UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221A **Quantum Field Theory** **Fall 2007**

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Course web page: http://www.kitp.ucsb.edu/∼joep/Web221A/221A.html

ASSIGNMENT #6

Due: Friday, Nov. 9, 5pm in TA's mailbox (5th floor Broida). See course web page for late homework policy.

1. Srednicki 14.1.

2. Srednicki 14.3

3. Srednicki 14.4 (As an aside, verify that the final result for Π does not depend on μ.)

4. Srednicki 14.5 (You will see that the result is much more trivial than for ϕ^3 theory - this is one of the reasons that we have taken ϕ^3 as our main example.)

5. 5. Calculate the one-loop graph of ch. 14 in $d = 6$ with a Pauli-Villars regulator. That is, multiply each propagator by a convergence factor,

$$\frac{-i}{k^2 + m^2 - i\epsilon} \to \frac{-i}{k^2 + m^2 - i\epsilon} \times \frac{\Lambda^2}{k^2 + \Lambda^2 - i\epsilon}$$

for some large Λ. Combine the two $(k^2 + m^2 - i\epsilon)$ factors in exactly the same way as before, and define the same shifted q. Write the regulator factors in terms of q, and then drop factors of m^2 and k in the regulator (because they are much less than Λ), but keep factors of q^2 since this can become large. Now do the q^2 integral. Show that as $\Lambda \to \infty$ there is a quadratic divergence $O(\Lambda^2)$ and a logarithmic divergence $O(\log \Lambda)$, and that both can be canceled against the counterterms Z_ϕ and Z_m to give a finite limit as $\Lambda \to \infty$. Show that the counterterms can be chosen so as to bring the graph to the same finite form (14.39) as in dimensional regularization.

To be honest, this approximation for the regulator factors is not really good enough, because of the quadratic divergence: we should expand them to order k^2 and m^2, not just zeroth order, but the difference is just some additional finite terms in Z_ϕ and Z_m.