Errata and Addenda for Group Theory In A Nutshell For Physicists

"Of making books there is no end, and much study is a weariness of the flesh." -- Ecclesiastes 12:12
Last update: Dec 29, 2020

Errors, typographical and otherwise, in Group Theory in a Nutshell for Physicists are listed here. Readers who find errors are urged to bring them to my attention by email (zee@kitp.ucsb.edu) using for subject "group nutshell errata" so that your email does not get treated as spam by the filter on my mailer. I would appreciate it if you would write them in exactly the same format as used here, including the (Thanks to ABC). Notation: Line -n means line n from the bottom of the page. It would be easiest for me if errata are sent to me as plain text in a email rather than in some other format.

Please check to see if the errata you found are not already listed here.

I am away on sabbatica and I have many other commitments, and so I may not get to your erratum for quite some time. If you feel that the erratum you sent in is particularly serious, please email me again and say so. Thanks.

My intention is to produce a functional errata page that would help the reader understand the material, rather than a nice looking errata page that is perfectly formatted and ordered, free from repetitions, etc. On the other hand, if you find a significant error on this errata page, of course I would appreciate your letting me know.

One thing that is useful for me to know is how complete the index is. If there are items in the index that you feel should be there but are not, please let me know. Most people do not know that the index is not prepared by the author but by a professional index compiler who often knows almost nothing about the subject of the book.

I would also like to take this opportunity to thank all of you who sent in errata, particularly those who also posted a favorable "Customer Review" on Amazon.com. Appreciative words from readers make the enormous effort that went into writing a book like this worthwhile.

Thanks to Menghang Wang, Qian Ye, Sharif Schulze Allen, Evan Gurnick for the update on September 18, 2019.

Errata

p10: In (37), cross out Script_D, replace two 1's on the diagonal with Script_D. (Thanks to John Dickinson)

p13: In (54), Script_D = ad-bc (not ac-bd). (Thanks to John Dickinson)

p14: In the line above equation (57), in the subscript, it should be 2Q(2) and not 2Q(1). (Thanks to Achint Kuma)

p27: In the power series expansion of log(x), the sum is should be over "n" and not "k". (see two lines above equation (93). (Thanks to Achint Kumar)

p33: In the formula for the Area of Triangle in Exercise 16, the absolute value of the determinant should be taken. (Thanks to John Dickinson)

p42: line 1: after equation 1 "...tanh \phi =v." should be "...tanh \phi=v/c.". (Thanks to Sourya Ray)

p48: line 1 “column” should be “row” (Thanks to Mark Weitzman)

p49: The words "But the two .... we obtain" preceding the table in the middle of the page should be deleted and replaced by the following. "You should verify that the second choice leads to Z_4 again (and A=B^2), with B^2=A, B^3=BA=C, and B^4=BC=I. With the first choice, we obtain". (Thanks to Sven Gnutzmann, Mark Weitzman)

p60: line 5, additional note: although for an element in A_4, n_1 = 1. We usually ignore the one cycle since adding this one cycle will not change the number of elements within the given structure. (Thanks to Petra Axolotl)

p61: typographical error: All the ”Quarternion” should be written as “Quaternion”.

p64: -line 4, for dependence of h_k, replace g_a with (g_b)^(-1). More concretely, h_k depends on (g_b)^(-1), g_b, h_i, h_j. (Thanks to Menghang Wang)

p66: Line 2 after (11), replace “H_k” by “H_(k-1)/H_k.” (Thanks to John Dickinson)

p67: line 7: more precisely, <a,b> = I + [A,B] - A^2 - B^2 (Thanks to Petra Axolotl)

p75: line 7 in section titled “distance squared between neighboring points”: “first equation from the second” should be “second equation from the first” (Thanks Mark Weitzman) p82: In equation 26 and line -4 and excercise 1 of page 84, The tensor product ⊗ means a cross-product x. (Thanks to Menghang Wang, Qian Ye, Mark Weitzman)

p103: Line 6, add “unitary” before “representation.” Also for p104: Line 1 after (2) (Thanks to John Dickinson)

p109: 4th sentence in 2nd paragraph should be "Hence U^dag U=I. But U^dag U=I implies that UU^dag =I." (Thanks to Chen Panyu)

