The n-cube is Rupert

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Abstract

An oval in R^n is called *Rupert* if a straight tunnel can be made in it through which a second congruent oval can be passed. We show that the n-cube is Rupert for each $n \geq 3$.

1. Introduction.

The surprising observation that a hole can be cut in one of two equal cubes large enough to permit the passage of the second cube was attributed by the contemporaneous John Wallis [15, pp. 470–471] to Prince Rupert of the Rhine (1619–1682), son of Frederick V, the Winter King of Bohemia, and Elizabeth, daughter of James I of England. This remarkable fact has been the subject of continuing interest ever since, and in recent years many polyhedra in R^3 have been shown to have this property.

An oval in R^n has the *Rupert property,* or is *Rupert,* if a straight tunnel can be made in it through which a second oval congruent to the first can be passed. We examine this property for ovals generally and give a sufficient condition for an oval to be Rupert. Then we specialize our results to hypercubes and show that for $n \geq 3$ the n-cube enjoys this curious property.

Throughout we take R^n, $n \geq 3$, to be our ambient space. Our arguments employ little more than analytic geometry (i.e., linear algebra) and rudimentary topology in R^n. For clarity we sometimes employ geometric language (point, line, plane, etc.) rather than the linear-algebraic equivalent (vector, one-dimensional affine subspace, etc.). Our reasoning is, in the main, geometric and descriptive.

2. Ovals.

An *oval* in a Euclidean space is a compact, convex set having nonempty interior. We begin by considering the Rupert property for ovals in R^n.

Not every oval is Rupert, of course. In R^3 an equilateral drum, a (solid) circular cylinder of height equal to its diameter, does not have the Rupert property, nor does a sphere or any solid of constant width. Many convex polyhedra in R^3 are known to be Rupert including the Platonic solids and others.

Tunnels.

To clarify the idea of "tunnel," recall that a (right) cylinder in R^3 is the collection of points on the lines through a fixed plane set (called the *directrix)* in the direction normal to the plane of the directrix. The definition in R^n is analogous, but we require that the directrix be an oval in a hyperplane (i.e., with nonempty $(n-1)$-dimensional interior).

Definition. Let D be an oval in a hyperplane H. For each point $p \in D$ let ℓ_p be the line through p in the direction of the vector $\boldsymbol{h}$ normal to H. The *(right) cylinder* $C_{\boldsymbol{h}}(D)$ with *direction* $\boldsymbol{h}$ and *directrix* D is the point set

$$C_{\boldsymbol{h}}\left(D\right) = \bigcup\nolimits_{p \in D} \ell_p.$$

The *tunnel* $T_{\boldsymbol{h}}\left(D\right)$ is the interior of this cylinder.

Thus a tunnel is an open set in R^n, and its direction is normal to the hyperplane H containing its directrix.

We will need the elementary observation that the intersection of the cylinder $C_{\boldsymbol{h}}\left(D\right)$ with a hyperplane K having normal $\boldsymbol{h}$ is a translate of D and consequently is congruent to D.

Projections.

Let O be an oval in R^n and H a hyperplane. The projection of a point p into H is the point q of H so that the vector $\overrightarrow{pq}$ is a scalar multiple of the normal vector $\boldsymbol{h}$ of H. The projection $\pi_H\left(O\right)$, the union of the projections of the points of O into H, is compact and convex and has nonempty interior relative to H, that is to say, it is an oval in H.

Definition. A set S in a hyperplane H *fits* in a set T in a hyperplane K if there is a motion (i.e., an isometry) of R^n that carries H to K and S to a subset of T.

Theorem 1 *Let O be an oval in R^n. If there are hyperplanes H and K (with normal $\boldsymbol{k}$) and an oval D in $K \cap \operatorname{int}(O)$ so that the projection $\pi_H(O)$ fits in* $\operatorname{int}(D)$, *then a copy of O can be translated through the tunnel $T_{\boldsymbol{k}}(D)$.*

Proof. The intersection of the cylinder $C_{\boldsymbol{k}}(\pi_K(D))$ and a hyperplane having $\boldsymbol{k}$ as normal vector is a "cross section" oval congruent to $\pi_K(D)$. Because O fits in the cylinder $C_{\boldsymbol{h}}\left(\pi_H(D)\right)$, it fits in the larger tunnel $T_{\boldsymbol{k}}\left(\pi_K(D)\right)$. Thus the oval O fits throughout in the tunnel $T_{\boldsymbol{k}}\left(\pi_K(D)\right)$, and its transit is fully described by

$$O_t = t\,\boldsymbol{k} + O \subset T_{\boldsymbol{k}}\left(\pi_K(D)\right, \qquad -\infty < t < \infty.$$

■

3. The n-cube.

An *orthotope* (or *box*) in R^n is, by definition, the product of n closed intervals. It is a centrally symmetric oval. If all the intervals have the same length s, the orthotope is an n*-cube* with edge s. We say that an n-cube is *central* if its center of symmetry is at the origin and its edges are parallel to the coordinate axes.

