CHAPTER 5

FORCE AND NEWTON'S LAWS

In Chapters 2 and 4, we studied the motion of a particle. We did not ask what "caused" the motion; we simply described it in terms of the vectors **r**, **v**, *and* **a**. *In this chapter and the next, we discuss the* causes *of motion, a field of study called* dynamics.

The approach to dynamics we consider in this chapter and the next, which is generally known as classical mechanics, *was developed and successfully tested in the 17th and 18th centuries. In our century, new theories (special and general relativity and quantum mechanics) have indicated certain realms far from our ordinary experiences where classical mechanics fails to give predictions that agree with experiment, but these new theories reduce to classical mechanics in the limits of ordinary objects.*

Without reference to special or general relativity or to quantum mechanics, we can build great skyscrapers and study the properties of their construction materials; build airplanes that can carry hundreds of people and fly halfway around the world; and send space probes on complex missions to the comets, the planets, and beyond. This is the stuff of classical mechanics.

5-1 CLASSICAL MECHANICS

We focus our attention on the motion of a particular body. It interacts with the surrounding bodies (its *environment*) so that its velocity changes: an acceleration is produced. Table 1 shows some common accelerated motions and the environment that is mostly responsible for the acceleration. The central problem of classical mechanics is this: (1) We are given a body whose characteristics (mass, volume, electric charge, etc.) we know. (2) We place this body, at a known initial location and with a known initial velocity, in an environment of which we have a complete description. (3) What is the subsequent motion of the body?

In previous chapters, we have treated physical objects as *particles,* that is, as bodies whose internal structures or motions can be ignored and whose parts all move in exactly the same way. In studying the interaction of a body with its environment, we often must consider extended objects whose different parts may interact with the environment in different ways. For example, a worker pushes a heavy crate along a rough surface. The worker pushes on one vertical side of the crate, while the horizontal bottom experiences the retarding effect of friction with the floor. The front surface may even experience air resistance.

Later in the text we treat the mechanics of extended bodies in detail. For the present, we continue to assume that all parts of the body move in the same way, so that we can treat the body as a particle. Under this assumption, it doesn't matter where the environment acts on the body;

TABLE 1 SOME ACCELERATED MOTIONS AND THEIR CAUSES

Object	*Change in Motion*	*Major Cause (Environment)*
Apple	Falls from tree	Gravity (Earth)
Billiard ball	Bounces off another	Other ball, table, gravity (Earth)
Skier	Slides down hill	Gravity (Earth), friction (snow), air resistance
Beam of electrons (in TV set)	Focusing and deflection	Electromagnetic fields (magnets and voltage differences)
Comet Halley	Round trip through solar system	Gravity (Sun)

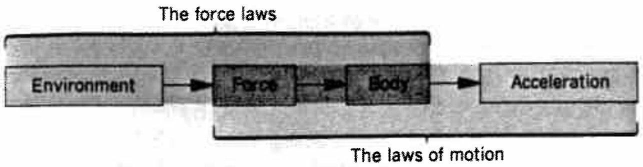

Figure 1 Our program for mechanics. The three boxes on the left suggest that force is an interaction between a body and its environment. The three boxes on the right suggest that a force acting on a body will accelerate it.

our primary concern is with the *net effect* of the environment.

This problem of classical mechanics was solved, at least for a large variety of environments, by Isaac Newton (1642–1727) when he put forward his laws of motion and formulated his law of universal gravitation. The procedure for solving this problem, in terms of our present framework of classical mechanics, is as follows: (1) We introduce the concept of *force* **F** (which we regard for now as a push or a pull), and we define it in terms of the acceleration **a** experienced by a particular standard body. (2) We develop a procedure for assigning a *mass m* to a body so that we may understand the fact that different bodies experience different accelerations in the same environment. (3) Finally, we try to find ways of calculating the forces that act on bodies from the properties of the body and of its environment; that is, we look for *force laws.* Force, which is basically a means of relating the environment to the motion of the body, appears both in the laws of motion (which tell us what acceleration a given body will experience under the action of a given force) and in the force laws (which tell us how to calculate the force that will act on a given body in a given environment). The laws of motion and the force laws, taken together, constitute the laws of mechanics, as Fig. 1 suggests.

This program of mechanics cannot be tested piecemeal. We must view it as a unit and we shall judge it to be successful if we can say "yes" to these two questions. (1) Does the program yield results that agree with experiment? (2) Are the force laws simple in form? It is the crowning glory of Newtonian mechanics that we can indeed answer each of these questions in the affirmative.

5-2 NEWTON'S FIRST LAW

For centuries the problem of motion and its causes was a central theme of natural philosophy, an early name for what we now call physics. It was not until the time of Galileo and Newton, however, that dramatic progress was made. Isaac Newton, born in England in the year of Galileo's death, is the principal architect of classical mechanics. He carried to full fruition the ideas of Galileo and others who preceded him. His three laws of motion were first presented (in 1686) in his *Philosophiae Naturalis Principia Mathematica,* usually called the *Principia.*

Before Galileo's time most philosophers thought that some influence or "force" was needed to keep a body moving. They thought that a body was in its "natural state" when it was at rest. For a body to move in a straight line at constant speed, for example, they believed that some external agent had to continually propel it; otherwise it would "naturally" stop moving.

