A new estimator of the deceleration parameter from galaxy rotation curves

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ABSTRACT

On a de Sitter background described by a cosmological constant Λ of holographic origin, non-Newtonian behavior is expected beyond a distance $r_t = 4.6M_{11}^{\frac{1}{2}}$kpc to a baryonic mass $M = M_{11}10^{11}M_\odot$ assocated with a background energy surface density $\sqrt{\Lambda}/(2\sqrt{2}\pi)$. The deceleration parameter satisfies $q = 1-(4\pi a_0/cH)^2$, where H denotes the Hubble parameter, c the velocity of light and a_0 is Milgrom's parameter in the asymptotic centriputal acceleration $a = \sqrt{a_0 a_N}$ in galaxy rotation curves in $r >> r_t$, where a_N denotes the Newtonian acceleration imparted by M. A detailed study of galaxy rotation curves at low redshifts may hereby provide a new estimator of dq/dz at $z = 0$ to identify a dynamical origin of dark energy.

1. Introduction

On a cosmological background dominated by dark energy (Riess et al. 1998; Perlmutter et al. 1999), the limit of weak gravitational attraction on galactic scales and beyond is defined by the Hubble acceleration scale $a_H = cH_0$ of a few Angstrom per second squared, where c is the velocity of light and $H_0 \simeq 70$ km s^{-1}Mpc^{-1} is the Hubble parameter. In this regime, we are off by several orders of magnitude from accelerations, where Newton's law has been rigorously tested (Famae & McGaugh 2012). The cosmological horizon at $R = c/H_0$ is a Lorentz invariant, that may affect Newton's law by its covariant embedding in general relativity. Spacetime may derive from a Planck scale structure of light modes. If so, these modes are potentially sensitive to any low energy scale introduced by the cosmological horizon. Non-Newton behavior is then expected in the regime of gravitational

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attractions on the order of a_H or less.

We start with gravitation from a holographic principle ('t Hooft 1993; Susskind 1995), that posits imaging of curved spacetime and matter within by superposition of modes encoded in a large number of Planck sized *surface elements*. In unitary holography (van Putten 2015a), the information that defines spacetime images is determined by enclosed mass and distances to a holographic surface derived from particle propagators. Wave functions of matter herein derive from superpositions of massless modes that obey the trivial dispersion relation in vacuum, representing a projection of modes in the screen.

Any low energy scale introduced by the cosmological horizon is illustrative for the holographic principle. Such is most apparent in geometrical units, in which Newton's constant G and the velocity of light c are set equal to 1. In these units, force is dimensionless. A force $F = 1$ on the cosmological horizon area $A = 4\pi R^2$ shows a pressure $p = 1/A = H^2/(4\pi)$ that, by Lorentz invariance of the cosmological horizon, carries an associated energy density $\rho = -p$. The result is a cosmological constant $\Lambda \equiv 8\pi\rho$ satisfying $\Lambda = 2H^2$, which recovers $\Omega_\Lambda = 2/3$ close to the present day value of about 0.7. For a deceleration paramter $q = -1$, the cosmological horizon also has a de Sitter temperature $T_{dS} = H/2\pi$ (Gibbons & Hawking 1977), equal to the Unruh temperature $\kappa_H = a_H/(2\pi)$ defined by its surface gravity H (Unruh 1976). T_{dS} is representative for a thermal energy *surface density* $\Sigma = \frac{1}{2}T = H/(4\pi)$. While the dark energy volume density $\Lambda/8\pi$ is notoriously small, $\Lambda \simeq 1.21 \times 10^{-56}\,\mathrm{cm}^{-2}$, its holographic origin is a dark energy surface density $\Sigma = \sqrt{\Lambda}/(4\sqrt{2}\pi) \simeq 6 \times 10^{-29}\,\mathrm{cm}^{-28}$ that is *not* small. An immediate implication is a critical transition radius for gravitational attraction. Consider a central mass $M = M_{11}10^{11}M_\odot$ of a typical galaxy. Then $A\Sigma = M$ with $A = 4\pi r^2$ defines a transition radius