The vertices of a central n-cube of edge 2 are the 2^n points $(\pm 1, \pm 1, \ldots, \pm 1)$. Its *facets* are $(n-1)$-cubes of edge 2 lying in the support hyperplanes $x_j = \pm 1$. An n-cube has $2n$ facets, parallel in pairs normal to the coordinate axes.

Much more about n-cubes can be found in [2], [8], and [12].

Specializing Theorem 1 to the Rupert situation gives the following basic result.

Theorem 2 *A unit n-cube that contains an $(n-1)$-cube of edge greater than one in its interior is Rupert.*

Proof. The projection of a central unit n-cube $\mathbf{Q}$ into the hyperplane H with equation $x_n = \frac{1}{2}$ is a facet of $\mathbf{Q}$, an $(n-1)$-cube $\mathbf{C}$ of edge 1. Let $\mathbf{C}_s$ be an $(n-1)$-cube of edge $s > 1$ in the interior of $\mathbf{Q}$. Since $\mathbf{C}$ fits inside the larger $\mathbf{C}_s$, the result follows from Theorem 1. ■

4. Cube in cube.

The proof that an n-cube $\mathbf{Q}$ is Rupert hinges on the fact that it contains an $(n-1)$-cube $\mathbf{C}$ with greater edge length. We establish this next. The following argument has been adapted from unpublished notes written many years ago by Huber [6] and by Shultz [14].

Suppose the n-cube $\mathbf{Q}$ is in central position and has edge 2. Our strategy is to describe an orthonormal basis

$$\mathcal{R} = \{\boldsymbol{r}_i : i = 1, 2, \ldots, n\}$$

for R^n so that the $(n-1)$-cube $\mathbf{C}$ of edge 2 centered at the origin and with edges parallel to the vectors $\boldsymbol{r}_1, \ldots, \boldsymbol{r}_{n-1}$ of $\mathcal{R}$ lies in the interior of $\mathbf{Q}$, where it may be enlarged and remain inside $\mathbf{Q}$. Then the desired result follows from Theorem 2.

Write $x_1, x_2, \ldots, x_n$ for the coordinates of R^n and $\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n$ for the corresponding unit vectors.

The case $n = 3$.

The result for $n = 3$ follows from Theorem 2 and the fact that the largest square in a cube of edge 2 has edge $\frac{3}{2}\sqrt{2}$ (see [4, p. 173]). Other arguments are possible, see [9] for an example. But to illustrate our reasoning in general, we argue the case $n = 3$ in detail.

Let $\mathbf{Q}$ be the cube with vertices $(\pm 1, \pm 1, \pm 1)$. The idea is to tip the cross-section square $\mathbf{C}_0$ in the plane $x_3 = 0$ about the x_2-axis by moving $\boldsymbol{e}_1$ through an angle ϑ_1 toward $\boldsymbol{e}_3$, leaving all four vertices of $\mathbf{C}_0$ on the faces $x_2 = \pm 1$ but moving them all off the faces $x_1 = \pm 1$. Then tipping the resulting square by moving $\boldsymbol{e}_2$ toward $\boldsymbol{e}_3$ through a suitable angle ϑ_2 moves all four vertices inside $\mathbf{Q}$.

Let ϑ_1 and ϑ_2 be acute angles, and shorten the notation by writing

$$\begin{aligned} s_1 &= \sin\vartheta_1 & c_1 &= \cos\vartheta_1 \\ s_2 &= \sin\vartheta_2 & c_2 &= \cos\vartheta_2 \end{aligned}$$

The first rotation is performed by the rotation matrix

$$\begin{bmatrix} c_1 & 0 & -s_1 \\ 0 & 1 & 0 \\ s_1 & 0 & c_1 \end{bmatrix},$$

the second by the rotation matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{bmatrix},$$

and the desired action is accomplished by their product

$$\Phi_3 = [\varphi_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{bmatrix} \begin{bmatrix} c_1 & 0 & -s_1 \\ 0 & 1 & 0 \\ s_1 & 0 & c_1 \end{bmatrix} = \begin{bmatrix} c_1 & 0 & -s_1 \\ -s_1 s_2 & c_2 & -c_1 s_2 \\ c_2 s_1 & s_2 & c_1 c_2 \end{bmatrix}.$$