If we wanted to test these ideas experimentally, we would first have to find a way to free a body from all influences of its environment or from all forces. This is hard to do, but in certain cases we can make the forces very small. If we study the motion as we make the forces smaller and smaller, we shall have some idea of what the motion would be like if the external forces were truly zero.

Let us place our test body, say a block, on a rigid horizontal plane. If we let the block slide along this plane, we note that it gradually slows down and stops. This observation was used, in fact, to support the idea that motion stopped when the external force, in this case the hand initially pushing the block, was removed. We can argue against this idea, however, by reasoning as follows. Let us repeat our experiment, now using a smoother block and a smoother plane and providing a lubricant. We note that the velocity decreases more slowly than before. Let us use still smoother blocks and surfaces and better lubricants. We find that the block decreases in velocity at a slower and slower rate and travels farther each time before coming to rest. You may have experimented with an air track, on which objects can be made to float on a film of air; such a device comes close to the limit of no friction, as even a slight tap on one of the gliders can send it moving along the track at a slow and almost constant speed. We can now extrapolate and say that if all friction could be eliminated, the body would continue indefinitely in a straight line with constant speed. An external force is needed to set the body in motion, but *no external force is needed to keep a body moving with constant velocity.*

It is difficult to find a situation in which no external force acts on a body. The force of gravity will act on an object on or near the Earth, and resistive forces such as friction or air resistance oppose motion on the ground or in the air. Fortunately, we need not go to the vacuum of distant space to study motion free of external force, because, as far as the overall translational motion of a body is concerned, *there is no distinction between a body on which no external force acts and a body on which the sum or resultant of all the external forces is zero.* We usually refer to the resultant of all the forces acting on a body as the "net" force. For example, the push of our hand on the sliding block can exert a force that counteracts the force of friction on the block, and an upward force of the horizontal plane counteracts the force of gravity. The net force on the block can then be zero, and the block can move with constant velocity.

This principle was adopted by Newton as the first of his three laws of motion:

Consider a body on which no net force acts. If the body is at rest, it will remain at rest. If the body is moving with constant velocity, it will continue to do so.

Newton's first law is really a statement about reference frames. In general, the acceleration of a body depends on the reference frame relative to which it is measured. However, the laws of classical mechanics are valid only in a certain set of reference frames, namely, those from which all observers would measure the *same* acceleration for a moving body. Newton's first law helps us to identify this family of reference frames if we express it as follows:

If the net force acting on a body is zero, then it is possible to find a set of reference frames in which that body has no acceleration.

The tendency of a body to remain at rest or in uniform linear motion is called *inertia*, and Newton's first law is often called the *law of inertia*. The reference frames to which it applies are called *inertial frames*, as we discussed in Section 4-6. You will recall from that discussion that observers in different inertial reference frames (moving with constant velocity relative to one another) all measure the same value of the acceleration. Thus there is not just one frame in which the acceleration happens to be zero; there is a set of all inertial frames in which the acceleration is zero.

To test whether a particular frame of reference is an inertial frame, we place a test body at rest in the frame and ascertain that no net force acts on it. If the body does not remain at rest, the frame is not an inertial frame. Similarly, we can put the body (again subject to no net force) in motion at constant velocity; if its velocity changes, either in magnitude or direction, the frame is not an inertial frame. A frame in which these tests are everywhere passed is an inertial frame. Once we have found one inertial frame, it is easy to find many more, because a frame of reference that moves at constant velocity relative to one inertial frame is also an inertial frame.

In this book we almost always apply the laws of classical mechanics from the point of view of an observer in an inertial frame. Occasionally, we discuss problems involving observers in noninertial reference frames, such as an accelerating car, a rotating merry-go-round, or an orbiting satellite. Even though the Earth is rotating, a reference frame attached to the Earth can be considered to be approximately an inertial reference frame for most practical purposes. For large-scale applications, such as analyzing the flight of ballistic missiles or studying wind and ocean currents, the noninertial character of the rotating Earth becomes important.

Notice that there is no distinction in the first law between a body at rest and one moving with a constant velocity. Both motions are "natural" if the net force acting on the body is zero. This becomes clear when a body at rest in one inertial frame is viewed from a second inertial frame, that is, a frame moving with constant velocity with respect to the first. An observer in the first frame finds the body to be at rest; an observer in the second frame finds the same body to be moving with constant velocity. Both observers find the body to have no acceleration, that is, no change in velocity, and both may conclude from the first law that no net force acts on the body.

If there is a net interaction between the body and objects present in the environment, the effect may be to change the "natural" state of the body's motion. To investigate this, we must now examine carefully the concept of force.

5-3 FORCE

We develop our concept of force by defining it operationally. In everyday language, a force is a push or a pull. To measure such forces quantitatively, we express them in terms of the acceleration that a given standard body experiences in response to that force.

As a standard body we find it convenient to use (or rather to imagine that we use!) the standard kilogram (see Fig. 5 of Chapter 1). This body has been assigned, by definition, a mass m_0 of exactly 1 kg. Later we shall describe how masses are assigned to other bodies.

