Turbulence structure in thick layers of fluid.

D. Byrne, H. Xia, and M. Shats[a)]

Research School of Physics and Engineering,

The Australian National University, Canberra ACT 0200,

Australia

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Measurements of spatial structure of turbulence in thick fluid layers driven electromagnetically at small scales are reported. It is shown that at modest Reynolds numbers the top sublayer behaves as quasi-2D and supports the inverse energy cascade regardless of the layer depth. The cascade survives even in the presence of strong 3D eddies developing in a layer whose depth exceeds half the forcing scale. In a bounded flow at low bottom dissipation, the initial development of the inverse cascade in the top sublayer, leads to the generation of a spectral condensate which destroys 3D eddies in the bulk flow and enforces its planarity.

[a)]Electronic mail: Michael.Shats@anu.edu.au

I. INTRODUCTION

There has been remarkable progress in the understanding of turbulence in fluid layers. Such layers, characterized by large aspect ratios, are ubiquitous in nature. 2D turbulence theory by Kraichnan[1], in particular the inverse energy cascade, has been confirmed in experiments in thin fluid layers[2–7]. More recent studies showed that theoretical results derived for idealized 2D turbulence are valid in a variety of conditions, even when the theory assumptions are violated[8,9].

In bounded turbulence, the inverse energy cascade may lead to the accumulation of spectral energy at the box size scale and the generation of a spectral condensate, a large vortex coherent over the flow domain[9]. Good agreement with the Kolmogorov-Kraichnan theory was found in the double layers of fluids in spectrally condensed turbulence[9,10].

It should be noted that though spectral condensation was observed in the double-layer configurations[3,7–9,11], in single layers of electrolytes not only spectral condensation, but also the very existence of the inverse energy cascade has been questioned. For example, in Ref.[12] the flow generated in a single layer was referred to as "spatio-temporal chaos" to stress the absence of the turbulent energy cascades. It is thus important to better understand spatial structure of turbulence in such layers as well as differences between turbulence in double and single layers. (Here we do not discuss here MHD flows which also show spectral condensation[2], but where two-dimensionality is imposed by homogeneous magnetic field and bottom dissipation is restricted to a thin Hartmann layer.)

A comparison of turbulent flows in a single and double-layer configuration is also important to improve our understanding of turbulence in atmospheric boundary layers. These layers are very different over terrain and the oceans, with the former being substantially thicker than the latter ones. Stable immiscible layers of fluids have been generated in the laboratory by placing a heavier non-conducting fluid at the bottom of the cell and a lighter layer of electrolyte resting on top of it[6,8,9]. In this case the electromagnetic forcing is detached from the solid bottom and it is maximal just above the interface between the two fluids. The structure of the flow in the top layer close to the interface with the bottom layer may be similar to that of the atmospheric boundary layer over the ocean.

In this paper we present new results on the spatial structure of turbulence in a single- and a double-layer turbulent flows. In contrast to the previous studies, we focus here on *thick*

layers. Recent 3D direct numerical simulations of the Navier-Stokes equations[13] have shown that the finite layer depth leads to splitting of the energy flux. In thin layers the energy flux injected by forcing is inverse. As the layer thickness h is increased, a larger fraction of the flux is redirected down scale, towards the wave numbers larger than the forcing wave number k_f. At $h/l_f > 0.5$, most of the injected flux cascades to small scales.

It was recently found that in double layers turbulence remains quasi-two-dimensional during spectral condensation. Large scale coherent flows in layers enforce the flow planarity by shearing off vertical eddies and thus secure upscale energy transfer[10]. In this case, the range of depths in which the flow remains planar is greatly extended.