$$r_t = \sqrt{MR_H} = 4.6\, M_{11}^{\frac{1}{2}}\,\mathrm{kpc}. \tag{1}$$

The transition radius (1) is commonly observed in galaxy rotation curves and bears out in a deviation of centripital accelerations a relative to the Newtonian acceleration a_N expected from the observed baryonic mass (Milgrom 1983; Famae & McGaugh 2012) as shown in Fig. 1. (1) defines strong gravitational interactions in $r << r_t$ and weak gravitational interactions in $r >> r_t$. This pertains to accelerations relative to M/r_t^2, i.e., $a << a_H$ and, respectively, $a >> a_H$. Holography hereby identifies a_H as a critical acceleration in galaxy rotation curves.

Holographic imaging is a function of area A and opening angle Ω by their product $A\Omega$. (More precisely, it is a function of the integral of Ω over the surface of the screen.) Factorization of $A\Omega$ is hereby an internal symmetry of holography (cf. 't Hooft 2015). Scaling of A and Ω corresponds to curvature and, respectively, lensing. These may be realized, repsectively, by a conformal factor or a deficit angle, giving different manifestations of the same.

In unitary holography, the information I in imaging a mass m is given by the distance to a screan measured in total phase $\Delta\varphi$, as defined by m's wave function. For a spherical screen of radius s, we have (van Putten 2015a)

$$I = 2\pi\Delta\varphi. \tag{2}$$

A heuristic counting argument shows a minimum of four bits in the imaginging of mass, electric charge, angular momentum, electromagnetic and gravitational waves. Mass alone is herein encoded in one out of four bits. It follows that I is encoded in the impression f of $A\Omega$ by $fA = A - A_E$ or $f\Omega = 4\pi - \Omega_E$, where $A_E = 8\pi ms$ and $\Omega_E = 8\pi m/s$ denote the Einstein area and opening angles, respectively, i.e.:

$$fA\Omega = 4\pi\left(A - A_E\right) = 4\pi A\left(4\pi - \Omega_E\right) = 16\pi^2 s^2 f. \tag{3}$$

A *minimum* screen size attains with $A = A_E$ or $\Omega_E = 4\pi$ at the Schwarzschild radius

$s = R_S$, $R_S = 2m = \sqrt{S/\pi}$ with $S = \min I = 4\pi m^2$ in $I = 2\pi m\,(s - R_S) + S$ the same as the Bekenstein-Hawking entropy. Accordingly, $f = 1 - 2m/s$ in (3).

Holographic imaging with factorization by a conformal factor Φ^4 is described by the isotropic line-element

$$ds^2 = -N^2 dt^2 + \Phi^4 \left(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\phi^2\right), \tag{4}$$

where $N = N(\Phi)$ denotes the gravitational redshift, i.e., the ratio of energy-at-infinity to locally measured energy. According to the above, $R_S = \sqrt{4S/\pi}$ expresses the mass-energy of a particle by its linear size, locally measured by the minimal surface area $4S$ of an enveloping holographic screen. In keeping the total energy-at-infinity constant, putting a test particle at various locations preserves a constant $N\sqrt{S}$:

$$N\Phi^2 \simeq \text{const.} \tag{5}$$

Here, we use the approximation of minor perturbations to the spherically symmetric line-element (4). (For a detailed consideration, see (van Putten 2012).) We shall refer to (5) as *Gibbs' condition*, following its use in thermodynamics. According to the equations of geodesic motion, Newton's law derives from N in the large distance limit. By (5), it equivalently derives from the conformal scale Φ^4. Since ρ reduces to the ordinary radial distance at large separations, (4-5) can be seen to embed Newton's law in

$$\Phi \simeq f^{-\frac{1}{4}} \simeq 1 + \frac{m}{2s} \quad (s >> 2m). \tag{6}$$

In §2, the non-Newtonian asymptotic behavior in $r >> r_t$ in (1) is detailed on a de Sitter background, based on aforementioned Gibbons-Hawking temperature T_{dS}. In §3, these results are extended to the more general Friedmann-Robertson-Walker background, parameterized by a finite deceleration parameter q. A finite sensitivity of the behavior in $r >> r_t$ to q is proposed as a new estimator of $q = q(z)$ as a function of redshift z. It gives a new method to determine dq/dz at $z = 0$, to discriminate between dynamic and static dark energy independent of supernova Type Ia supernovae.