For an environment that exerts a force, we place the standard body on a horizontal table having negligible friction and we attach a spring to it. We hold the other end of the spring in our hand, as in Fig. 2*a*. Now we pull the spring horizontally to the right so that by trial and error we are able to give the standard body a measured constant acceleration of exactly 1 m/s². We then declare, as a matter of definition, that the spring (which is the significant body in the environment) is exerting on the standard kilogram a constant force whose magnitude we call "1 newton" (abbreviated 1 N). We note that, in imparting this

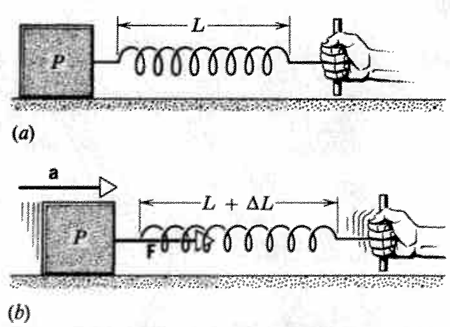

Figure 2 (*a*) A "particle" *P* (the standard kilogram) at rest on a horizontal frictionless surface. (*b*) The body is accelerated by pulling the spring to the right.

force, the spring is stretched an amount ΔL beyond its normal unextended length L, as Fig. 2*b* shows.

We can repeat the experiment, either stretching the spring more or using a stiffer spring, so that we measure an acceleration of 2 m/s^2 for the standard body. We now declare that the spring is exerting a force of 2 N on the standard body. In general, if we observe this particular standard body to have an acceleration a in a particular environment, we then say that the environment is exerting a force F on the standard 1-kg body, where F (in newtons) is numerically equal to a (in m/s^2).

Now let us see whether force, as we have defined it, is a *vector* quantity. In Fig. 2*b* we assigned a magnitude to the force F, and it is a simple matter to assign a direction to it as well, namely, the direction of the acceleration that the force produces. However, to be a vector it is not enough for a quantity to have magnitude and direction; it must also obey the laws of vector addition described in Chapter 3. We can learn only from experiment whether forces, as we defined them, do indeed obey these laws.

Let us arrange to exert a force of 4 N along the x axis and a force of 3 N along the y axis. We apply these forces first separately and then simultaneously to the standard body placed, as before, on a horizontal, frictionless surface. What will be the acceleration of the standard body? We would find by experiment that the 4-N force in the x direction produced an acceleration of 4 m/s^2 in the x direction, and that the 3-N force in the y direction produced an acceleration of 3 m/s^2 in the y direction (Fig. 3*a*). When the forces are applied simultaneously, as shown in Fig. 3*b*, we find that the acceleration is 5 m/s^2 directed along a line that makes an angle of 37° with the x axis. This is the same acceleration that would be produced if the standard body were experiencing a force of 5 N in that direction. This same result can be obtained if we first add the 4-N and 3-N forces vectorially (Fig. 3*c*) to a 5-N resultant directed at 37° from the x axis, and then apply that single 5-N net force to the body. Experiments of this kind show conclusively that forces are vectors: they have magnitude and direction, *and* they add according to the vector addition law.

Note that we have two methods of analysis available, which should produce identical results: (1) Find the acceleration produced by each separate force, and add the resultant accelerations vectorially. (2) Add the forces vectorially to a single resultant, and then find the acceleration when that single net force is applied to the body.

5-4 MASS

In Section 5-3 we considered only the accelerations given to one particular body, the standard kilogram. We were able thereby to define forces quantitatively. What effect would these forces have on other bodies? Because our standard body was chosen arbitrarily in the first place, we know that for any given body the acceleration will be directly proportional to the force applied. The significant question remaining then is: What effect will the *same force* have on *different bodies*?

Everyday experience gives us a qualitative answer. The same force will produce different accelerations on different bodies. A baseball will be accelerated more by a given force than will an automobile. In order to obtain a quantitative answer to this question, we need a method to measure mass, *the property of a body which determines its resistance to a change in its motion.*

Let us attach a spring to our standard body (the standard kilogram, to which we have arbitrarily assigned a mass $m_0 = 1$ kg, exactly) and arrange to give it an acceleration a_0 of, say, 2.00 m/s^2, using the method of Fig. 2*b*. Let us measure carefully the extension ΔL of the spring associated with the force that the spring is exerting on the block.

We now attach two identical standard bodies to the spring and apply the same force as before (that is, we pull on the two bodies until the spring stretches by the same

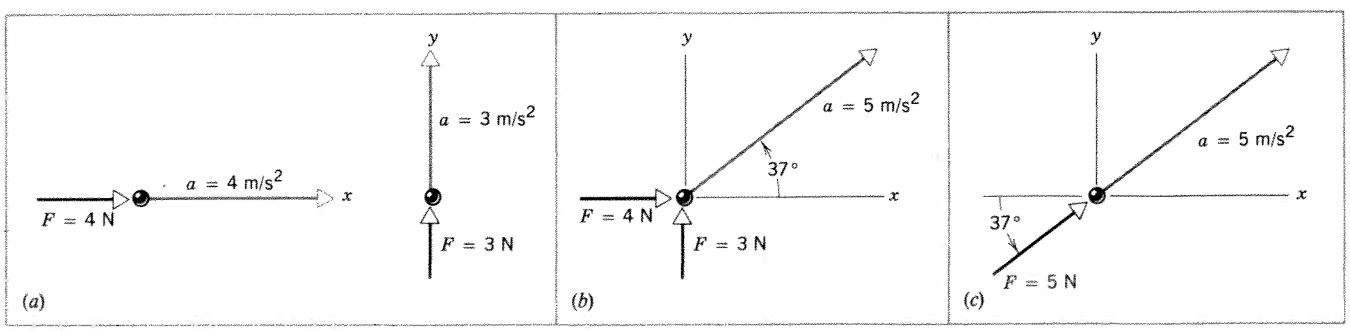

Figure 3 (*a*) A 4-N force in the x direction gives an acceleration of 4 m/s^2 in the x direction, and a 3-N force in the y direction gives an acceleration of 3 m/s^2 in the y direction. (*b*) When the forces are applied simultaneously, the resultant acceleration is 5 m/s^2 in the direction shown. (*c*) The same acceleration can be produced by a single 5-N force in the direction shown.

