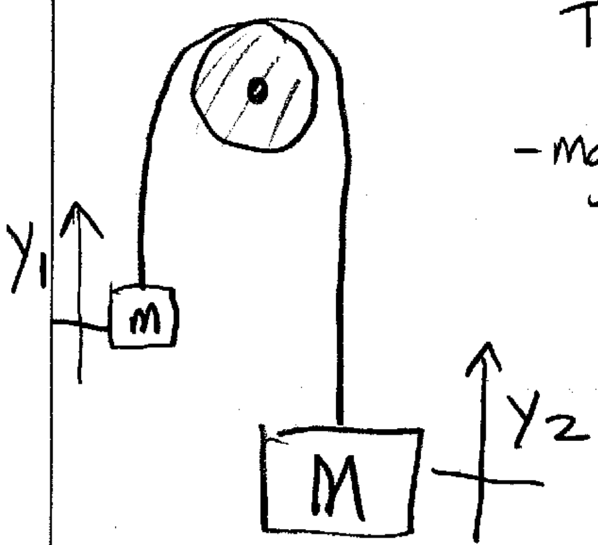
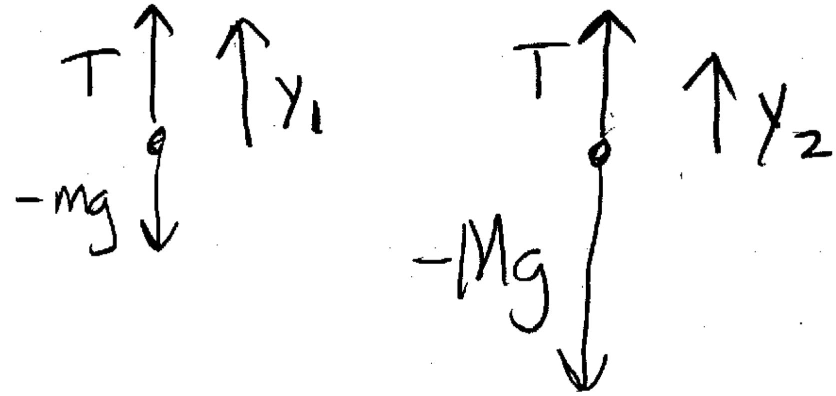
1 KK 2.5





$$T - mg = m\ddot{y}_1$$

$$T - Mg = M\ddot{y}_2$$

string doesn't stretch $\quad -\ddot{y}_1 = \ddot{y}_2$

want T, $\ddot{y}_2$

$$T - mg = -m\ddot{y}_2$$

$$\ddot{y}_2 = -\frac{T}{m} + g$$

$$T - Mg = M\left(-\frac{T}{m} + g\right)$$

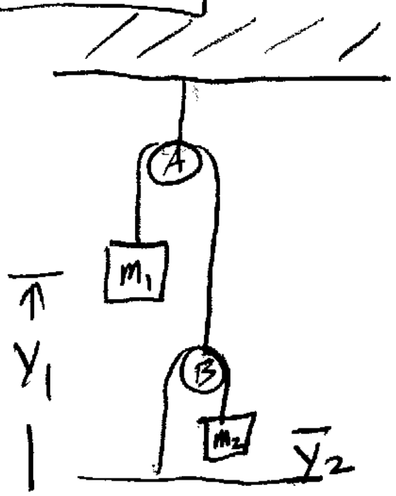
$$\left(1 + \frac{M}{m}\right)T = 2Mg \qquad M = 2m$$

$$\boxed{T = \frac{2M}{1 + \frac{M}{m}} g = \frac{2Mm}{m + M} g} = \frac{4m^2}{3m} g = \frac{4}{3} mg = \frac{2}{3} Mg$$

$$\ddot{y}_2 = -\frac{m}{3m} g = -\frac{g}{3}$$

$$\boxed{\ddot{y}_2 = -\frac{T}{m} + g = \left(\frac{-2M}{m + M} + 1\right) g = \left(\frac{m - M}{m + M}\right) g}$$

② $\boxed{\text{KK } 2.13}$



$$2\ddot{y}_1 = -\ddot{y}_2$$

think: if y_1 goes up by Δ, pulley B goes down by Δ, leaving 2Δ of slop for m_2.