In single layers, where spectral condensation does not usually occur, the reduction in the inverse energy flux with the increase in the layer thickness has been observed in experiments in thick layers[14]. It was found that the energy flux in the inertial interval was reduced by almost an order of magnitude as the layer thickness was increased from $h/l_f \approx 0.37$ to $h/l_f \approx 1.25$. This result however could not be explained by the splitting of the turbulent cascade and the redirection of the energy flux to small scales, as in[13], since no signs of the direct cascade (e.g. no $k^{-5/3}$ spectrum) was observed at $k > k_f$. The absence of the direct cascade was probably due to low Reynolds numbers typical for such experiments ($Re < 200$). In this paper we show that at low Reynolds numbers turbulence supports robust inverse energy cascade even in thick layers.

II. EXPERIMENTAL SETUP AND FLOW CHARACTERIZATION

In these experiments turbulence is generated through the interaction of a large number of electromagnetically driven vortices[7–9]. An electric current flowing through the conducting fluid layer interacts with the spatially varying vertical magnetic field produced by a 24×24 or 30×30 array of magnetic dipoles (10 mm and 8 mm separation respectively), Figure 1. The magnet arrays are placed under the bottom of the 0.3×0.3 m^2 fluid cell. To ensure that turbulence is forced monochromatically at $k = k_f$, vertical magnetic field produced by the array has been measured using a Hall probe scanned in horizontal planes at several heights above the array. The measured magnetic field (Fig. 1(b)) has then been Fourier transformed in 2D, Fig. 1(c). The spectrum shows that $J \times B$ forcing is localized in k-space in a narrow spectral range (in this example, for a 10 mm magnet separation, the spectrum

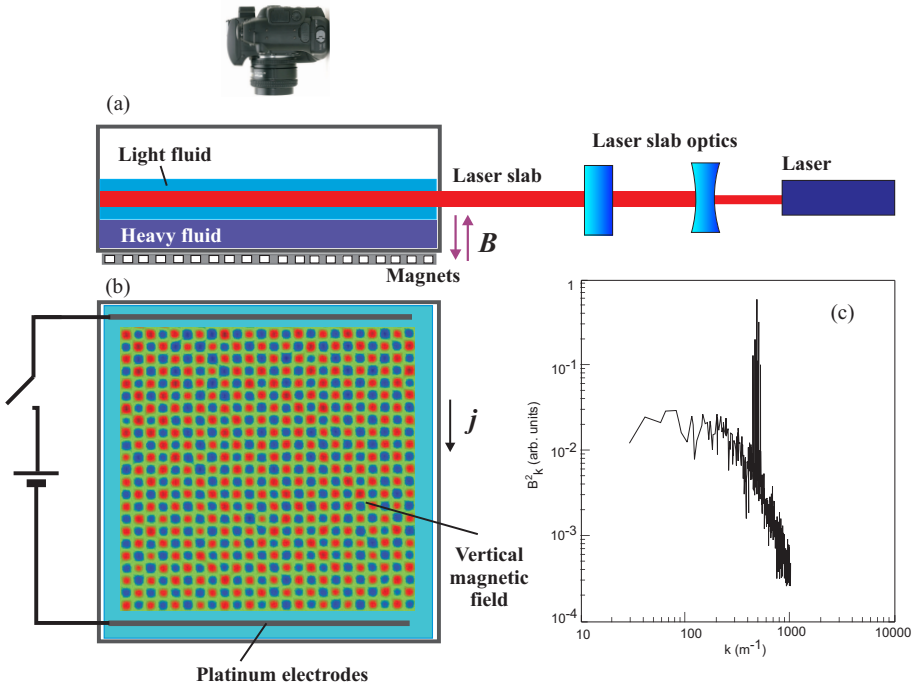

FIG. 1. Schematic of experimental setup. (a) Neutrally buoyant seeding particles in the top (conducting) layer are illuminated using a laser slab and their motion is filmed from above. (b) Measured vertical magnetic field produced by the magnet array: blue and red dots indicate upward and downward B direction. (c) Wave number spectrum of the measured magnetic field.

peaks at $k \approx 630$ m^{-1}). The forcing strength is controlled by adjusting electric current through the layer in the range 0.5-5 A.