p111: just after (18): "positive integers" should be replaced by "positive numbers" (Thanks to Menghang Wang, Petra Axolotl)

p111: row orthogonality is proved in Appendix 4, not 3 (Thanks to Sharif Schulze Allen)

p112: the 2nd last line: Replace "," with ";" for \Gamma (c,d, \bar{e}). (Thanks to Menghang Wang, Xiaoyang Shi)

p130: In equation (23), the entry in the third row (corresponding to element (123)) under column 5_bar should be -1 instead of 0. (Thanks to John Dickinson, Mark Weitzman)

p132: Line 3 in Appendix 4, H is an invariant subgroup of G. Replace the subset symbol by the funny-looking triangular symbol. (Thanks to John Dickinson)

p132: line 13: 60=5! -> 60=5!/2; line 18: 4! -> (4!/2) (Thanks to Mark Weitzman)

p143: line -3: f2 g2 = k instead of f2 g2 = k2. (Thanks to Kevin Cortes Gutierrez)

p173:line 2, the characters should be \chi=(6,0,0), not (6,0,2). Consequently, this and the next three paragraphs have to be rewritten. Both (4) and (5) now yield the decomposition n_1=1, n_1 bar=1, and n_2=2. In (11), one of the (1)'s should be changed to (1 bar). From that point on, the discussion is correct, including figure 3. (Thanks to Daniel Poldosky.)

p188: line 1: after (5), D(R_2)D(R_2) should be D(R_2)D(R_1) (thanks to Burak IIhan)

p196: line 7: after (19), there should not be arrows above E_x and E_y. (Thanks to John Dickinson) p200: line -5: in the round parenthesis to the right of T, a minus sign is missing. (Thanks to Nick Murphy)

p200: line -4: T_b should be A_b (Thanks to Nick Murphy)

p200: last line: a minus sign is missing which trivially propagates into some subsequent equations. (Thanks to Nick Murphy)

p200; Second paragraph I think it should be ”which is N-by-N instead of ”which is N(N -1)-by-N(N -1). (Thanks to Samuel Lorin)

p201: line 2 of Exercise 7: replace cos(theta) by sin(theta). (Thanks to John Dickinson)

p210: line 7: delete the factor i_\eps_ijk just after the first "=" sign. (Thanks to Gunther Lippens)

p214: see comment for page 363 below

p214: Line 3 after (31): replace j by 2j. (Thanks to John Dickinson)

P220: third equation of (7), should be for I0,0>, not I1,0>; (thanks to Burak Ilhan)

p220; In equation 7 there are two states ”|1,0 >. One should be ”|0,0 >. (Thanks to Samuel Lorin)

p222; In the first footnote, the second to last expression there should be 1 6 instead of √ 1 6 . (Thanks to Samuel Lorin)

p222: line 12, eliminate the reference ket |-1, 2>. That state is not possible since for j-prime = 1, the maximum value for m-prime is 1. (Thanks to John Dickinson)

p247; In the first paragraph under section ”Representing the Lie Algebra of SU(2) You refer to section IV. 4. I Think it should be iV.2. (Thanks to Samuel Lorin)

p228: line 4: A^T + A=0 (Thanks to Dominique Hirondell, John Dickinson)

p238: line 8: T^a 's (Thanks to Dominique Hirondell)

p238: \varphi \sigma is to be understood as \varphi \cdot \sigma as in (11) on page 248. (Thanks to Dominique Hirondell)

p245: the 6th last line: when we calculate the determinant, we don’t need to use det(U) = 1 to finish the proof. We can use the property that det(U^+ X U) = det(X)det(U^+ U) = det(X). For instance, reflection also leaves a length of a vector invariant though reflection is not in SO(3) but in O(3). Therefore, these example doesn’t just work in SO(3) Group, but also work in O(3) group. (Thanks to Jyotirmoy Bhattacharya)

p247: the last paragraph and p248 the second paragraph: "chapter IV.4" seems like to be read as "chapter IV.2". (Thanks to Nam-Kyu Park.)

p248: eq.10: u×v instead of u⊗v (it is a cross product, not a direct product, between u and v). (Thanks to Qian Ye)

p252: line 4 after (16), delete the /2

p252: line 6 after (18), the matrix element of S = i sigma_2 should have an overall minus sign, namely S = MatrixForm[{{0, 1}, {-1, 0}}]. (Thanks to Jyong-Hao Chen.)

