

223c Final Exam (due 6/12 by 10am)

Problem 1 (40) Consider a population of $n(t)$ individuals undergoing a stochastic birth/death process. Each individual might duplicate itself with the probability γ per unit time or might die with the same probability per unit time.

a) Write down the Master Equation describing the probability $p_n(t)$ for the population to contain exactly n individuals at time t .

b) Show that the dynamics of the generating function: $Q(\lambda, t) = \sum_{n=0}^{\infty} \lambda^n p_n(t)$ is governed by

$$\frac{\partial}{\partial t} Q(\lambda, t) = \gamma(\lambda - 1)^2 \frac{\partial}{\partial \lambda} Q(\lambda, t) \quad (1)$$

c) What is the behavior of the average population size: $N(t) = \sum_{n=0}^{\infty} n p_n(t)$?

d) Solve the PDE for $Q(\lambda, t)$ with the initial condition corresponding to $p_n(0) = \delta_{n,1}$. (Hint: use method of characteristics.)

e) What is the probability of extinction (before time t): $p_0(t)$?

Problem 2 (20) Consider a surface immersed in an electrolyte containing $NaCl$ and HCl . The surface contains SH molecules, at an areal density ρ , which can release a proton H^+ into the solution ($SH \rightleftharpoons S^- + H^+$ with the dissociation constant K_d).

What is the surface charge density (as a function of salt concentration and the pH) in the limit of low and high pH? (Hint: use Grahame equation).

Problem 3 (40) Consider a *real* polymer chain consisting of N monomers of length a adsorbed from good solvent onto a solid substrate. A monomer in contact with the substrate attracts to it with energy u .

a) What is the thickness of the adsorbed chain? (Hint: start with a scaling theory estimate of the free energy cost of "confinement" near the surface.)

b) What is the free energy of the adsorbed chain?

c) Suppose that one of the ends of the adsorbed chain is attached to an atomic force microscope tip and is pulled away from the surface. What is the minimal force necessary to pull away the chain?