Don’t Stress! (or Strain)

Determining Young’s Modulus

Relation to geophysics

While rocks appear inflexible and rigid, they actually have elastic properties. These properties underlie many seismological principles such as seismic wave propogation and earthquake cycles. Terms such as stress and strain are very applicable to geologic concepts.

Definitions

Normal (or direct) stress (σ) is the force applied normal to the surface (F_n) divided by the surface area (A).

$$\sigma = F_n/A$$

Possible units: Pascals (Pa), N/m^2, psi

Strain (ε) is stretch per unit length of a material. It is found by determining the change in length (ΔL) divided by the initial length (L) of a sample.

$$\varepsilon = \Delta L/L$$

Strain is a dimensionless quantity (no units)

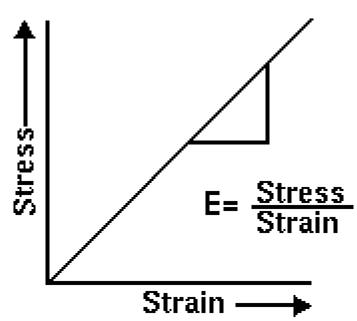
Young’s modulus (E) is the ratio of the stress to the strain and is also known as the elastic modulus.

$$E = \sigma/\varepsilon = (F_n/A)/(\Delta L/L)$$

Possible units: Pascals (Pa), N/m^2, psi

Since E = stress/strain, the slope of a graph of stress (y-axis) vs. strain (x-axis) for a sample can give the value of Young’s modulus for the material. If the graph is linear, the material is elastic and if the stress is removed, the material will return to its original size and not remain deformed.

Remember: Hooke’s Law [$F = -k(\Delta L)$] also describes elastic materials, where F is the force exerted, ΔL is the elongation or compression, and k is the spring constant which describes the stiffness. (note: The minus sign is a convention which allows k to be a positive value.) Therefore, k is related to Young’s modulus as k = AE/L.



http://homepage.usask.ca/~mjr347/prog/geoe118/geoe118.035.html

Terms and Equations to know

- modulus = measure
- isotropic = uniform in all directions; properties don't vary with orientation
- homogeneous = uniform structure throughout
- normal stress = σ = F_n/A = force applied normal to surface/surface area (Pa, N/m^2, psi)
- strain = ε = $\Delta L/L$ = compression/original height (no units)
- Young's modulus = E = σ/ε = slope of the stress vs strain graph (Pa, N/m^2, psi)
- elastic modulus = another name for Young's modulus
- elastic materials = return to their original shape when the load is removed
- linear materials = strain increases regularly with the stress. This means that the ratio of load to deformation remains constant.

The following terms describe materials

- Stiffness – large force is needed to deform it
- Strength – returns to its original shape even after a large force is applied
- Hardness – resistance to permanent deformation by compression
- Toughness – amount of energy that can be absorbed before fracturing

Just to clarify

Material stiffness is not the same thing as geometric stiffness. The geometric stiffness depends on shape. For example, the stiffness of an I-beam is much larger than that of a spring made of the same steel.

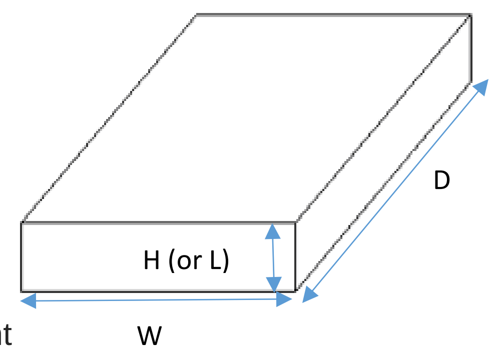
Activity Instructions

Materials needed:

- compressible sponges (one is needed for procedure 1, but at least 2 types of approximately the same top area are needed for procedure 2)
- various weights (bricks, books, and weight sets are possibilities, depending upon the compressibility of the sponges supplied)
- balance
- rule
- level (not required, but may help with even distribution of weight)

Procedure 1 – Determining Young's modulus

Steps to follow…



- Choose one sponge.
- Measure the height, width, depth and mass of the sponge. Record in data table 1.
- Select a weight to place on top of the sponge which covers the entire top area of the sponge. Determine its mass. Record in data table 2.
- Place a weight on the sponge (which is horizontally positioned on a sturdy, flat surface), covering the entire top area of the sponge. Make sure the weight is distributed as evenly as possible.
- Measure the new height of the sponge. Record in data table 3.
- Place another (known) weight on the sponge. Increase weight from smaller to larger values in small but noticeable increments, recording the values in table 2.
- Measure the new heights, recording the values in table 3.
- Repeat until you have tested 5 weights.