Turbulence is generated either in a single layer of Na_2SO_4 water solution, or it is driven in the top layer of the electrolyte which rests upon a layer of heavier (1820 kg/m^3) nonconducting liquid (FC-3283 by 3M) which is not soluble in water. In the latter case, forcing is the strongest just above the interface between the layers.

The flow is visualized using neutrally buoyant seeding particles 50 μm in diameter illuminated by a horizontal laser slab. Turbulent velocity fields are derived using particle image velocimetry (PIV). The thickness of the laser slab and its height relative to the free surface are adjusted to visualize different regions of the layers. In all reported experiments the free surface is unperturbed (flat). The absence of the surface perturbations is monitored by reflecting a thin laser beam off the free surface onto a distant screen.

To characterize the vertical structure of the flow, defocusing PIV is used[15]. This technique uses a single CCD camera with a multiple pinhole mask to measure three-dimensional veloc-

ity components of individual seeding particles in the flow[14]. Particle pairs are matched from frame to frame throughout the illuminated volume using a PIV/PTV hybrid algorithm. Derived velocities are then averaged over hundreds of frame pairs to generate converged statistics of the root-mean-square velocities $\langle V_{x,y,z} \rangle_{rms}$ throughout the layer. The technique allows one to resolve vertical velocities $\langle V_z \rangle_{rms} \geq 0.4$ mm/s.

We also study vertical motion of seeding particles by illuminating the layer using vertical laser slabs. Streaks of the particles in $z - x$ plane are filmed with the exposure time of 1-2 s through a transparent side wall of the fluid cell.

III. TURBULENT FLOW IN A THICK SINGLE LAYER

We first describe turbulent flows in a single layer. The flow is forced at $k_f \approx 800$ m^{-1} ($l_f \approx 7.8$ mm) in a layer of thickness $h_l = 10$ mm. According to numerical simulations, turbulence in such a layer should show substantial three-dimensionality[13], even when the forcing is 2D. Figure 2 shows vertical profiles of vertical (a) and horizontal (b) velocities along with the snapshot of the particle streaks in the vertical ($z - x$) plane. RMS vertical velocities $< V_z >$ are low in the top sublayer, 2 mm below the free surface, as well as in the bottom boundary sublayer. In the bulk of the flow (2 - 8 mm) $< V_z >$ is high, being only a factor of two lower than horizontal velocities $< V_{x,y} >$.

$< V_{x,y} >$ shows a maximum at $h =$ (3 - 4) mm, which is indicative of the competition between the forcing and the bottom drag. The forcing $f \sim J \times B$ is the strongest near the bottom (magnets underneath the cell, the current density J is constant through the layer) and it decreases inversely proportional to the distance from the bottom, $f \propto 1/h$. The bottom drag is also the strongest near the bottom; in a quasi-2D flow it should scale faster[16], $\alpha \propto 1/h^2$, resulting in the maximum of $< V_{x,y} >$ being at $h = (3 - 4)$ mm. In the top sublayer, $h = (8 - 10)$ mm, turbulence is expected to behave as quasi-2D due to the lower vertical velocities and the absence of vertical gradients of the horizontal velocities. The planarity of the top layer can be seen qualitatively in the particle streaks of Fig. 2(c).

To test if the nature of the turbulent energy transfer changes between the top sublayer and the bulk flow we perform PIV measurements of the horizontal velocities using laser slabs which illuminate different ranges of heights: $h_{top} = (8 - 10)$ mm and $h_{bulk} = (4 - 7)$ mm. The velocity fields are analyzed as described, for example, in Ref.[9]. We compute the wave

