# Lecture 18

## Spontaneous symmetry breaking

- Discrete
  - Consequences of broken and unbroken symmetry.
  - Kinks
- Continuous
  - Mermin-Wagner-Coleman theorem

~~Complex~~ Real scalar $\phi$, with $\phi \to -\phi$ sym.

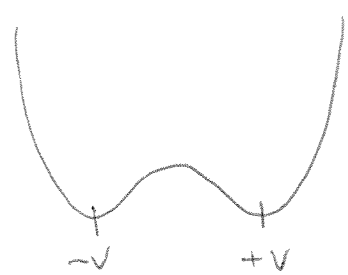
$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$

Nothing requires $m^2 > 0$!

Potential energy:

$m^2 > 0$


$m^2 < 0$




$\leftarrow \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^3$

$$\frac{\partial V}{\partial \phi} = m^2\phi + \frac{1}{3!}\lambda\phi^3, \quad \text{~~minima~~}$$

minima at $\phi^2 = -\frac{6m^2}{\lambda} \equiv v^2$

if $m^2 < 0$

$\phi = \pm v.$

$$V(\phi) = \frac{1}{4!}\lambda(\phi^2 - v^2)^2 - \underline{\frac{1}{4!}\lambda v^4}$$

$\llcorner$ doesn't affect S-matrix.

Two classical minima → two vacua.

$$\langle 0+|\phi(x)|0+\rangle = +v$$

$$\langle 0-|\phi(x)|0-\rangle = -v$$

$$Z^{-1}\phi(x)Z = -\phi(x) \qquad Z|0+\rangle = |0-\rangle.$$

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But in QM there would be tunnelling: true ground state would be $\sim |0+\rangle + |0-\rangle$, and $|0+\rangle - |0-\rangle$ would be slightly higher.

Tunneling $\sim e^{-\underline{\text{barrier penetration fact.}}} \propto$ volume $\to 0$ for QFT in infinite volume.

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To analyze spectrum, write Feynman rules, define (for $|\phi+\rangle$) $\quad \rho(x) = \phi(x) - (v) \qquad \phi = \rho + v$

$$V(\phi) = \frac{1}{4!}\lambda(\rho^2+2v\rho)^2 = \frac{1}{4!}\lambda\rho^4 + \frac{1}{6}\lambda\rho^3 v + \frac{1}{6}\lambda v^2\rho^2$$

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\rho\,\partial^\mu\rho - V$$

$$m_\rho^2 = \frac{1}{3}\lambda v^2 = -2m^2 \quad (m^2<0).$$

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~~Unbr.~~ Consequences of symmetry:

unbroken: $\quad Z|0\rangle = |0\rangle$

$$\langle 0|\phi(x_1)\dots\phi(x_n)|0\rangle = \langle 0|Z^{-1}\phi(x_1)\dots\phi(x_n)Z|0\rangle(-1)^n$$

$= \langle 0|\phi(x_1)\ldots\phi(x_n)|0\rangle \Rightarrow = 0$ for $n$ odd.

$2 \to 2$ but never $2 \to 3$. cont→

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For broken sym this fails for $n=1$:

$$\langle 0+|\phi(x)|0+\rangle = v \neq 0, \quad \text{because } Z|0+\rangle = |0-\rangle.$$

Any consequences? $\quad Y = \lambda v \qquad X = \lambda$

$$m_\rho^2 = \frac{1}{3}\lambda v^2$$

So $\quad X \cdot m_\rho^2 = \frac{1}{3}(Y)^2 \quad \leftarrow$ pretty subtle.

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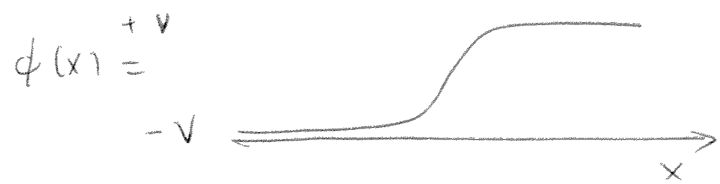
Kinks: consider in ~~dim~~ configuration




(independent of $y, z$).

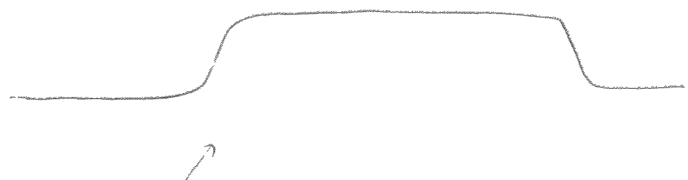
Stable (lowest energy configuration with given boundary conditions).

Finite energy/area $\equiv$ tension

· Might be produced cosmologically (and in some cases lead to unacceptable effects - too much energy density).

Two interesting effects of kinks in 1+1 dim

#1) $T \neq 0$

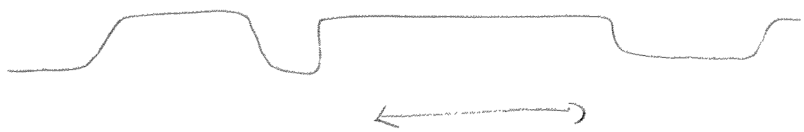


$|0\rangle \to$

Boltzmann weight $= e^{-2M_{\text{kink}}/T} \cdot \frac{1}{2}\underset{\text{volume}}{\underline{V}}^2$

$\gg 1$ in $\infty$ volume limit

Typical config



$O(e^{2M_{\text{kin}}/T})$

$\to$ symmetry is <u>restored</u>, $\langle \phi(x)\phi(0)\rangle \to 0$ as $x \to \infty$.