amount ΔL). We measure the acceleration of the two bodies, and obtain the value of 1.00 m/s². If we used three identical standard bodies and applied the same force, we would obtain an acceleration of 0.667 m/s².

From these observations, it appears that, for a given force, the greater the mass, the smaller the acceleration. More precisely, we conclude from many such experiments that *the acceleration produced by a given force is inversely proportional to the mass being accelerated.* Another way to put this is: *the mass of a body is inversely proportional to the acceleration it receives from the application of a given force.* The mass of a body can thus be regarded as *a quantitative measure of the resistance of a body to acceleration by a given force.*

This observation gives us a direct way to compare the masses of different bodies: we simply compare the accelerations we measure from the application of a given force to each body. The ratio of the masses of the two bodies is then the same as the *inverse* ratio of the accelerations given to these bodies by that force, or

$$\frac{m_1}{m_0} = \frac{a_0}{a_1} \quad \text{(same force } F \text{ acting).}$$

Here we are comparing the acceleration a_1 of the body of unknown mass m_1 with the acceleration a_0 imparted to the standard body of mass m_0.

For example, suppose as above we use a force that gives an acceleration of 2.00 m/s² to the standard body. We apply the same force (by stretching the spring by the same amount ΔL) to a body of unknown mass m_1, and we measure an acceleration a_1 of, say, 0.50 m/s². We can then solve for the unknown mass, which gives

$$m_1 = m_0\left(\frac{a_0}{a_1}\right) = (1.00\text{ kg})\left(\frac{2.00\text{ m/s}^2}{0.50\text{ m/s}^2}\right) = 4.00\text{ kg}.$$

The second body, which has only one-fourth the acceleration of the first body when the same force acts on it, has four times the mass of the first body. This illustrates the inverse relationship between mass and acceleration for a given force.

Let us now repeat the preceding experiment on the same two bodies using a common force F' different from that used above. This force will give the standard body an acceleration of a_0' and the unknown body an acceleration of a_1'. From our measurement we would find that the ratio of the accelerations, a_0'/a_1', is the same as in the previous experiment, or

$$\frac{m_1}{m_0} = \frac{a_0}{a_1} = \frac{a_0'}{a_1'}.$$

For example, we apply a greater force so that the extension of the spring is $1.5\Delta L$. We would then find that the standard mass m_0 is accelerated to 3.00 m/s² and the unknown mass m_1 is accelerated to 0.75 m/s². We would deduce the unknown mass to be

$$m_1 = m_0\left(\frac{a_0'}{a_1'}\right) = (1.00\text{ kg})\left(\frac{3.00\text{ m/s}^2}{0.75\text{ m/s}^2}\right) = 4.00\text{ kg}.$$

We obtain the same value for the unknown mass m_1, no matter what the value of the common force. The mass ratio m_1/m_0 is independent of the common force used; the mass is a fundamental property of the object, unrelated to the value of the force used to compare the unknown mass to the standard mass. In effect, this procedure allows us to measure mass by comparison with the standard kilogram.

We can extend this procedure to a direct comparison of the masses of any two bodies. For example, let us first use our previous procedure to compare a second arbitrary body with the standard body, and thus determine its mass, say m_2. We can now compare the two arbitrary bodies, m_2 and m_1, directly, obtaining accelerations a_2'' and a_1'' when the same force F'' is applied. The mass ratio, defined as usual from

$$\frac{m_2}{m_1} = \frac{a_1''}{a_2''} \quad \text{(same force acting),}$$

turns out to have the same value that we obtain by using the masses m_2 and m_1 previously determined by direct comparison with the standard.

We can show, in still another experiment of this type, that if objects of mass m_1 and m_2 are fastened together, they behave mechanically as a single object of mass $(m_1 + m_2)$. In other words, *masses add like (and are) scalar quantities.*

One practical example of the use of this technique—assigning masses by comparison of the relative accelerations produced by a given force—is in the precise measurement of the masses of atoms. The force in this case is a magnetic deflecting force and the acceleration is centripetal, but the principle is exactly the same. For a common magnetic force acting on two atoms, the ratio of their masses is equal to the inverse ratio of their accelerations. Measuring the deflection, as in the mass spectrometer shown in Fig. 6 of Chapter 1, permits precise mass ratios to be measured, and defining ^{12}C as the standard then permits precise values of masses, such as those shown in Table 6 of Chapter 1, to be obtained.