Calculations:

Sponge description - ____

Data table 1 – general sponge properties

Height (m) this is "L"	Width (m)	Depth (m)	Area of the top when laying flat = W X D (m^2)	Volume (m^3) = W x D x H	Sponge Mass (kg)	Sponge Density (kg/m^3) = mass/volume

Data table 2 – determining the stress

Mass placed on top (kg)	Weight (force) of mass (N)	Stress (σ) = Force/Area (N/m^2 or Pascals)

Data table 3 – determining the strain

Added force (N)	$L_{original}$ (or height) (m) (stays the same)	L_{new} (compressed height) (m)	ΔL = $L_{original} - L_{new}$ (m)	Strain (ε) = $\Delta L/L_{original}$ (no units)

Analysis of Data:

Graph STRESS (y-axis) vs STRAIN (x-axis). Find the line of best fit (and it's equation if you can). **What is the slope (with units)?** This is Young's modulus.

Is the graph really linear? If so, the sponge is elastic.

What would a non-linear stress vs. strain relationship tell us?

Is this a large Young's modulus? Compare it with some other materials.

What does a small Young's modulus imply?

What does a large Young's modulus imply?

According to the terms and equations above, if a material has a **constant** Young's modulus, it must be both ______________________ and ______________________.

Extension:

Complete the following…

1. Crushing two faces of a sponge straight toward each other shows _______________ modulus.
2. Pushing one end of a sponge upward and the opposite end downward shows the _______________ modulus.
3. Crushing the sponge into a ball, pushing from all directions shows the _________________ modulus.

Word bank: Young's, Bulk, Shear

Procedure 2 – Determining the effects of a composite material

Relation to geophysics

Because the Earth is comprised of layers of various rocks and minerals with differing elastic properties, geologic occurrences are influenced by the composite properties of these materials.

What do you think…
How would you expect the stress, strain, and Young's modulus values for a layered set of different sponges to compare with the values for one individual sponge?

Steps to follow…

- Determine the Young's modulus values for at least 2 varying sponge materials individually (using the process from Procedure 1).
- Horizontally stack the two (or more) sponges on top of each other.
- Using the combination of sponges as your base, determine Young's modulus for this composite material.

Questions to consider:

How does the composite stress value compare to the two (or more) values for the individual sponges?

How does the composite strain value compare to the two (or more) values for the individual sponges?

How does the composite Young's modulus value compare to the two (or more) values for the individual sponges?

Is this what you expected?

Does Young's modulus seem to be related to the sponge density? Explain.

What do you think would happen if you layered the sponges vertically instead?

(Optional extension - Determine Young's modulus for a composite of vertically layered sponges).

Sponge description - ____

Data table 1 – general sponge properties

Height (m) this is "L"	Width (m)	Depth (m)	Area of the top when laying flat = W X D (m^2)	Volume (m^3) = W x D x H	Sponge Mass (kg)	Sponge Density (kg/m^3) = mass/volume

Data table 2 – determining the stress

Mass placed on top (kg)	Weight (force) of mass (N)	Stress (σ) = Force/Area (N/m^2 or Pascals)

Data table 3 – determining the strain

Added force (N)	Original L (or height) (m)	New L (compressed height) (m)	ΔL = $L_{original} - L_{new}$ (m)	Strain (ε) = $\Delta L/L$ (no units)

Referenced materials:

http://web.mit.edu/course/3/3.225/book.pdf

https://www.iris.edu/hq/inclass/downloads/download/28

https://www.iris.edu/hq/inclass/search#type=3&4

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