')(ie\gamma^\rho)u(p) \tag{0.3}$$

The second amplitude is almost the identical except there is one exchange $k \leftrightarrow l$ in the denominator

$$i\mathcal{M}_{(2)} = -e^4 \int \frac{d^4k d^4l}{(2\pi)^8} \frac{1}{k^2 l^2} \frac{(p' \cdot p)^2}{(p' \cdot l)(p \cdot k)(p \cdot (k + l))(p' \cdot (k + l))} \bar{u}(p')(ie\gamma^\rho)u(p) \tag{0.4}$$

Using the identity

$$\frac{1}{p \cdot (k + l) p \cdot k} + \frac{1}{p \cdot (k + l) p \cdot l} = \frac{1}{(p \cdot k)(p \cdot l)}$$

we find the total amplitude is

$$\begin{aligned} i\mathcal{M} &= -e^4 \int \frac{d^4k d^4l}{(2\pi)^8} \frac{1}{k^2 l^2} \frac{(p' \cdot p)^2}{(p \cdot k)(p \cdot l)(p' \cdot k)(p' \cdot l)} \bar{u}(p')(ie\gamma^\rho)u(p) \\ &= \frac{1}{2} I^2 \bar{u}(p')(ie\gamma^\rho)u(p) \end{aligned}$$

where

$$I \equiv -ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{(p' \cdot p)}{(p \cdot k)(p' \cdot k)}$$