Discrete sym are always unbroken at $T>0$ in 1+1.

" " " " " " $T=0$ in 0+1.

#2) ($T=0$ again). This theory has two kinds of "particles": the $\rho$-particles and the kinks $\leftarrow$ much heavier

(higher-dimensional examples: vortex, monopole).

At weak coupling, $\rho$ is much lighter than the kink (by $O(\frac{1}{\lambda})$)

However, there are cases ~~where~~ (e.g. $V(\phi) =$  )
where at strong coupling ~~$t$~~ kink $\rightarrow$ fundamental field
$\phi \rightarrow$ bound state

Sine-Gordon $\leftrightarrow$ Thirring

(simple example of duality).

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Note: $S = -\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$

$\phi \rightarrow \phi/\sqrt{\lambda}$ $\quad S = \frac{1}{\lambda}\left(-\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\phi^4\right)$

$e^{iS/\hbar}$ depends on $\hbar$ and $\lambda$ only in the combination $\hbar\lambda$. So loop expansion = $\hbar$ expansion, with this scaling of fields.

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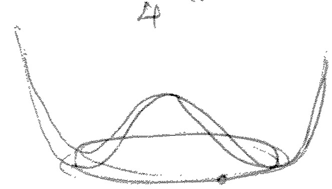
Now the same, but with a continuous symmetry:

$$\mathcal{L} = -\partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2$$

$m^2 < 0$ $\qquad V(\phi) \sim$



$\phi = v e^{i\theta}/\sqrt{2}$ $\quad v^2 = \frac{-4m^2}{\lambda}$

$\hookrightarrow$ $|\theta\rangle$

For $\phi = \frac{1}{\sqrt{2}}(v + a(x) + i b(x))$

insert: find $m_a^2 = -\frac{m^2}{2}$ $\quad m_b^2 = 0 \leftarrow$ Goldstone boson.

potential is always flat in symmetry direction
$\rightarrow$ massless degree of freedom.

(Can also use $\phi(x) = \frac{1}{\sqrt{2}}(v + \rho(x))\, e^{-i\chi(x)/v}$:

potential is manifestly independent of $v$).

~~[Quantum corrected mass: sum over all 2~~

Interactions of $\chi \propto \partial_\mu \chi$ so vanish at $k^\mu = 0$:

no correction to mass.

CMW: $\quad \langle \chi(x)\chi(0)\rangle = \int \frac{d^d k}{(2\pi)^d} \frac{e^{ik\cdot x}}{k^2}$.

In $d = 2$ ($1+1$) this diverges (at $k=0$): quantum fluctuations always

smear the state out over the whole ring 

($T=0$)

At $T>0$ this happens in $2+1$ (or less)

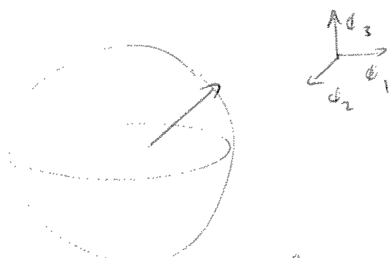
Minimum dimension in which symmetry breaking is possible:

| | $T=0$ | $T>0$ |
|---|---|---|
| discrete | 1+1 | 2+1 |
| continuous | 2+1 | 3+1 |

Non-Abelian example: $\phi_i$ $i=1,2,3$ SO(3). 3 generators

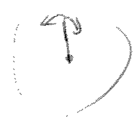
$$V = \frac{1}{2} m^2 \phi_i\phi_i + \frac{1}{8}\lambda(\phi_i\phi_i)^2$$

$$\phi_i\phi_i = -\frac{2m^2}{\lambda} \equiv \frac{v^2}{2} \quad (\lambda < 0)$$




e.g. $\phi_i = \begin{pmatrix} 0 \\ 0 \\ v/\sqrt{2} \end{pmatrix}$

expand $\phi_3(x) = v/\sqrt{2} + \rho(x)$



Find $\phi_1$, $\phi_2$ massless $\rho$ massive

$$\overset{3\text{ gen}}{SO(3)} \longrightarrow \underbrace{\overset{1\text{ gen}}{SO(2)}}_{\text{unbroken, acts on } \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}}$$

$T^a_{ij}\phi_j = 0$ : unbroken generators $\neq 0$ broken generators

$$\begin{pmatrix} \phi_1 \\ \phi_N \end{pmatrix} \leftarrow N-1 \text{ Goldstone bosons}$$

$$\begin{array}{ccc} SO(N) & \to & SO(N-1) \\ \uparrow & & \uparrow \\ \frac{1}{2}N(N-1) & & \frac{1}{2}(N-1)(N-2) \end{array} \leftarrow \text{difference} = N-1$$

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$U(2)$ sym, $V = m^2 \phi_i^\dagger \phi_i + \frac{1}{4}\lambda(\phi_i^\dagger \phi_i)^2$
$= SU(2) \times U(1)$

~~what is the unbroken symmetry~~ for $m^2 < 0$ ?

Use $SU(2)$ to rotate to "spin up" $\begin{pmatrix} \phi_1 \\ 0 \end{pmatrix}$

and then to make it real and posit. $\begin{pmatrix} v \\ 0 \end{pmatrix}$

Unbroken: $U(1)$ of phase rotation on lower component

$U(2) \to U(1)$.