This matrix is orthonormal, and its rows form the desired basis $\mathcal{R}$. In particular, the first two row vectors

$$\begin{aligned}\boldsymbol{r}_1 &= c_1\boldsymbol{e}_1 - s_1\boldsymbol{e}_3,\\ \boldsymbol{r}_2 &= -s_1s_2\boldsymbol{e}_1 + c_2\boldsymbol{e}_2 - s_2c_1\boldsymbol{e}_3,\end{aligned}$$

are orthogonal and of unit length.

Let $\mathbf{C}$ be the square of side 2 centered at the origin with sides parallel to $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. With a glance at the general case to come, we take

$$\begin{aligned}\vartheta_1 &= \arcsin \tfrac{1}{6}\sqrt{2}\\ \vartheta_2 &= \tfrac{1}{4}\pi.\end{aligned}$$

The vertices of $\mathbf{C}$, the endpoints of the vectors $\pm\boldsymbol{r}_1 \pm \boldsymbol{r}_2$, are the columns of the product

$$\begin{bmatrix} c_1 & 0 & -s_1 \\ -s_1s_2 & c_2 & -c_1s_2 \\ c_2s_1 & s_2 & c_1c_2 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.97 & -0.97 & -0.97 & 0.97 \\ 0.54 & 0.87 & -0.54 & -0.87 \\ 0.87 & 0.54 & -0.87 & -0.54 \end{bmatrix},$$

and $\mathbf{C}$ clearly lies inside $\mathbf{Q}$. Hence $\mathbf{C}$ can be enlarged to a square inside $\mathbf{Q}$ whose side is greater than 2, and the desired result follows from Theorem 2.

Alternatively, because the corners of $\mathbf{C_0}$ are $(\pm1, \pm1, 0)$, we could establish the same result by showing that $\rho_i = |\varphi_{i1}| + |\varphi_{i2}| < 1$ for each i. We find that

$$\begin{aligned}\rho_1 &= |c_1| < 0.98\\ \rho_2 &= |s_1s_2| + |c_2| < 0.88\\ \rho_3 &= |s_1c_2| + |s_2| < 0.88,\end{aligned}$$

so that $\mathbf{C}$ lies in the box $B = [-\rho_1, \rho_1] \times [-\rho_2, \rho_2] \times [-\rho_3, \rho_3]$, which fits inside $\mathbf{Q}$, and consequently $\mathbf{C}$ also fits inside $\mathbf{Q}$.

The general case $n \geq 3$.

Here we give a proof of the general result.

Theorem 3 The n-cube is Rupert for each $n \geq 3$.

Proof. The argument parallels and generalizes that given for $n = 3$. Let $\mathbf{Q}$ be an n-cube with edge 2 in central position, and suppose ϑ_1, $\vartheta_2, \ldots, \vartheta_{n-1}$ are acute angles. As before, we abbreviate $\sin\vartheta_k$ by s_k and $\cos\vartheta_k$ by c_k. Let $\Phi_n^k(\vartheta_k)$ be the rotation matrix obtained by setting $\varphi_{kk} = c_k$, $\varphi_{kn} = -s_k$, $\varphi_{nk} = s_k$, and $\varphi_{nn} = c_k$ in the identity matrix I_n, leaving the other entries of I_n unchanged. This matrix performs the operation of rotation about the $(n-2)$-dimensional subspace spanned by the basis vectors other than $\boldsymbol{e}_k$ and $\boldsymbol{e}_n$ taking $\boldsymbol{e}_k$ through the angle ϑ_k toward $\boldsymbol{e}_n$. The matrix that executes all the desired rotations is the product

$$\Phi_n = [\varphi_{ij}] = \Phi_n^{n-1}(\vartheta_{n-1})\Phi_n^{n-2}(\vartheta_{n-2})\cdots\Phi_n^1(\vartheta_1).$$

As a product of orthonormal matrices, Φ_n is orthonormal. The desired basis $\mathcal{R}$ is formed by the row vectors of Φ_n.