2. Sensitivity to a de Sitter background

In what follows, A will refer to surface area as well as the number of Planck sized surface elements A/l_p^2, where $l_p = \sqrt{G\hbar/c^3}$ denotes the Planck length.

In holography, the wave function of a particle m results from a superposition of a large number A of low energy Planck sized harmonic oscillators. Ordinarily, there is one mode in the image for each mode in the surface on the screen. (The number of degrees of freedom in the image is then equal to the number of degrees of freedom in the holographic screen.) Quantum mechanically, m is the time rate-of-change of total phase as measured at infinity, whereby

$$m = \frac{1}{2}A\omega \tag{7}$$

of the ground state energies $(1/2)\omega$ of harmonic oscillators in the screen. Distance encoding (2) derives from $\Delta\varphi = ks$ with the total wave number k given by the superposition of wave numbers of massless modes imaged by these harmonic oscillators,

$$k = \frac{1}{2}A\kappa. \tag{8}$$

This identification (8) follows by application of the trivial dispersion relation $\omega = \kappa$, which recovers the Compton wave number $k = k_C$, $k_C = m$, with the low energy wave numbers

$$\kappa_N = \frac{2m}{A} = \frac{a_N}{2\pi} \tag{9}$$

defined by the Newtonian acceleration $a_N = m/s^2$.

It will be appreciated that recovering the Compton relation $k = m$ by the trivial dispersion associates κ_N with the Unruh temperature of Newtonian acceleration. In entropic gravity (Verlinde 2011), the above implies entropic forces at the temperature $T = m/2\pi s^2$ by $dS = -dI = -2\pi m' ds$, giving $F = -dU/ds = TdS/ds = -mm'/s^2$. In keeping with (5-6), however, we shall not use entropic force arguments.

Following our introduction, (9) may become susceptable to the low energy de Sitter temperature of the cosmological horizon. Screen modes satisfy the dispersion relation

$$\omega = \sqrt{\kappa^2 + \kappa_H^2} \tag{10}$$

representing an incoherent sum of a momenta κ and the background de Sitter tempreature, $\kappa_H = T_{dS}$ (Narnhofer et al. 1996; Deser & Levin 1997; Jacobsonb 1998). A spherical screen imaging a mass m at its center hereby assumes

$$\kappa_N = \omega - \kappa_H : \quad \kappa = \sqrt{\kappa_N^2 + 2\kappa_N \kappa_H}, \tag{11}$$

giving $\kappa \simeq \kappa_N$ $(r >> r_t)$ and $\kappa \simeq \sqrt{a_0 a_N}$ $(r << r_t)$ with $a_0 = 2a_H$ as proposed in (Klinkhamer & Kopp 2011). However, (11) overestimates the Milgrom parameter a_0 (Milgrom 1983) by about *one order of magnitude* according to the data shown in (Fig. 1). The momentum κ in (11) is *not* representative for the wave number of the modes in the holographic image.