<u>mass m_2</u>: $\uparrow T_2$, $\downarrow -m_2 g$

$$T_2 - m_2 g = m_2 \ddot{y}_2$$

<u>Pulley B</u>: $\uparrow T_1$, $\downarrow -T_2$, $\downarrow -T_2$

$$T_1 - 2T_2 = \cancel{m}\cdot(-\ddot{y}_1) \quad 0$$

$$T_1 - 2T_2 = 0$$

mass m_1: $\uparrow T_1$, $\downarrow m_1 g$

$$T_1 - m_1 g = m_1 \ddot{y}_1$$

want to solve for $\ddot{y}_1$

$$T_2 - m_2 g = -2m_2\ddot{y}_1$$

$$T_1 = 2T_2 = 2m_2(g - 2\ddot{y}_1)$$

$$2m_2(g-2\ddot{y}_1) - m_1 g = m_1\ddot{y}_1$$

$$(2m_2 - m_1)g = (4m_2 + m_1)\ddot{y}_1$$

HINT

$$\boxed{\ddot{y}_1 = \frac{2m_2 - m_1}{4m_2 + m_1}\, g} \Rightarrow m_1 = m_2,\ \frac{2m_1 - m_1}{4m_1 + m_1} g = \frac{1}{5} g$$

(3) 1.17 $\dot{r} = 4\ m/s$ $\dot{\theta} = 2\ rad/s$ 3/7

(a) $\vec{v} = \underset{\substack{\uparrow \\ 4\frac{m}{s}}}{\dot{r}}\hat{r} + \underbrace{\underset{\substack{\uparrow \\ 3m}}{r}\underset{\substack{\uparrow \\ 2\ rad/s}}{\dot{\theta}}}_{6}\hat{\theta}$

$$\boxed{v = |\vec{v}| = \sqrt{4^2 + 6^2} = \sqrt{16+36} = \sqrt{52}\ m/s}$$

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(b) $\vec{a} = (\underbrace{\cancel{\ddot{r}}}_{0} - r\dot{\theta}^2)\hat{r} + (\underbrace{r\cancel{\ddot{\theta}}}_{0} + 2\dot{r}\dot{\theta})\hat{\theta}$

$\dot{r} = \text{constant} \qquad \dot{\theta} = \text{constant}$

$$\vec{a} = -r\dot{\theta}^2\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta}$$
$$= (-3\cdot 2^2\hat{r} + 2\cdot 4\cdot 2\,\hat{\theta})\ \text{m/s}^2$$
$$= (-12\hat{r} + 16\hat{\theta})\ \text{m/s}^2$$

$$|\vec{a}| = \sqrt{12^2 + 16^2}\ \text{m/s}^2$$
$$= \sqrt{(-3\cdot 2^2)^2 + (4\cdot 2^2)^2}\ \text{m/s}^2$$
$$= 2^2\underbrace{\sqrt{(-3)^2 + (4)^2}}\ \text{m/s}^2$$

$$\boxed{|\vec{a}| = 2\cdot 2\cdot 5\ \text{m/s}^2 = 20\ \text{m/s}^2}$$

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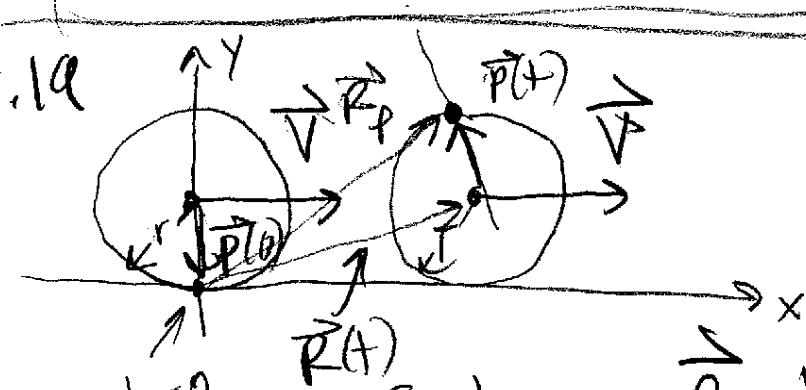
4) 1.19



r = radius of tire

$V\cancel{t} = r\omega\cancel{t}$

$V = r\omega$

<u>Center</u>: $\vec{R} = Vt\,\hat{\imath} + r\hat{\jmath}$
$= r\omega t\,\hat{\imath} + r\hat{\jmath}$

<u>Pebble</u>: $\vec{p}(t) = -r\sin(\omega t)\hat{\imath} - r\cos(\omega t)\hat{\jmath}$

$$\vec{R}_p(t) = \vec{R}(t) + \vec{p}(t)$$

position: $\boxed{\vec{R}_p(t) = r[(\omega t - \sin(\omega t))\hat{\imath} + (1 - \cos(\omega t))\hat{\jmath}]}$