5-5 NEWTON'S SECOND LAW

We can now summarize all the previously described experiments and definitions in one equation, the fundamental equation of classical mechanics,

$$\sum \mathbf{F} = m\mathbf{a}. \tag{1}$$

In this equation $\Sigma\,\mathbf{F}$ is the (vector) *sum* of *all* the forces acting *on* the body, m is the mass of the body, and $\mathbf{a}$ is its (vector) acceleration. We shall usually refer to $\Sigma\,\mathbf{F}$ as the *resultant* force or *net* force.

Equation 1 is a statement of Newton's second law. If we write it in the form $\mathbf{a} = (\Sigma\,\mathbf{F})/m$, we can easily see that the acceleration of the body is in magnitude directly proportional to the resultant force acting on it and in direction parallel to this force. We also see that the acceleration, for a given force, is inversely proportional to the mass of the body.

Note that the first law of motion appears to be contained in the second law as a special case, for if $\Sigma \mathbf{F} = 0$, then $\mathbf{a} = 0$. In other words, if the resultant force on a body is zero, the acceleration of the body is zero and the body moves with constant velocity, as stated by the first law. However, the first law has an independent and important role in defining inertial reference frames. Without that definition, we would not be able to choose the frames of reference in which to apply the second law. We therefore need *both laws* for a complete system of mechanics.

Equation 1 is a vector equation. As in the case of all vector equations, we can write this single vector equation as three scalar equations,

$$\sum F_x = ma_x, \quad \sum F_y = ma_y, \quad \text{and} \quad \sum F_z = ma_z, \quad (2)$$

relating the x, y, and z components of the resultant force (ΣF_x, ΣF_y, and ΣF_z) to the x, y, and z components of acceleration (a_x, a_y, and a_z) for the mass m. It should be emphasized that ΣF_x is the *algebraic* sum of the x components of *all* the forces, ΣF_y is the *algebraic* sum of the y components of *all* the forces, and ΣF_z is the *algebraic* sum of the z components of *all* the forces acting on m. In taking the algebraic sum, the signs of the components (that is, the relative directions of the forces) must be taken into account.

In analyzing situations using Newton's second law, it is helpful to draw a diagram showing the body in question as a particle and showing all forces as vectors that act on the particle. Such a drawing is called a *free-body diagram* and is an essential first step both in the analysis of a problem and in the visualization of the physical situation.

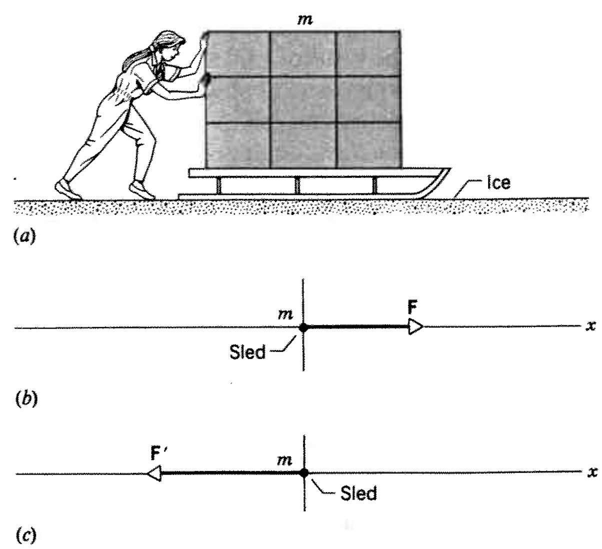

Figure 4 Sample Problems 1 and 2. (*a*) A student pushing a loaded sled over a frictionless surface. (*b*) A free-body diagram, showing the sled as a "particle" and the force acting on it. (*c*) A second free-body diagram, showing the force acting when the student pushes in the opposite direction.

Sample Problem 1 A student pushes a loaded sled whose mass m is 240 kg for a distance d of 2.3 m over the frictionless surface of a frozen lake. She exerts a constant horizontal force F of 130 N (= 29 lb) as she does so; see Fig. 4*a*. If the sled starts from rest, what is its final velocity?

Solution As Fig. 4*b* shows, we lay out a horizontal x axis, we take the direction of increasing x to be to the right, and we treat the sled as a particle. Figure 4*b* is a *partial* free-body diagram. In drawing free-body diagrams, it is important always to include *all* forces that act on the particle, but here we have omitted two vertical forces that will be discussed later in this chapter and that do not affect our solution. We assume that the force F exerted by the student is the only horizontal force acting on the sled. We can then find the acceleration of the sled from Newton's second law, or

$$a = \frac{F}{m} = \frac{130\ \text{N}}{240\ \text{kg}} = 0.54\ \text{m/s}^2.$$

Because the acceleration is constant, we can use Eq. 20 of Chapter 2 [$v^2 = v_0^2 + 2a(x - x_0)$] to find the final velocity. Putting $v_0 = 0$ and $x - x_0 = d$ and solving for v, we obtain

$$v = \sqrt{2ad} = \sqrt{(2)(0.54\ \text{m/s}^2)(2.3\ \text{m})} = 1.6\ \text{m/s}.$$

The force, acceleration, displacement, and final velocity of the sled are all positive, which means that they all point to the right in Fig. 4*b*.

Note that to continue applying the constant force, the student would have to run faster and faster to keep up with the accelerating sled. Eventually, the velocity of the sled would exceed the fastest speed at which the student could run, and thereafter the student would no longer be able to apply a force to the sled. The sled would then continue (in the absence of friction) to coast at constant velocity.