p252: last line: k should go from 0, not from 1. (Qian Ye)

p252: In both Line 1 and Line 2 after (18), II.3 should be II.4. (Thanks to John Dickinson)

p254: should be spelled quaternion (Thanks to Dominique Hirondell)

p256: In spin-precession section, you didn't include the gyromagnetic ratio in the Hamiltonian. (Thanks to Achint Kumar) [It is suppressed. Actually, I was purposely vague about what \mu is.]

p259: line 1 after (11): “But according to (10), T \Psi is ….” should be “But according to (10), T^2 \Psi is ….” . (thanks to Burak Ilhan)

p268: line 7: Insert a division sign between (1+cos ψ)ε) and (2sin ψ) for n_2. n_1 and n_2 shall be switched. n_2 = [(1+cos(ψ)) σ + ε sin(ψ)]/(2 sin(ψ)); n_1 = [(1+cos(ψ)) ε - σ sin(ψ)]/(2 sin(ψ)). I would add the following sentence to make it more clear: "Originally, the denominator is 2(sin(ψ) + δ cos(ψ)). Since δ << 1, we just keep 2 sin(ψ)". (Thanks to Petros Roditis, Menghang Wang, John Dickinson)

p270: line 3: replace -ς^(j-k) with -ς^(-j+k). (Thanks to John Dickinson)

p275: line 1: in section “Running a reality check …,” replace II.3 with II.4. (Thanks to John Dickinson)

p281: line 2: after (13), replace I ⊗ iA with iA ⊗ I and replace σ_3 ⊗ S with S ⊗ σ_3. (Thanks to John Dickinson)

p308: line 7: For p + p \rightarrow p + p, we have g^2. For n + n

\rightarrow n + n, we have (-g)^2 = g^2. (Thanks to Mao Tian Tan)

p313: lines 20 and 21 : replace the last three terms of the tensor \tilda T_kl^îj (the terms of 1/20) by 1/25 * (\delta_k^i \delta_l^j + \delta_k^j \delta_l^i + \delta_l^i \delta_k^j) (thanks to Gunther Lippens and Heinz Knoedlseder)

p315: in the table on top of the page, the (1,1) should be listed as : 1/2 * 2 * 2 * 4 = 8 (thanks to Gunther Lippens)

p321: "Consider the fundamental representation 3 of SU(2)" should be written as "Consider the fundamental representation 3 of SU(3)". (Thanks to Sashwat Tanay)

p328: in (14), I not T. (Thanks to Nam-Kyu Park.)

p331: should be i_3, i_8, not i_3, y

p331: In the sentence "Similarly, ..." I_- and U_+ should be interchanged.

p331: In the list of five facts, the third fact cannot be disproved for a general Lie algebra. It is always true.

p352: should be chapter V.3 (twice)

p358: first paragraph (recall the SO(6) example) is the wrong example. (Thanks to Samuel Lorin) p361: top and bottom, should be chapter IV.8

p363: line 1, better to have apostrophe between e and s, but this looks like arecurring stylistic choice done by the typesetter, see for example Js on page 214, Es on page 367, etc.

P368: the last paragraph: H^l, not H^r (Thanks to Nam-Kyu Park.) Also, note that I remark upon l versus r in a footnote on page 327.

p370: SO(4k+3) has (4k+3)(4k+2)/2 generators

P371: In. (20), the left-hand side should be N_beta, alpha. (Thanks to Tianhong Wang)

P372 4th paragraph from bottom: "[[U_," seems to be single bracket, as "[U_". (Thanks to Nam-Kyu Park.)

p372: line -8, a [is missing

p381: notation, should be l, not r

p384: line 15: replace "the integers q and q' " by "the integers p and p' " (Remark: this is due to notation switch p-> q from chapter VI.4

-> VI.5) (Thanks to Gunther Lippens)

p387: line -8 and line -10, "angle" should be replaced by "dot product"; alternatively, line -7 and line -9, "vanishes" should be replaced by "equals 90^o." (Thanks to Gunther Lippens)

p394: line -3: "n=m=p=1" should be "n=m=p=2" (Thanks to Chia-Min Lin)

p399: The second paragraph under the section, Cartan Matrix (α1,α2) = (α2,α3) = 1. (Thanks to Samuel Lorin)

p407: line after (8), the result of the second commutator should be -2i \gamma _1

p409: First footnote, Spin(5) should be replaced by Spin (6).