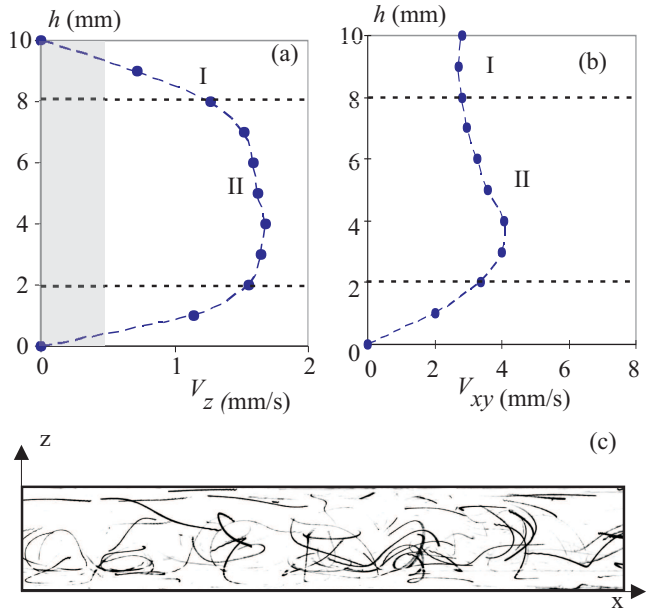

FIG. 2. Vertical profiles of (a) vertical, V_z, and (b) horizontal, $V_{x,y}$, velocities. A grey box in (a) indicates the sensitivity of the defocusing PIV technique. (c) A snapshot of of the particle streaks taken at the exposure time of 2 s.

number energy spectra $E_k(k)$ and the third-order structure function $S_{3L} = \langle(\delta V_L)^3\rangle$, where δV_L is the increment across the distance r of the velocity component parallel to r. The third-order structure function is related to the energy flux in k-space as $\epsilon = -(2/3)S_{3L}/r$. Positive S_{3L} corresponds to negative ϵ and the inverse energy cascade.

Figures (3)(a,b) show the kinetic energy spectrum and the third-order structure function as a function of the separation distance measured in the top sublayer. At $k < k_f$ the spectrum scales close to $k^{-5/3}$, while S_{3L} is positive at $l > l_f$ and is a linear function of r. This is in agreement with the expectation of the quasi-2D turbulence in the top sublayer.

In the bulk flow, which is dominated by 3D motions seen in Fig.3(c), the spectrum is still close to $k^{-5/3}$, though it flattens at low wave numbers. Consistently with this, the range of scales for which S_{3L} is positive and linear is reduced to about $r \approx 4$ cm. Such a behavior of $S_{3L}(r)$ is also typical for thin single layers. The reduction in the inverse energy cascade range is correlated with the increased damping. 3D motions present in the bulk flow (Fig.3(c)) increase damping to the bottom due to the increased flux to the bottom of the horizontal momentum, as has been discussed in[14].

The above results suggest that despite the presence of substantial 3D motion in a thick ($h_l/l_f = 1.28$) single layer, statistics of the horizontal velocity fluctuations remain consistent

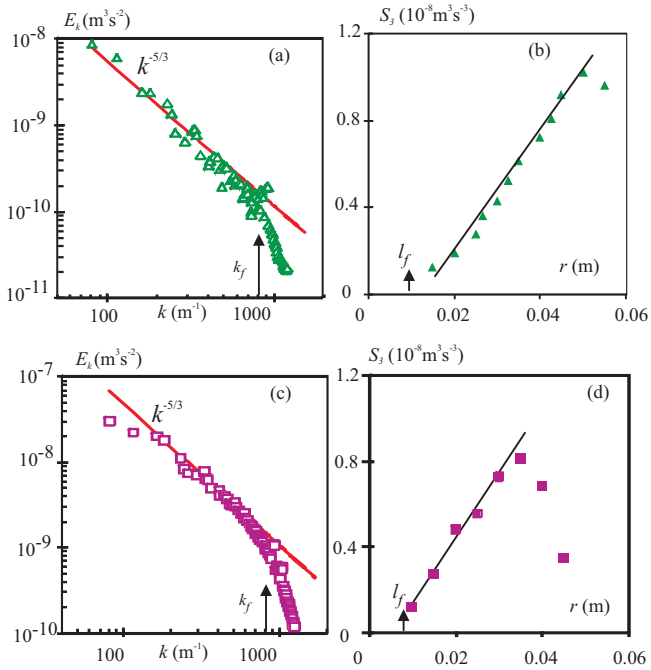

FIG. 3. (a,c) Wave-number spectra and (b,d) the third order structure functions S_{3L} for (a,b) the surface sublayer ($h = (8 - 10)$ mm), and for (a,b) the surface sublayer ($h = (4 - 7)$ mm) sublayer regions.