Sample Problem 2 The student in Sample Problem 1 wants to reverse the direction of the velocity of the sled in 4.5 s. With what constant force must she push on the sled to do so?

Solution Let us find the (constant) acceleration, using Eq. 15 of Chapter 2 ($v = v_0 + at$). Solving for a gives

$$a = \frac{v - v_0}{t} = \frac{(-1.6\ \text{m/s}) - (1.6\ \text{m/s})}{4.5\ \text{s}} = -0.71\ \text{m/s}^2.$$

This is larger in magnitude than the acceleration in Sample Problem 1 (0.54 m/s²) so it stands to reason that the student must push harder this time. We find this (constant) force F' from

$$F' = ma = (240\ \text{kg})(-0.71\ \text{m/s}^2) = -170\ \text{N}\ (= -38\ \text{lb}).$$

The negative sign shows that the student is pushing the sled in the direction of decreasing x, that is, to the left as shown in the free-body diagram of Fig. 4c.

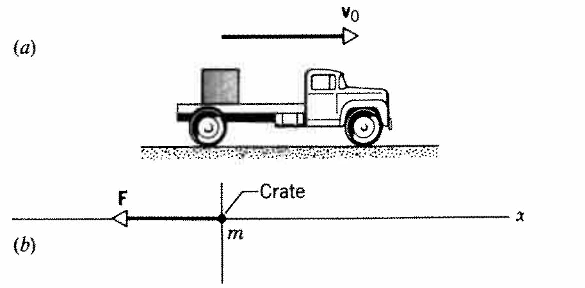

Figure 5 Sample Problem 3. (*a*) A crate on a truck that is slowing down. (*b*) The free-body diagram of the crate.

Sample Problem 3 A crate whose mass m is 360 kg rests on the bed of a truck that is moving at a speed v_0 of 120 km/h, as in Fig. 5*a*. The driver applies the brakes and slows to a speed v of 62 km/h in 17 s. What force (assumed constant) acts on the crate during this time? Assume that the crate does not slide on the truck bed.

Solution We first find the (constant) acceleration of the crate. Solving Eq. 15 of Chapter 2 ($v = v_0 + at$) for a yields

$$a = \frac{v - v_0}{t} = \frac{(62\ \text{km/h}) - (120\ \text{km/h})}{17\ \text{s}}$$
$$= \left(-3.41\ \frac{\text{km}}{\text{h}\cdot\text{s}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right) = -0.95\ \text{m/s}^2.$$

Because we have taken the positive sense of the horizontal direction to the right, the acceleration vector must point to the left.

The force on the crate follows from Newton's second law:

$$F = ma$$
$$= (360\ \text{kg})(-0.95\ \text{m/s}^2) = -340\ \text{N}.$$

This force acts in the same direction as the acceleration, namely, to the left in Fig. 5*b*. The force must be supplied by an external agent, such as the straps or other mechanical means used to secure the crate to the truck bed. If the crate is not secured, then friction between the crate and the truck bed must supply the required force. If there is not enough friction to provide a force of 340 N, the crate will slide on the truck bed because, as measured by a ground-based observer, it will slow down less rapidly than the truck.

5-6 NEWTON'S THIRD LAW

Forces acting on a body result from other bodies that make up its environment. If we examine the forces acting on a second body, one that was formerly considered part of the environment, then the first body is part of the environment of the second body and is in part responsible for the forces acting on the second body. Any single force is therefore part of the mutual interaction between *two* bodies. We find by experiment that when one body exerts a force on a second body, the second body always exerts a

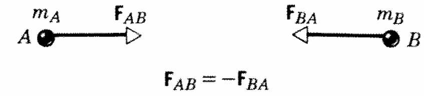

Figure 6 Newton's third law. Body A exerts a force $\mathbf{F}_{BA}$ on body B. Body B must then exert a force $\mathbf{F}_{AB}$ on body A, and $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$.

force on the first. Furthermore, we find these forces *always* to be equal in magnitude but opposite in direction. A single isolated force is therefore an impossibility.

Suppose this were not true. Consider two isolated bodies A and B, and suppose that body A exerts a force on body B, while no force is exerted by B on A. The total force on the combination $A + B$ is nonzero, and the combined mass must accelerate. If such a situation could occur, then we would have a limitless source of energy that could propel $A + B$ through space at no cost: sailboats could sail by passengers blowing on the sails, and spaceships could be accelerated by astronauts pushing on the walls. The impossibility of these actions is a consequence of Newton's third law.

We arbitrarily label one of the forces of the mutual interaction between two bodies as the "action" force, and the other is called the "reaction" force. Newton's third law can then be stated in traditional form:

To every action there is an equal and opposite reaction.

A more modern version of the third law concerns the mutual force exerted by two bodies on one another:

When two bodies exert mutual forces on one another, the two forces are always equal in magnitude and opposite in direction.

Formally (see Fig. 6) let body A exert a force $\mathbf{F}_{BA}$ on body B; experiment then shows that body B exerts a force $\mathbf{F}_{AB}$ on body A. (Note the order of subscripts; the force is exerted *on* the body represented by the first subscript *by* the body represented by the second.) In terms of a vector equation,

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}. \qquad (3)$$

It is important to remember that the action and reaction forces always act on *different* bodies, as the differing first subscripts remind us. If they acted on the same body, there would be no net force on that body and no accelerated motion.