P411: Eqn (28) the lower "S+" seems like to be read as "S-". (Thanks to Nam-Kyu Park.)

p411: According to your convention, there should be a sign after the equal sign σ34|+ + >= |+ + > (Thanks to Samuel Lorin)

p411: equation 28 defines S + twice. (Thanks to Samuel Lorin)

p412: Second paragraph under the section “Complex conjugation: the SO(2n)s are not created equal”. In the sentece we ask whether there... it should be ζTCψ, not ζCψ (Thanks to Samuel Lorin)

p443; in the middle of the page it should be. It says “Wμ = J˜ ρσP p ”. Here the indices don’t match. (Thanks to Samuel Lorin)

P444: 2nd sentence in 4th paragraph, J and K are antisymmetric and symmetric, respectively, not symmetric and antisymmetric, respectively (Thanks to Heinz Knoedlseder)

p466: X_R^ and X_R

p470: 2 lines above (15), a term is missing after p \cdot q. There should be a term involving \sigma^{\mu \nu} p_\mu q_\nu

p480: exercise 8, should be "in terms of u and w". (Thanks to Gunther Lippens)

p484: eqn(10), remove the outer parenthesis.

p485: Line -10, the equation of \bar{psi_c} shouuld have a negative sign (\bar{psi_c} = - psi^T \gamma^2 \gamma^0). The equation after it follows. (Thanks to John Stroughair) page 501: The last commutator should read $\{f^\dagger_n, f_m\} =\delta_{n m}$ instead of being equal to 1. (Thanks to Mariano Hermida de La Rica)

p516: Sentence after (3), should read =\lam^{-1} x^\mu, ... =\lamˆ2 g_{\rho\sigma}(x). (Thanks to H. Haber.)

p533; In the enumeration under “Constructing gauge theories” second point I think it should be N^2 −1 instead of 1 2 N(N−1). (Thanks to Samuel Lorin)

P533 2nd paragraph from bottom: "1/2 N(N−1)" seems like to be read as "(N square − 1)”. (Thanks to Nam-Kyu Park.)

P572: the solution to excercise 2 of II.2: K(R)K(R) = 1/4(R^2+I+I+R^8) = 1/2K(I) + 1/2K(R^2) (Thanks to Menghang Wang, Panyu Chen, Petra Axolotl, Shi Xiaoyang)

p573: eq (18) should be corrected to: \sigma_{(12)(34)}=1+1+2−1−1=2 (Thanks to Mark Weitzman)

p573: eq (25) should be corrected to: \sigma_{(12345)}=\sigma_{12354} = 1+ \xi +1 −\xi −1 + 0 =1 (Thanks to Mark Weitzman)

P575: In the solution to Exercise 2 of IV.4, replace V.2 with V.3. (Thanks to John Dickinson)

P576: Line 5 of the solution to IV.5 Exercise 2, R_ab tr T σ_2 σ_a should be R_ab tr T σ_2 σ_b. (Thanks to John Dickinson)

Addenda

p22: for real symmetric matrix, we can choose the eigenvectors to be real. (This is not an errata, since multiplying an imaginary number to make it complex is not our purpose here.) (Thanks to Menghang Wang, Petra Axolotl)

p29: The direct product in the book is usually ⊗, which is different from conventional $\times$ notation. (Thanks to Mark Weitzman)

p39: In the last footnote, to be more precise, the electromagnetic interactions is based on the abelian gauge symmetry. (Thanks to Mark Weitzman)

p66: line 8 after (12), “The claim is that H is not contained in some larger invariant subgroup.” Here, we clearly consider the propper subgroup instead of trivial case such as identity or G itself. (Thanks to Mark Weitzman)

p96: line 2 under Unitary representations section, "for all representation" mainly means that all representations can be equivalent to corresponding unitary representations by similarity tranformation. (Thanks to Mark Weitzman)

p152: The proof of Wilson's theorem would be clearer if it is emphasized that "the inverse of each of the remaining integers {2, . . . , n − 2} is to be found among this subset of n − 3 integers", and none of the inverse of each equals itself; for example, the inverse of 2 should not be 2 itself (otherwise, 2 would not be able to find its partner in the set). This remark takes care of the exceptional case n=4 which is certainly not a prime, but 3!=6 is certainly not equal to 0 mod 4. (Thanks to Fujun Du)

p205: line -12, after <m|m> = 1, add (for all m). (Thanks to Lewis Robinson)
page 369, after first three paragraphs, add these clarifying remarks

