UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221C **Quantum Field Theory** **Spring 2007**

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ASSIGNMENT #2
Due: Friday, April 20, 5pm in TA's mailbox

1. Peskin & Schroeder 6.3 (posted at http://www.kitp.ucsb.edu/ joep/W221C/PS6.3). In evaluating the integral you can use the fact that the Higgs mass is much greater than the electron mass. Use the current limits, which are stronger than in Peskin, 10^{-11} for a_e, and 3×10^{-9} for a_μ.

2. Consider a real scalar field with the symmetry $\phi(x) \to \phi(x) + \epsilon$, for constant ϵ.

a) Show that there are no renormalizable interactions.

b) Show also that there are no nontrivial Lorentz invariant terms of dimension 5, 6, or 7 that can be added to the action. Note that a term is trivial if it is a total derivative or can be removed by a field redefinition as in Srednicki 10.5.

c) Show that there is a dimension 8 interaction which is nontrivial — either show that it cannot be removed, or calculate explicitly an S-matrix element. So this theory is like gravity, in that the symmetries require even the lowest-dimension interaction to be nonrenormalizable (but it will still appear in the context of effective field theory). Here it is suppressed by four powers of the high energy scale, rather than two as in gravity.

3. Consider two real scalars with Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{\lambda}{8}(\phi_1^2 + \phi_2^2 - v^2)^2$$

with v^2 a positive constant. This theory has a $U(1) = SO(2)$ symmetry of rotations in (ϕ_1, ϕ_2) field space, which is spontaneously broken: any point with $\phi_1^2 + \phi_2^2 = v^2$ gives a vacuum, and none of these are invariant under the symmetry. If we write $\phi_1 + i\phi_2 = (\rho + v)e^{i\theta}$ we see that ρ has a potential and is massive, but that θ is massless, and moreover has the symmetry discussed in problem 3 of the last set. If follows from the earlier assignment that at low momenta the S-matrix for the massless quanta should be suppressed by four powers of the heavy mass scale.

Show this explicitly by expanding around one particular vacuum, $(\phi_1, \phi_2) = (v, 0)$, so shift to $\phi_1' = \phi_1 - v$ to get the Feynman rules. Calculate the tree level S-matrix for scattering of four massless ϕ_2 particles and verify explicitly the expected suppression.

In the next problem set I will ask you to do the following (I am telling you now because you might want to combine it with PS 6.3).

1. a) In PS 6.3, how should h transform so as to make the interaction in PS 6.3(a) CP invariant? How should a transform so as to make the interaction in PS 6.3(c) CP invariant?

b) Suppose there is a real scalar ϕ with *both* kinds of interaction. What can you say about CP?

c) Calculate the contribution to the electric dipole moment of the electron and the muon from a virtual ϕ.