When a bat strikes a baseball, the bat exerts a force on the ball (the action), and the ball exerts an equal and opposite force on the bat. When a soccer player kicks the ball, the foot exerts a force on the ball (the action), and the ball exerts an opposite reaction force on the foot. When you push a stalled car, you can feel the car pushing back on you. In each case the action and reaction forces act on different bodies. If our goal were to study the dynamics of one body—the baseball, for instance—only one force of the action-reaction pair would be considered; the other is felt by a different body and would be considered only if we were studying the dynamics of that body.

The following examples illustrate applications of the third law.

1. *An orbiting satellite.* Figure 7 shows a satellite orbiting the Earth. The only force that acts on it is $\mathbf{F}_{SE}$, the force exerted *on* the satellite *by* the gravitational pull of the Earth. Where is the corresponding reaction force? It is $\mathbf{F}_{ES}$, the force acting on the Earth owing to the gravitational pull of the satellite.

You may think that the tiny satellite cannot exert much of a gravitational pull on the Earth but it does, exactly as Newton's third law requires. That is, considering magnitudes only, $F_{ES} = F_{SE}$. (Recall that the magnitude of any vector quantity is always positive.) The force $\mathbf{F}_{ES}$ causes the Earth to accelerate, but, because of the Earth's large mass, its acceleration is so small that it cannot easily be detected.

2. *A book resting on a table.* Figure 8*a* shows a book resting on a table. The Earth pulls downward on the book with a force $\mathbf{F}_{BE}$. The book does not accelerate because this force is canceled by an equal and opposite contact force $\mathbf{F}_{BT}$ exerted on the book by the table.

Even though $\mathbf{F}_{BE}$ and $\mathbf{F}_{BT}$ are equal in magnitude and oppositely directed, they do *not* form an action-reaction pair. Why not? *Because they act on the same body—the book.* They cancel each other and thus account for the fact that the book is not accelerating.

Each of these forces must then have a corresponding reaction force somewhere. Where are they? The reaction to $\mathbf{F}_{BE}$ is $\mathbf{F}_{EB}$, the (gravitational) force with which the book attracts the Earth. We show this action-reaction pair in Fig. 8*b*.

Figure 8*c* shows the reaction force to $\mathbf{F}_{BT}$. It is $\mathbf{F}_{TB}$, the contact force on the table owing to the book. The action-reaction pairs involving the book in this problem, and the bodies on which they act, are

first pair: $\mathbf{F}_{BE} = -\mathbf{F}_{EB}$ (book and Earth)

and

second pair: $\mathbf{F}_{BT} = -\mathbf{F}_{TB}$ (book and table).

3. *Pushing a row of crates.* Figure 9 shows a worker W pushing two crates, each of which rests on a wheeled cart that can roll with negligible friction. The worker exerts a force $\mathbf{F}_{1W}$ on crate 1, which in turn pushes back on the worker with a reaction force $\mathbf{F}_{W1}$. Crate 1 pushes on crate 2 with a force $\mathbf{F}_{21}$, and crate 2 pushes back on crate 1 with a force $\mathbf{F}_{12}$. (Note that the worker exerts no force on crate 2 directly.) To move forward, the worker must push backward against the ground. The worker exerts a force $\mathbf{F}_{GW}$ on the ground, and the rection force of the ground on the worker, $\mathbf{F}_{WG}$, pushes the worker forward. The figure shows three action-reaction pairs:

$$\mathbf{F}_{21} = -\mathbf{F}_{12} \quad \text{(crate 1 and crate 2)},$$

$$\mathbf{F}_{1W} = -\mathbf{F}_{W1} \quad \text{(worker and crate 1)},$$

$$\mathbf{F}_{WG} = -\mathbf{F}_{GW} \quad \text{(worker and ground)}.$$

The acceleration of crate 2 is determined, according to Newton's second law, by the net force applied to it:

$$F_{21} = m_2 a_2.$$

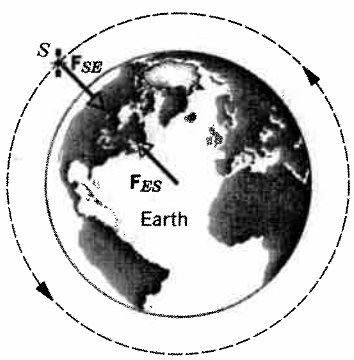

Figure 7 A satellite in Earth orbit. The forces shown are an action-reaction pair. Note that they act on different bodies.

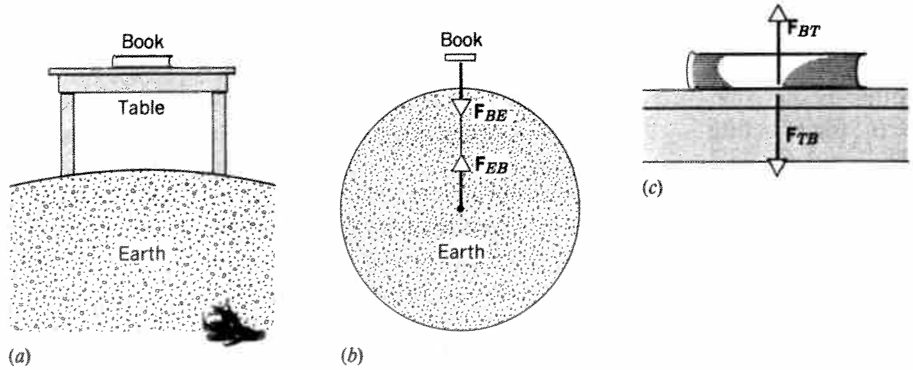

Figure 8 (*a*) A book rests on a table, which in turn rests on the Earth. (*b*) The book and the Earth exert gravitational forces on each other, forming an action-reaction pair. (*c*) The table and book exert action-reaction contact forces on each other.

