UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221C **Quantum Field Theory** **Spring 2007**

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ASSIGNMENT #4
Due: Friday, May 4, 5pm in TA's mailbox

1. a) Assuming that $\mathrm{Tr}(T^a_R T^b_R) = T(R)\delta^{ab}$, show that the structure constant f^{abc} appearing in the Lie algebra of the T^a_R is totally antisymmetric.

b) Show that $[T^a_R T^a_R, T^b_R] = 0$ (repeated index summed).

Note: It follows (from Schur's lemma from linear algebra) that for an *irreducible* representation, $T^a_R T^a_R$ is a constant $C(R)$ times the identity. For a reducible representation, $R = R_1 \oplus \ldots \oplus R_k$ (where the R_i are irreducible), we would have

$$T^a_R T^a_R = \sum_i C(R_i) P_i$$

where P_i projects onto the subspace tranforming as the irrep R_i.

c) For the fundamental representation of $SU(N)$, give a basis T^a with the conventional normalization $T(\text{fund.}) = \frac{1}{2}$. Use this to calculate $T^a T^a$ explicitly and obtain $C(\text{fund.})$. Verify Sred. 70.10.

2. a) Show by writing out all terms that $[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$ for any three matrices (the Jacobi identity).

b) From the Jacobi identity for T^a, T^b, and T^c obtain an identity involving the f^{abc}.

c) A general field transforms $\delta\phi_i(x) = i\alpha^a(x) T^a_{ij}\phi_j(x)$. For the field strength we found (in matrix form) that $\delta F_{\mu\nu} = i[\alpha(x), F_{\mu\nu}]$. Write out the transformation for $F^a_{\mu\nu}$ and deduce the corresponding (adjoint) representation matrices. Note that in this case the indices a, b play the role of the i, j. Verify that these matrices have the correct commutator algebra.

3. Srednicki 70.2.

4. Srednicki 70.5.

5. Srednicki 70.6.