UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221C **Quantum Field Theory** **Spring 2007**

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ASSIGNMENT #1
Due: Friday, April 13, 5pm in TA's mailbox

1. Sred. 62.1.

2. Sred. 62.2. In this problem and in 66.3, you may use the results from the book for Feynman gauge ($\xi = 1$) without rederiving them, so you need only include the ξ-dependent term in the photon propagator when you do the graphs. *Also,* obtain the ξ-dependent part of $F_2(q^2)$.

3. Sred. 66.1 (The graphs are already done, you just have to extract the γ's).

4. Sred. 66.3.

5. The *Callan-Symanzik equation* is important. Verify the following, using the results for the various quantities as given in the text (for simplicity, just Feynman gauge):

$$\Big(\mu\partial_\mu + \beta(\alpha)\partial_\alpha + \gamma_m m\partial_m + 2\gamma_A\Big)\tilde{\mathbf{\Delta}}_{\mu\nu}(k) = 0$$
$$\Big(\mu\partial_\mu + \beta(\alpha)\partial_\alpha + \gamma_m m\partial_m - 2\gamma_A\Big)(k^2 P^{\mu\nu} - \Pi^{\mu\nu})(k) = 0$$
$$\Big(\mu\partial_\mu + \beta(\alpha)\partial_\alpha + \gamma_m m\partial_m - 2\gamma_\psi\Big)\tilde{\mathbf{S}}(\not{p})^{-1} = 0$$
$$\Big(\mu\partial_\mu + \beta(\alpha)\partial_\alpha + \gamma_m m\partial_m - 2\gamma_\psi - \gamma_A\Big)\tilde{\mathbf{V}}^\mu(p', p) = 0$$

(Use the RG functions and renormalized amplitudes all in the $\overline{\rm MS}$ scheme). You are looking for the first nontrivial terms, which are of order α in the first three equations and $\alpha^{3/2}$ in the last. Notice the pattern: the anomalous dimension terms count the number of fields in the amplitude, with a minus sign for 1PI amplitudes and a plus sign for the full amplitudes with external propagators. In each equation, the μ-dependence of the one-loop amplitude should cancel the tree amplitude times the one-loop RG functions.

This equation determines the μ-dependence of any amplitude: it can be absorbed in rescalings of the fields and shifts of the masses and couplings. The way this is used is as follows. If we have an amplitude where all momenta are of some order M, then by setting $\mu \sim M$ we optimize perturbation theory, automatically summing the largest parts of the higher order corrections. (If we have an amplitude with more than one characteristic scale, we first study the highest energy subamplitudes using the RG, then express them as effective interactions and go to the next lowest scale, etc.).