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Fluid Dynamics Problem Set

1. Sedimentation along the Andean Margin in South America.

The basic question here is how far offshore suspended sediment from the Andes will travel before it falls to the bottom of the ocean. The ocean quickly drops to a depth of about 4000 m (4 km) offshore and there is an ocean current that moves with an average velocity of 0.2 m/s to the west. We will consider 3 different size classes of sediment – clay (radius = 0.001 mm), fine silt (radius = 0.005 mm), and coarse silt (radius = 0.025 mm) – and we will pretend they are all perfectly spherical. Assume a sediment particle density of 2650 kg/m^3 and a water viscosity of 1.5e-3 Pa*s.

A. Find the settling velocity of each size of sediment giving your answer in m/s. Include the Reynold's number for each case to justify your choice of turbulent or viscous settling velocity. Be wary of your units and remember to write them down within each step of the problem to ensure the correct answer.

This is an excellent problem to have students derive the equation for the settling velocity (Stoke's Law of Settling). When the viscous drag force (F_D) equals the buoyancy force (F_B) the acceleration equals zero and therefore the particle has reached its terminal velocity. The terminal velocity can then be used in the Reynold's Number equation to determine what type of flow regime this is.

$$F_D = (6\pi r)\mu u \quad (1)$$
$$F_B = \rho V g \quad (2)$$

(1) is the viscous drag force equation where r is the radius, μ is the viscosity of the fluid that the sediment is flowing through – in this case it is water, and u is the velocity of the particle.
(2) Is the buoyancy force equation where ρ is the density of the particle, V is the volume of the particle, and g is gravity. The important thing to notice here is that density is mass divided by volume, so the buoyancy force is simply mass times gravity, Newton's second law, since density time volume is equal to mass.

These can be set equal to each other and rearranged to find the terminal velocity:

$$U_T = \frac{2r^2\rho g}{9\mu} \quad (3)$$

(3) is the settling velocity for a particle sinking through some fluid medium. Volume has been substituted by the equation for the volume of a sphere.

The final step for this part of the problem is to substitute the radius of each respective particle in equation 3 and then use equation 4 to determine the Reynold's Number for the flow. I highly recommend making the dimensional analysis for this a part of the problem (i.e. make them work the units out separately, this helps give better understanding to what units actually mean such as a pascal).

$$R_e = \frac{Inertial\ Forces}{Viscous\ Forces} = \frac{\rho_f ul}{\mu} \qquad (4)$$

(4) is the equation for Reynold's Number where ρ_f is the density of the fluid, u is the velocity of the particle, l is the characteristic length (radius of the particle), and μ is the viscosity of the fluid. When Re > 0.5 the flow regime is laminar, when Re < 0.5 the flow is turbulent.

Answers:

u1 = 2.4e-6 m/s	Re1 = 1.6e-6	viscous	(clay)
u2 = 6.0e-5 m/s	Re2 = 2.0e-4	viscous	(fine silt)
u3 = 0.0015 m/s	Re3 = 0.025	viscous	(coarse silt)

B. Find the time required (in seconds and years) to reach the bottom of the ocean for each sediment size.

Simply the depth of the ocean divided by the settling velocity, but WATCH UNITS

Answers:

T1 = 1.67e9 s or 53 years
T2 = 6.67e7 s or 2.11 years
T3 = 2.67e6 s or 0.087 years

Wow! Look at the difference in time to settle to the ocean bottom from the clast sizes!

C. How far offshore will the sediments travel through the Pacific Ocean? Is the Pacific Ocean wide enough to accommodate this distance? (Average width of the Pacific is ~11,000 km).

Simply multiply the time from part b by the current of the ocean given.

Answers:

D1 = 3.4e5 km, not wide enough
D2 = 1.4e4 km, not wide enough
D3 = 534 km, easily wide enough

D. Observations show that the Andean sediment does not extend past 150 kilometers offshore, offer an explanation to this paradox.

It could be that the current dies off with depth or reberses or is dominated by eddies; or it could be that sediments form larger aggregate particles such as fecal pellets, which then settle much faster. This portion of the problem is intended for students to think critically why the equations are not always ideal and why models do not always explain things perfectly.