with that of quasi-2D turbulence and supports the inverse energy cascade. As seen from the energy spectra of Fig.3(a,c), there is no evidence of the direct energy cascade at $k > k_f$. The spectrum shows a decrease in E_k much steeper than the 3D Kolmogorov scaling of $k^{-5/3}$.

IV. TURBULENCE STRUCTURE IN A DOUBLE LAYER FLOW

In the light of the results presented in the previous section, in particular the sustainment of the inverse energy cascade in the presence of 3D motions, one could expect that in bounded turbulence such a cascade could lead to spectral condensation and to the generation of a large-scale coherent structure. Such a structure could then impose two-dimensionality on the flow in the layer, as has been found in recent experiments[10]. In this section we study spatial structure of the flow during spectral condensation in thick layers subject to a low bottom drag.

The condensate forms when the bottom energy dissipation is low. The bottom drag can be reduced by generating two immiscible layers in which the bottom layer (heavier, non-conducting liquid) isolates the conduction layer from the bottom. We keep the bottom

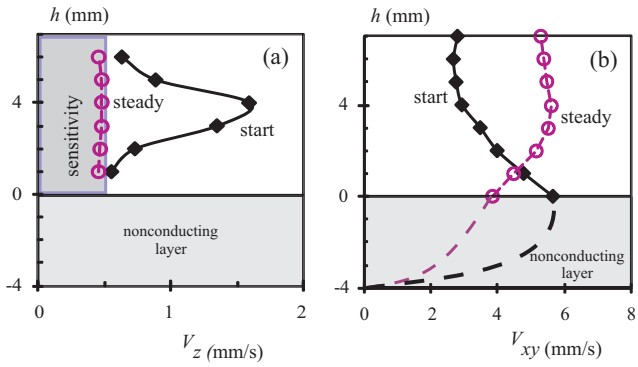

FIG. 4. Vertical profiles of (a) vertical, and (b) horizontal RMS velocities measured at $t = 5$ s after forcing start, solid diamonds, and at $t = 20$ s, open circles.

layer relatively thin, $h_b/l_f < 0.5$ to avoid 3D motions in the layer. The top layer, on the other hand, is thick, $h_t/l_f > 0.5$, to allow three-dimensionality to develop, even with 2D forcing. Here we study the layer configuration described in[10], namely $h_t = 7$ mm and $h_t = 4$ mm, which correspond to $h_t/l_f = 0.44$ and $h_t/l_f = 0.78$ respectively. It has been reported that flow in the top layer shows substantial 3D motions shortly after turbulence is forced. However in the steady state, the development of spectral condensate leads to a substantial reduction of 3D eddies and planarization of the flow.

Figure 4 shows vertical profiles of (a) vertical, and (b) horizontal RMS velocities measured using defocusing PIV in the top layer. The grey box in Fig.4(a) indicates the method sensitivity. Shortly after the flow is forced (t =5 s), vertical velocities peak in the middle of the layer at $\langle V_z \rangle \approx 1.7$ mm/s, solid diamonds in Fig.4(a). 20 seconds later vertical velocity fluctuations are substantially reduced, to below the resolution level, $\langle V_z \rangle \leq 0.5$ mm/s. Horizontal velocities during the initial stage of the flow development peak at the interface between the two layers, i.e. in the region of strongest forcing. In the steady state, after the large coherent vortex develops the horizontal velocity profile becomes flat over most of the top layer, showing $\langle V_{x,y} \rangle \approx 6$ mm/s at $h = 2 - 7$ mm above the interface, open circles in Fig.4(b). A reduction in $\langle V_{x,y} \rangle$ near the interface in the steady state is probably related to the effect of sweeping of the forcing scale vortices by the developing condensate[11]. Thus, a substantial fraction of the top layer is quasi-2D, i.e. $\langle V_z \rangle \approx 0$ and $\partial \langle V_{x,y} \rangle / \partial z \approx 0$.