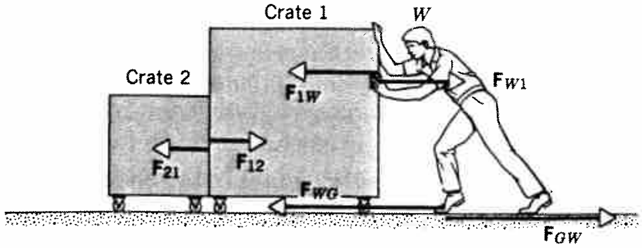

Figure 9 A worker pushes against crate 1, which in turn pushes on crate 2. The crates are on wheels that move freely, so there is no friction between the crates and the ground.

The net force on crate 1 determines its acceleration,

$$F_{1W} - F_{12} = m_1 a_1,$$

where we have written the vector sum of the forces as the difference in their magnitudes, because they act on crate 1 in opposite directions. If the two crates remain in contact, their accelerations must be equal. Letting a represent the common acceleration and adding the equations gives

$$F_{1W} = (m_1 + m_2)a.$$

This same equation would result if we considered crates 1 and 2 to be a single object of mass $m_1 + m_2$. The net external force acting on the combined object is F_{1W}. The two contact forces at the boundary between crates 1 and 2 do not appear in the equation describing the *combined* object. Nor do the internal atomic forces that bind the object together; each internal force forms an action–reaction pair acting on separate parts (individual atoms, perhaps) and such pairs sum to zero when we add together the separate parts to make the combined whole.

Note that in this example the worker is the active agent that is responsible for the motion, but it is the reaction force of the ground that makes this possible. If there were no friction between the worker's shoes and the ground, the worker could not move the system forward.

4. *Block hanging from a spring.* Figure 10*a* shows a block hanging at rest from a spring, the other end of which is fixed to the ceiling. The forces on the block, shown separately in Fig. 10*b*, are its weight **W** (acting down) and the force **F** exerted by the spring (acting up). The block is at rest under the influence of these forces, but they are *not* an action–reaction pair, because once again they act on the same body. The reaction force to the weight **W** is the gravitational force that the block exerts on the Earth, which is not shown.

The reaction force to **F** (the force exerted *on* the block *by* the spring) is the force exerted *by* the block *on* the spring. To show this force, we illustrate the forces acting on the spring in Fig. 10*c*. These forces include the reaction to **F**, which we show as a force **F**′ (= −**F**) acting downward, the weight **w** of the spring (usually negligible), and

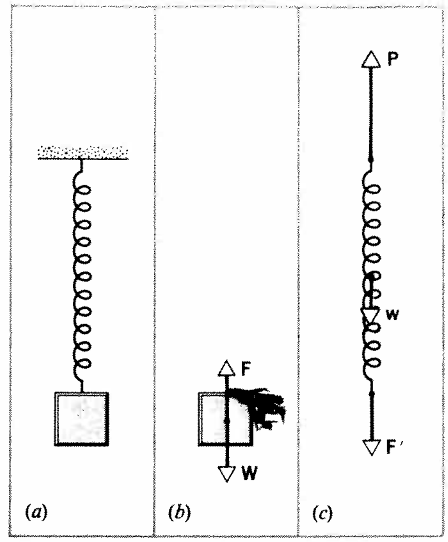

Figure 10 (*a*) A block hangs at rest supported by a stretched spring. (*b*) The forces on the block. (*c*) The forces on the spring.

the upward pull **P** of the ceiling. If the spring is at rest, the net force must be zero: $\mathbf{P} + \mathbf{w} + \mathbf{F}' = 0$.

The reaction force to **P** acts *on* the ceiling. Since we are not showing the ceiling as an independent body in this diagram, the reaction to **P** does not appear.

5-7 UNITS OF FORCE

Like all equations, Newton's second law ($F = ma$) must be dimensionally consistent. On the right side, the dimensions are, recalling from Chapter 1 that [ ] denotes *the dimensions of*, $[m][a] = \text{ML/T}^2$, and therefore these must also be the dimensions of force:

$$[F] = \text{ML/T}^2.$$

No matter what the origin of the force—gravitational, electrical, nuclear, or whatever—and no matter how complicated the equation describing the force, these dimensions must hold for it.

In the SI system of units, mass is measured in kg and acceleration in m/s^2. To impart an acceleration of 1 m/s^2 to a mass of 1 kg requires a force of 1 $\text{kg}\cdot\text{m/s}^2$. This somewhat inconvenient combination of units is given the name of newton (abbreviated N):

$$1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2.$$

If we measure the mass in kg and the acceleration in m/s^2, Newton's second law gives the force in N.

Two other systems of units in common use are the cgs (centimeter–gram–second) and the British systems. In the cgs system, mass is measured in grams and acceleration in cm/s^2. The force unit in this system is the *dyne*