The modes in the image satisfy the dispersion relation

$$\omega' = \sqrt{\kappa^2 + \Lambda}, \tag{12}$$

as follows directly from writing the wave equation of a vector field in curved spacetime, e.g., of the electromagnetic vector potential (e.g. Wald 1984) or the Riemann-Cartan connections in a Lorenz gauge (van Putten & Eardley 1996), where Λ derives from coupling to the Ricci tensor. It implies an effective rest mass energy $\sqrt{\Lambda}$ of the graviton and photon alike. Though an effective mass arising from background curvature is not the same as true mass, we mention in passing that the problem of consistent general relativity with massive gravitons has recently received considerable attention (de Rham et al. 2011; Bernard et al. 2014). With $q_0 H^2 = H^2 + \dot{H}$, the generalized Higuchi constraint $m^2 \geq 2(H^2 + \dot{H})$ (Higuchi 1987; Deser & Waldron 2011; Grisa & Sorbo 2010) reduces to $\Omega_\Lambda \geq 2q_0$. Based on observations, $1 < q_0 < 0.5$ (Riess et al. 2004; Wu & Yu 2008; Giostri et al. 2012), whereby $q_0 > -1$ appears secure at any rate.

Comparing low energy modes in the image and the screen in the regime $r >> r_t$, we encounter different effective masses in (10) and (12), namely

$$\Delta\omega' \simeq \frac{\kappa^2}{2\sqrt{\Lambda}}, \quad \Delta\omega \simeq \frac{\kappa^2}{2\kappa_H}. \tag{13}$$

In the limit of weak gravitation in de Sitter space, therefore, *a direct correspondence between screen modes and image modes is lost*, in striking departure from ordinary holography in $r << r_t$. Specifically, (13) shows a discrepancy by a factor of $2\sqrt{2}\pi$ in effective masses $\sqrt{\Lambda}$ and κ_H.

With screen momenta defined by total energy in (11), (13) defines an reduced image momenta

$$\kappa_N' = \omega' - \sqrt{\Lambda} = \sqrt{\kappa^2 + \Lambda} - \sqrt{\Lambda}. \tag{14}$$

By (11) once more, the associated reduced screen momenta are

$$\kappa' = \sqrt{\kappa_N'^2 + 2\kappa_N'\kappa_H}. \tag{15}$$

The resulting graph $\kappa'(\kappa_N)$ shown in Fig. 1 is in agreement with the data. Specifically, we arrive at Milgrom's constant

$$a_0 = \left(\frac{\kappa_H}{\sqrt{\Lambda}}\right) 2cH_0 = \frac{cH_0}{\sqrt{2}\pi} \simeq 1.5 \times 10^{-8}\mathrm{cm\,s}^{-2}, \tag{16}$$

where we restored dimensions in cgs units.

3. Sensitivity to $q(z)$

The above generalizes to Friedmann-Robertson-Walker (FRW) universes characterized by a Hubble parameter H and a deceleration parameter q with modified de Sitter temperature (Cai & Kim 2005; van Putten 2015b)

$$T_{dS} = \frac{1-q}{2}\frac{H}{2\pi}. \tag{17}$$

Milgrom's constant hereby generalizes to

$$a_0 = \frac{\sqrt{1-q}}{4\pi}cH_0, \tag{18}$$

allowing measurement of q from a_0 as a function of redshift:

$$q(z) = 1 - \left(\frac{4\pi a_0(z)}{cH(z)}\right)^2. \tag{19}$$

The low redshift sample of galaxies of (Famae & McGaugh 2012) accurately recovers the value $q_0 \simeq -0.75$ (Fig. 2) consistent with Type Ia supernova surveys (Riess et al. 2004; Wu & Yu 2008; Giostri et al. 2012)

The rather well-defined minimum in Fig. 2 suggests that galaxy rotation curves of samples at different redshifts may be used as probes of $q = q(z)$. This prospect seems particularly worthwhile to measure $dq(z)/dz$ at $z = 0$, by obtaining $a_0(z)$ in a nearby redshift range $z << 1$ to circumvent the limitations in systematic errors of existing supernova surveys. A principle objective is an accurate determination of $dq(z)/dz$ at $z = 0$, sufficient to discriminate between dynamical dark energy ($dq(z)/dz \simeq 2$) or a static dark energy conform ΛCDM ($dq(z)/dz \simeq 1$). The first would indicate the manifestation of a holographic dark energy $\Lambda = H^2(1-q)$ with a cosmological distribution of light dark matter with negligible clustering on galactic scales (van Putten 2015b).