The first $n-1$ row vectors $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, …, $\boldsymbol{r}_{n-1}$ are pairwise orthogonal, and all have unit length. Let $\mathbf{C}$ be the central $(n-1)$-cube with edges of length 2 parallel to the vectors $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ …, $\boldsymbol{r}_{n-1}$. The 2^{n-1} vertices of $\mathbf{C}$ are the endpoints of the vectors $\pm\boldsymbol{r}_1 \pm \boldsymbol{r}_2 \pm \cdots \pm \boldsymbol{r}_{n-1}$.

For each i, let

$$\rho_i^n = |\varphi_{i1}| + |\varphi_{i2}| + \cdots + |\varphi_{i,n-1}| \,. \tag{1}$$

To show that $\mathbf{C}$ lies inside $\mathbf{Q}$, we choose the angles so that $\rho_j^n < 1$ for each j. Then the box

$$\mathbf{B} = \prod_{i=1}^{n-1} [-\rho_i^n, \rho_i^n],$$

which contains $\mathbf{C}$, lies within $\mathbf{Q}$.

The matrices Φ_5 and Φ_6 exhibit the structure of the rotation matrix Φ_n more clearly than is visible in the case $n = 3$ (above):

$$\Phi_5 = \begin{bmatrix} c_1 & 0 & 0 & 0 & -s_1 \\ -s_1s_2 & c_2 & 0 & 0 & -c_1s_2 \\ -c_2s_1s_3 & -s_2s_3 & c_3 & 0 & -c_1c_2s_3 \\ -c_2c_3s_1s_4 & -c_3s_2s_4 & -s_3s_4 & c_4 & -c_1c_2c_3s_4 \\ c_2c_3c_4s_1 & c_3c_4s_2 & c_4s_3 & s_4 & c_1c_2c_3c_4 \end{bmatrix}$$

$$\Phi_6 = \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & -s_1 \\ -s_1s_2 & c_2 & 0 & 0 & 0 & -c_1s_2 \\ -c_2s_1s_3 & -s_2s_3 & c_3 & 0 & 0 & -c_1c_2s_3 \\ -c_2c_3s_1s_4 & -c_3s_2s_4 & -s_3s_4 & c_4 & 0 & -c_1c_2c_3s_4 \\ -c_2c_3c_4s_1s_5 & -c_3c_4s_2s_5 & -c_4s_3s_5 & -s_4s_5 & c_5 & -c_1c_2c_3c_4s_5 \\ c_2c_3c_4c_5s_1 & c_3c_4c_5s_2 & c_4c_5s_3 & c_5s_4 & s_5 & c_1c_2c_3c_4c_5 \end{bmatrix}.$$

The rotation matrix $\Phi_n = [\varphi_{ij}]$ in general has this same structure except in the last row and the last column, for which:

$$\varphi_{ij} = \begin{cases} (-1)^n s_j \prod_{k=j+1}^{n-1} c_k & \text{for} \quad i = 1,\, 2,\, \ldots,\, n-1, \text{ and } j = n \\ (-1)^{n+1} s_j \prod_{k=j+1}^{n-1} c_k & \text{for} \quad i = n, \text{ and } j = 1,\, 2,\, \ldots,\, n-1 \\ \prod_{k=1}^{n-1} c_k & \text{for} \quad i = n, \text{ and } j = n. \end{cases} \quad .$$

The row sums ρ_i^n for $i < n$ (1) have the following form:

$$\begin{array}{ll} i = 1 & c_1 \\ i = 2 & c_2 + s_1 s_2 \\ i = 3 & c_3 + s_2 s_3 + s_1 c_2 s_3 \\ i = 4 & c_4 + s_3 s_4 + s_2 c_3 s_4 + s_1 c_2 c_3 s_4 \\ i = 5 & c_5 + s_4 s_5 + s_3 c_4 s_5 + s_2 c_3 c_4 s_5 + s_1 c_2 c_3 c_4 s_5 \\ i = 6 & c_6 + s_5 s_6 + s_4 c_5 s_6 + s_3 c_4 c_5 s_6 + s_2 c_3 c_4 c_5 s_6 + s_1 c_2 c_3 c_4 c_5 s_6 \\ \vdots & \qquad\qquad\qquad\qquad\qquad \vdots \end{array}$$

For $i = n$ each summand of ρ_n^n has some s_k as a factor. The last term in the nth column is $\varphi_{nn} = c_1 c_2 \cdots c_{n-1}$.