2. While visiting your friend's vacation home at the foot of Mt. Rainier in Washington, an eruption begins. An andesitic lava flow has issued forth from the summit crater and is flowing down the 28° flanks – your friend's vacation home is in the expected path just 3 kilometers from the crater! Should you make a run for it, or should you calmly finish your dinner of grilled salmon, wild leeks, and Island Vintage volcano espresso? While your friend panics, you calmly get out a piece of paper and your calculator and get to work recalling your knowledge from fluids class. You also recall from your earth science courses that the viscosity of andesitic lava is 1.5e4 Pa*s, and its density is 2500 kg/m^3. Assume a flow depth of 2 meters.
   a. What is its velocity at the top of the flow? Give the answer in m/s and km/hr.

      Using the equation for the flow velocity at different depths:

      $$u(y) = \frac{-1}{2\mu}\rho g sin\alpha(y^2 - 2yL)$$

      Where μ is the viscosity of the flow, ρ is the density of the flow, g is gravity, α is the slope that the path is traveling down, y is the height in the flow of interest, and L is the total height of the flow. Since we are interested in the velocity at the top then y is the same thing as L and the term in the parenthesis can be changed to ($-y^2$), or it can be solved by simply plugging in 2 for y and 2 for L.

      Answer:

      u(2) = 1.54 m/s or 5.5 km/hr or 92.4 m/min (for part b)

      Wow! This lava flow is only moving 5.5 km/hr, which is roughly 3.5 mph, easily able to outrun it!

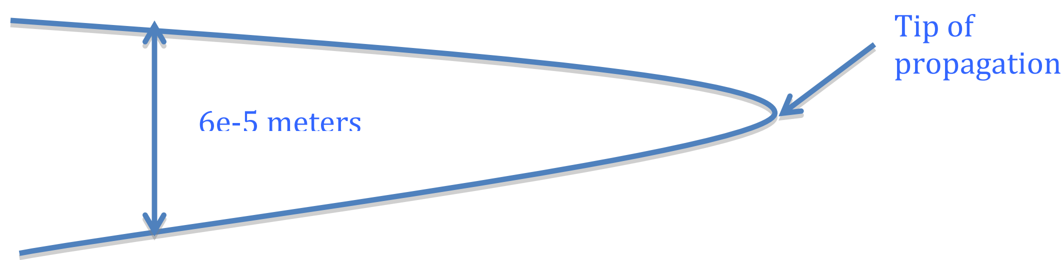
   b. How much time do you have (in minutes) to finish your dinner before making a run for it? Show your work and explain.

      Simply divide the distance from the crater by the velocity to get the time, which is roughly 32.5 minutes, plenty of time to enjoy your dinner before you briskly walk away from the lava flow!

3. You have just been hired by one of the most prestigious, Toret Energy Incorporated, as a staff scientist. The company plans to install several new well pads but needs to bring its "secret" fracking formula up to the DEP code, and you have been tasked with determining the viscosity needed for the formula to efficiently fracture the rock layers. The "breakdown pressure" for hydraulic fracturing is about 7500 psi, which is about 50

MPa, this is the fluid pressure needed to get fractures to propagate. For the fractures to continue to grow, the fluid must be able to advance into the new crack as fast as the crack tip is advancing, about 300 m/s. This means that the viscosity of the fluid has to be low enough to permit the fluid to flow fast enough in the crack. On the other hand, the fluid iscosity needs to be high enough to keep the proppants entrained. You recall from your fluids course that you can model the situation as viscous flow in a channel, where the channel height is the width of these cracks during fracking, typically 6e-5 m. The pressure gradient in this case is the breakdown pressure. Now you know exactly how to solve for the viscosity.

First it is important to understand what is happening by visualizing the fracking process. Remember that fracking is horizontal drilling at depths of thousands of feet.



This can be treated as viscous flow in a channel driven by a pressure gradient. So the equation below can be used to determine the velocity with other variables known:

$$u(y) = \frac{1}{2\mu}\frac{dP}{dx}(y^2 - yL)$$

Where μ is the viscosity of the fluid, dP/dx is the pressure gradient, y is the position of interest within the flow, and L is the width of the channel. With the information given, you know all but the viscosity therefore you can rearrange the equation by swapping velocity with viscosity to solve for viscosity. This is another excellent problem to add a step of dimensional analysis because the pressure gradient must be in pascals but is given in megapascals. The important part of the schematic is to show that the tip is located in the middle of the flow channel, so the 300 m/s velocity is occurring at the location of y/2. So simply sub in y/2 or use half of L for y and plug in the numbers.

Answer:

μ = 7.5e-5 Pa*s

Water has a viscosity of 8.9e-4 Pa*s so this is an order of magnitude smaller than that of water!

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