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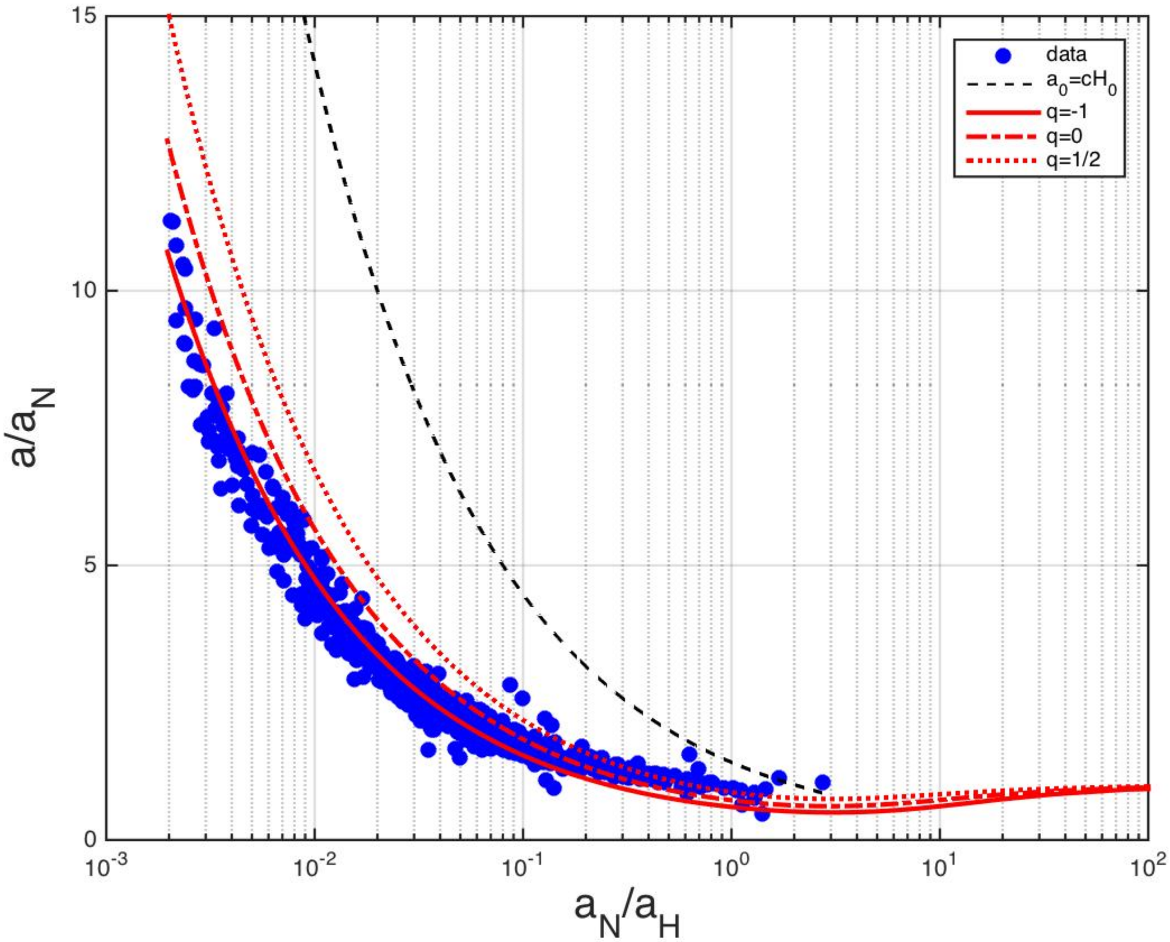

Fig. 1.— Galaxy rotation curves (blue dots) reveal a transition to a $1/r$ force law at weak accelerations asymptotically in $a << a_H$ away from Newtonian forces in $a >> a_H$ based on the observed baryonic matter. Shown is a theoretical curve (red) in unitary holography with a good match in a cosmological background with deceleration parameter close q in the range $-1 < q < -0.5$. Data are from galaxy curves with essentially zero redshifts from (Famae & McGaugh 2012).