Note that in going from (11) to (15), we are repeating what we did for the defining or fundamental representation in the preceding chapter for the adjoint representation. Write the H^i as diag(\be^i(1), \cdots, \be^i(a),\cdots, \be^i(n)) for i=1, \cdots, l. Line them up and read off (see for example (14) to (15) in the preceding chapter) the weights (which are l-dimensional vectors). For the adjoint representation the weight vectors are the same as the root vectors. Thus, the l-dimensional root vectors {\vec \be}(a) are as given in (15).

By construction, the root vectors are in one-to-one correspondence with the E's. Otherwise, the Cartan notation E_\be would not make sense. There are (n-l) root vectors and (n-l) E's.

Beginning students might also find it helpful to refer back to our discussion of SU(3): the H^i's correspond to I_3 and Y, while the E_\be's correspond to I_\pm, U_\pm, and V_\pm.

page 370: The discussion after (18) should be elaborated and clarified as follows. (Thanks to H. Haber for pointing out that the discussion in the text is incomplete.)

Add after (18) the following. (Thanks to J. Feinberg for discussions.)

Logically, either the right hand side does not vanish or it vanishes.

If it does not, then [E_\al, E_\be] is associated with the root vector \vec \al+\vec \be. We can add two root vectors to get another root vector.

If the right hand side vanishes; then there are three possibilities: (1) \vec \al+\vec \be =0, or (2) [E_\al, E_\be]=0, or (3) \vec \al+ \vec \be =0 and both [E_\al, E_\be]=0.

In case (1), \vec \be =-\vec \al, but with [E_\al, E_-\al] \neq 0. Since [E_\al, E_-\al] commutes with all the H^i's, and since the H^i's form by assumption a maximal subalgebra, [E_\al, E_-\al] must be a linear combination of the H^i's. This case is treated later around (21).

In case (2), [E_\al, E_\be]=0 but \vec \al+\vec \be \neq 0. Since \vec \al+\vec \be does not correspond to a generator E, it is not a root vector.

For an example of this possibility, recall from (14) in chapter V.3 that [I_+, V_+]=0, but the sum of their corresponding root vectors (see (21) in chapter V.3), namely (1,0)+(1/2, \sqrt{3}/2), certainly does not vanish, and that is not a root vector.

In case (3), E_\al commutes with its hermitean conjugate, and so its real and imaginary part could both be diagonalized. By the logic discussed later around (21) and around (250, \vec al=0. Both E_\al and E_-\al commute with all the H^i's, contradicting the assumption that the H^i's form a maximal commuting subalgebra. So this case is completely degenerate and not allowed.

This then more or less joins onto the existing text.

p495: three lines after (20), for what it is worth, what I called "funny business" is known to the smart set as Madelung's rule.

Regarding the clarifying remarks on page 369, the following could perhaps be added as an endnote in a future edition.

We could also provide an explicit proof that the root vectors and the E's are in one-to-one correspondence.

In the adjoint representation, Ti are diagonalized in the circular or polar basis (as explained on page 368) and represented by (11). We form \vec \be (a) as in (15).
Suppose the generators Ta and Tc are associated with the same root \vec \be. But on page 368 we have the explicit result if^{ia} _b=\be^i(a)\del^a_b. (As was pointed out in the text, this is awkward notation but perfectly sensible.)

The hypothesis is thatif^{ia}_b=if^{ia}_c for a \neq c, but this makes no sense since according to the Kronecker delta we have a = b and c = b.
Two different generators cannot be associated with the same root.
Can two different roots be associated with the same generator? No. The question does not even make sense. It would mean the same generator Ta commuted with the Hi’s would give different results.