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velocity: $\boxed{\vec{V}_p(t) = \frac{d\vec{R}_p}{dt} = r\omega\left[(1 - \cos(\omega t))\hat{\imath} + \sin(\omega t)\hat{\jmath}\right]}$

acceleration $\boxed{\vec{a}_p(t) = \frac{d\vec{V}_p}{dt} = r\omega^2\left[\sin(\omega t)\hat{\imath} + \cos(\omega t)\hat{\jmath}\right]}$

sometimes useful: $1 - \cos(\omega t)$

$$= 1 - \left(\cos^2\left(\frac{\omega t}{2}\right) - \sin^2\left(\frac{\omega t}{2}\right)\right)$$

$$= \sin^2\left(\frac{\omega t}{2}\right) + \sin^2\left(\frac{\omega t}{2}\right)$$

$$1 - \cos(\omega t) = 2\sin^2\left(\frac{\omega t}{2}\right)$$

$$\vec{R}_p(t) = r\left[(\omega t - \sin(\omega t))\hat{\imath} + 2\sin^2\left(\frac{\omega t}{2}\right)\hat{\jmath}\right]$$

$$\vec{V}_p = r\omega\left[\underbrace{2\sin^2\left(\frac{\omega t}{2}\right)\hat{\imath} + \sin(\omega t)\hat{\jmath}}_{t=0,\ \vec{V}_p(0)=0}\right]$$

$$\vec{a}_p = r\omega^2\left[\sin(\omega t)\hat{\imath} + \cos(\omega t)\hat{\jmath}\right]$$

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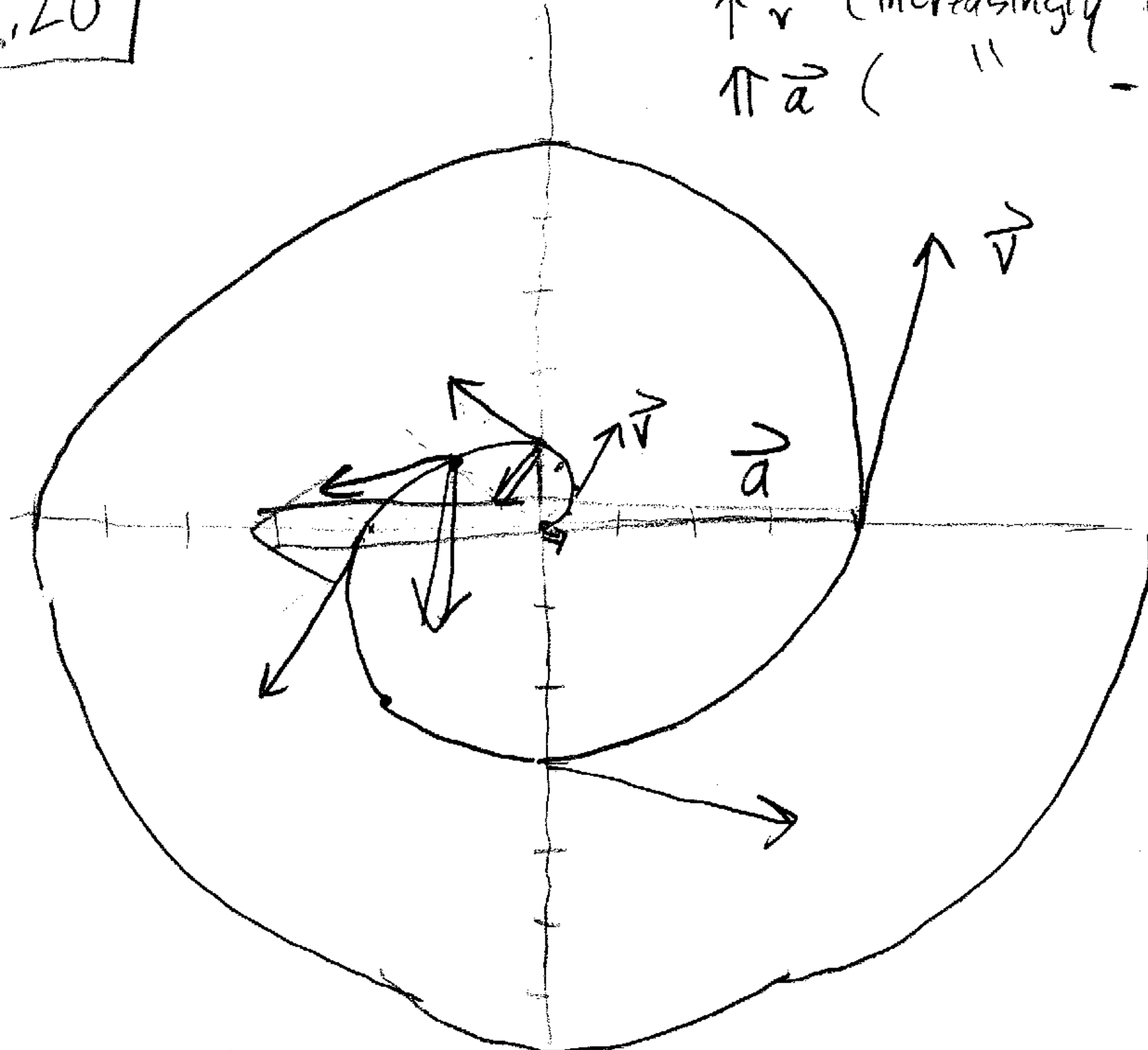
5) 1.20