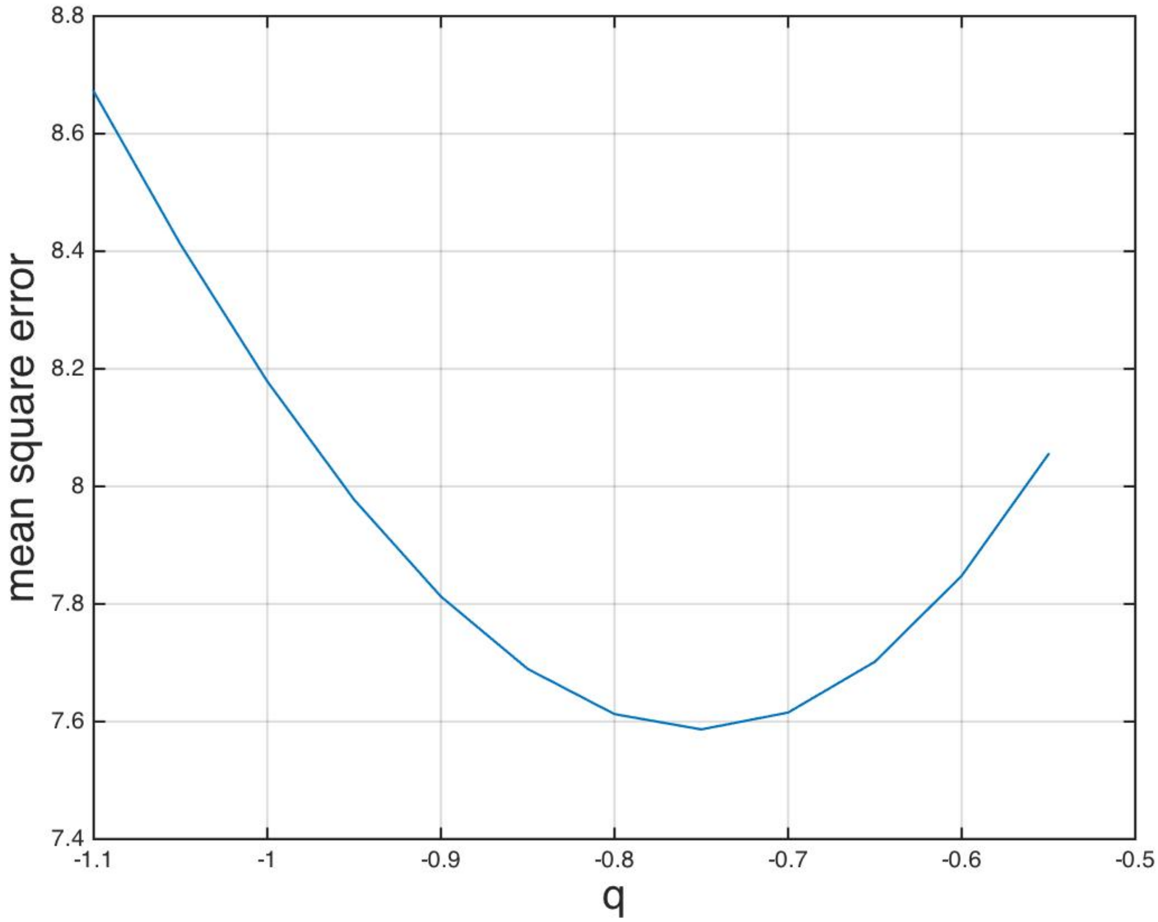

Fig. 2.— Estimation of $q_0 \simeq -0.75$ based on a sample of rotation curves of low redshift galaxies shown in Fig. 1.

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