UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221C **Quantum Field Theory** **Spring 2007**

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ASSIGNMENT #6
Due: Friday, May 18, 5pm in TA's mailbox

1. Peskin & Schroeder 19.1 (scanned version on course home page).

2. The fact that the anomaly equation is exact allows us to solve several non-trivial field theories in 1+1 dimensions.

a) Consider the path integral over a massless fermion in 1+1 dimensions interacting with a fixed gauge field $A_\mu(x)$. Consider the current $j^\mu(k)$ in momentum space. The conservation law for j^μ and the anomalous conservation law for j_5^μ give two equations for the two components of the current (since these two currents are related). Solve for $j^\mu(k)$ in terms of the gauge field; your result it exact, because the anomaly is exact. (When $k^2 = 0$ the equation is ill-defined, you can fix it by taking the usual Feynman ϵ-prescription in your answer).

b) Now consider the massless Schwinger model (1+1 electrodynamics with a massless fermion). Write the field equation for A_μ in Feynman gauge, and use your result from part (b) for the current. What can you conclude about the spectrum?

c) Now keep the same fermionic action (set $e = 1$) but replace the gauge field kinetic term with $A^\mu A_\mu/2g$ (this theory does not have a gauge invariance). Do the gaussian path integral over A^μ and show that this can be written as a purely fermionic theory with a local action (the Thirring model). The coupling g can have either sign - positive is attractive for fermion-antifermion. Normally when you integrate out a field the result is nonlocal, but here because the action for A_μ contains no derivatives the result is local; A_μ is known as an auxiliary field.

d) Solve the Thirring model by doing things the other way around, integrating out the fermions first and proceeding as in part (a). What can you say about the spectrum?

e) Add a source term $i\delta^2(x)$ to your field equation for A_μ: this gives the A_μ propagator. Find this in momentum space. You should get pathological behavior when g is too large and positive. This is connected with chiral symmetry breaking, though the physics is a little complicated in the Thirring model.

With a nontrivial bit of cleverness one can also solve the field equations for the fermions, and find all correlation functions, in both models.

3. Srednicki 83.3