We choose the angles as follows.[1] For each $k = 1,\, 2,\, \ldots,\, n-1$,

$$0 < \frac{1}{2}\sqrt{2} \cdot 3^{k+1-n} < 1,$$

and we take

$$\vartheta_k = \arcsin \tfrac{1}{2}\sqrt{2}\, 3^{k+1-n}.$$

In particular, $\vartheta_{n-1} = 45^\circ$.

Because $s_k = \frac{1}{2}\sqrt{2}\, 3^{k+1-n} = 3^{k-1} s_1$, for each $i = 1, 2, \ldots, n-1$,

$$\sum\nolimits_{k=1}^{i-1} s_k = \tfrac{1}{2} s_1 \left(3^{i-1} - 1\right),$$

so that

$$s_i - 2 \sum\nolimits_{k=1}^{i-1} s_k = s_1.$$

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[1] This choice of angles was given in [6]. A quite different algorithm for choosing the angles seriatim appears in [14].

Using the inequality $\cos\vartheta < 1 - \frac{1}{2}\sin^2\vartheta$ (valid for $0 < \vartheta < 2\pi$), we find for $1 \le i \le n-1$ that

$$\begin{aligned}\rho_i^n &< c_i + s_i \sum\nolimits_{k=1}^{i-1} s_k \\ &< 1 - \tfrac{1}{2}s_i^2 + s_i \sum\nolimits_{k=1}^{i-1} s_k \\ &< 1 - \tfrac{1}{2}s_i \left\{ s_i - 2\sum\nolimits_{k=1}^{i-1} s_k \right\} = 1 - \tfrac{1}{2}s_i s_1 < 1 - \tfrac{1}{2}s_1^2 < 1.\end{aligned}$$

But $\rho_n^n = \rho_{n-1}^n < 1$ as well, because $s_{n-1} = c_{n-1}$. Consequently the orthotope **B** fits in the interior of **Q**, and with it, **C**. So **C** may be enlarged within **Q**. This completes the proof. ■

5. A miscellany of comments.

Rupert polytopes. The five Platonic polyhedra in R^3 are all Rupert [11], as are at least eight of the 13 Archimedean solids [1]. A few other polyhedra in R^3 are also known to have the Rupert property, including all tetrahedra and all parallelepipeds [9], [10].

Shultz has recently shown that the duals of the n-cubes, the n-cross-polytopes or n-orthoplexes, are Rupert. Little more appears to be known in R^n for $n > 3$.

DeVicci cube. Gardner [5], [4, p. 172] wondered about the side of the largest cube that fits in a unit tesseract (i.e., the 4-cube). As reported by Huber in his Preface to [14, pp. 1a, 1b], several people responded to Gardner's question with the correct value, but only DeVicci[2] proved that it is maximal. She showed that the largest cube in a unit tesseract has side $\sqrt{x_0}$, where x_0 is the zero of the quartic

$$4x^4 - 28x^3 - 7x^2 + 16x + 16$$

in the range $(1, \frac{4}{3})$, namely

$$x_0 \approx 1.00743. \tag{2}$$

The details of the argument are available in her informal writeup [14]. Finch [3] referred to this value as the DeVicci constant, and the cube has since been called the DeVicci cube.

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[2] Now Kay R. Pechenick DeVicci Shultz.

Largest cube in cube. More generally, for $1 \leq m \leq n$ let $f(m,n)$ be the side of the largest m-cube that fits in a unit n-cube. Beyond the elementary results that $f(1,n) = \sqrt{n}$ and $f(2,3) = \frac{3}{4}\sqrt{2}$ and the value of $f(3,4)$ reported above, it is known only that $f(m,n) > 1$, that

$$f(2,n) = \begin{cases} \sqrt{k} & \text{if } n = 2k \\ \sqrt{k + \frac{1}{8}} & \text{if } n = 2k+1, \end{cases} \tag{3}$$

and that $f(m,n) = \sqrt{n/m}$ when n is a multiple of m (see [14]). Some additional results including some asymptotics are to be reported in [7].

Two conjectures. (1) An n-cube is Rupert, and a tunnel through which it can pass can be built on any $(n-1)$-cube in the unit n-cube whose edge is larger than 1. The edge of the *largest* n-cube that can pass through a unit n-cube is called the *Nieuwland constant* $\nu(n)$ of the n-cube (see [13]). Evidently $\nu(n) \geq f(n-1,n)$, and it seems likely that the equality holds.

(2) In the absence of an example of a convex polyhedron in R^3 that does not have the Rupert property, it has been suggested [11, p. 87] that perhaps *every* convex polyhedron in R^3 is Rupert. A counterexample would be of considerable interest.

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