Problem Set #5 due Thu. March 9

#1) Starting with the Mayer cluster expansion for the grand canonical partition function compute 3 first terms in the expansion of fugacity $\zeta(\rho)$ as a function of density ρ leaving the answer in terms of the cluster coefficients, b_i.

#2) Develop cluster expansion for the 2-particle (or radial) distribution function to the second give the two leading terms as a function of density. A good starting point is the 2-particle distribution in the grand canonical ensemble:

$$n(r_1,r_2) = Z_G^{-1} \sum_{N=2} \zeta^N Q_N^{(2)}(r_1,r_2)$$

where

$$Q_N^{(2)}(r_1,r_2) \equiv \frac{1}{(N-2)!} \int d^3r_3 ... \int d^3r_N e^{-\beta \sum_{i>j} \phi(r_{ij})}$$

(note: $Q_2^{(2)}(r_1,r_2) = e^{-\beta\phi(r_{12})}$)

and

$$Z_G = \sum_{N=0} \zeta^N Q_N$$

with

$$Q_N \equiv \frac{1}{N!} \int d^3r_1 ... \int d^3r_N e^{-\beta \sum \phi(r_{ij})}$$

(as always, $\zeta = \mathrm{e}^{\beta\mu} / \Lambda^3$)

Note: when developing expansion in ζ assume that it is smaller than anything, e.g. $\zeta V << 1$