Such a quasi-two-dimensionality of the flow is attributed to the shearing of vertical eddies by a strong condensate[10]. Indeed, in the steady state, the statistics of horizontal velocities is in a good agreement with the Kraichnan theory. The spectra and the third-order structure

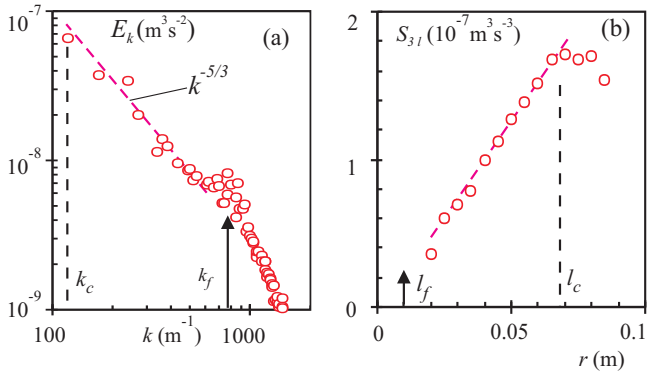

FIG. 5. Statistics of horizontal velocities in a double layer configuration: top layer thickness $h_t = 7$ mm, bottom layer thickness $h_t = 4$ mm. The forcing scale is $l_f = 7.8$ mm. (a) Wave number spectrum of the kinetic velocity E_k, and (b) the third-order structure function S_{3L}, computed after subtracting time-averaged mean velocity field. The laser slab illuminates the entire top layer.

functions in the presence of spectral condensate are computed after subtracting the time-average velocity field from the instantaneous velocity field, as discussed in[8–10]. After the mean subtraction, the spectrum shows $E_k \sim k^{-5/3}$, as seen in Fig.fig5(a). The third-order structure function is positive and is a linear function of the separation distance r up to $r \approx 7$ cm, Fig.5(b). Thus the spectra and the structure functions in such a flow agree with quasi-2D expectations and are consistent with the vertical structure of the flow of Fig. 4.

V. SUMMARY

We have studied spatial structure of turbulent flows in thick layers at low Reynolds numbers ($Re \sim 100$). If the free surface of a layer is unperturbed, there is a finite thickness layer close to the surface, which remains quasi-2D regardless of the the total layer thickness. Two dimensional turbulent velocities in the top layer show kinetic energy spectra and the third-order structure functions consistent with the Kraichnan theory of 2D turbulence. The vertical eddies which appear in the vertical plane when the layer is sufficiently thick, $h/l_f > 0.5$, introduce additional bottom drag due to the eddy viscosity[14], but they do not qualitatively change the statistics of the horizontal velocity fluctuations, which remains quasi-2D even in the presence of 3D motions. If the bottom drag is reduced by introducing an immiscible thin bottom layer, the inverse energy cascade leads to spectral condensation and to the formation of the large scale coherent structures. Such flows, as has recently been

shown[10], shear off eddies in the vertical plane and reinforce quasi-two-dimensionality of the flow. Measurements presented here, in particular Fig.5 confirm this.

One may thus speculate that flows in thick layers of fluids with an unperturbed free surface always support the inverse energy cascade at Reynolds numbers below the transition to 3D turbulence. Since the spectral energy of the forcing is not redirected in this case into the direct cascade, such flows, from the turbulent spectral transfer point of view, remain quasi-2D, at least in the top part of the layer. In a bounded domain at low damping the inverse cascade leads to spectral condensation of turbulence which reinforces the flow planarity.

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