$\uparrow \vec{v}$ (increasingly $\hat{\theta}$)
$\Uparrow \vec{a}$ (" $-\hat{r}$)



$\theta = \frac{1}{2}\alpha t^2 \Rightarrow t = \sqrt{\frac{2\theta}{\alpha}}$

$r = A\theta = \frac{1}{2}A\alpha t^2$

$\dot{\theta} = \alpha t = \alpha \cdot \sqrt{\frac{2\theta}{\alpha}}$

$\dot{r} = A\alpha t = A\alpha \cdot \sqrt{\frac{2\theta}{\alpha}}$

$\dot{\theta} = \sqrt{2\alpha\theta}$

$\dot{r} = A\sqrt{2\alpha\theta}$

$\ddot{\theta} = \alpha$

$\ddot{r} = A\alpha$

note $\frac{r}{\theta} = A = \frac{\dot{r}}{\dot{\theta}} = \frac{\ddot{r}}{\ddot{\theta}}$

(a) $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = A\sqrt{2\alpha\theta}\,\hat{r} + A\theta\sqrt{2\alpha\theta}\,\hat{\theta}$

$= A\sqrt{2\alpha\theta}\,(\hat{r} + \theta\hat{\theta})$

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θ	$\vec{v}\ (A\cdot\sqrt{2\alpha})$	$\vec{a}\ (A\alpha)$
0	0	$\hat{r}$
$\pi/4$	$\sqrt{\frac{\pi}{4}}\left(\hat{r}+\frac{\pi}{4}\hat{\theta}\right)$	$\left(1-2\left(\frac{\pi}{4}\right)^2\right)\hat{r}+5\left(\frac{\pi}{4}\right)\hat{\theta}$
$\pi/2$	$\sqrt{\frac{\pi}{2}}\left(\hat{r}+\frac{\pi}{2}\hat{\theta}\right)$	$\left(1-2\left(\frac{\pi}{2}\right)^2\right)\hat{r}+5\left(\frac{\pi}{2}\right)\hat{\theta}$
$3\pi/4$	$\sqrt{\frac{3\pi}{4}}\left(\hat{r}+\frac{3\pi}{4}\hat{\theta}\right)$	$\left(1-2\left(\frac{3\pi}{4}\right)^2\right)\hat{r}+5\left(\frac{3\pi}{4}\right)\hat{\theta}$
π	$\sqrt{\pi}\left(\hat{r}+\pi\hat{\theta}\right)$	
$3\pi/2$	$\sqrt{\frac{3\pi}{2}}\left(\hat{r}+\frac{3\pi}{2}\hat{\theta}\right)$	
2π	$\sqrt{2\pi}\left(\hat{r}+2\pi\hat{\theta}\right)$	$\left(1-2(2\pi)^2\right)\hat{r}+5(2\pi)\hat{\theta}$

$$\vec{a}=(\ddot{r}-r\dot{\theta}^2)\hat{r}+(r\ddot{\theta}+2\dot{r}\dot{\theta})\hat{\theta}$$

$$=(A\alpha-A\theta\cdot 2\alpha\theta)\hat{r}+\left(A\theta\alpha+2A\sqrt{2\alpha\theta}\sqrt{2\alpha\theta}\right)\hat{\theta}$$

(b) $$\vec{a}=A\alpha\underbrace{(1-2\theta^2)}\hat{r}+5A\theta\alpha\hat{\theta}$$

$=0$ when

$$1-2\theta^2=0$$

$$\theta=1/\sqrt{2}$$

$$\simeq 0.7\simeq \pi/4$$

as $\theta\to\infty$, $\vec{a}\to -A\alpha 2\dot{\theta}^2\hat{r}$

centripetal

(c) when $\cancel{A\alpha}(1-2\theta^2)=5\cancel{A}\theta\cancel{\alpha}$

$$2\theta^2-5\theta-1=0\Rightarrow\theta=\frac{5\pm\sqrt{25-8}}{4}$$

$$\boxed{\theta_\pm=\frac{5\pm\sqrt{17}}{4}}$$

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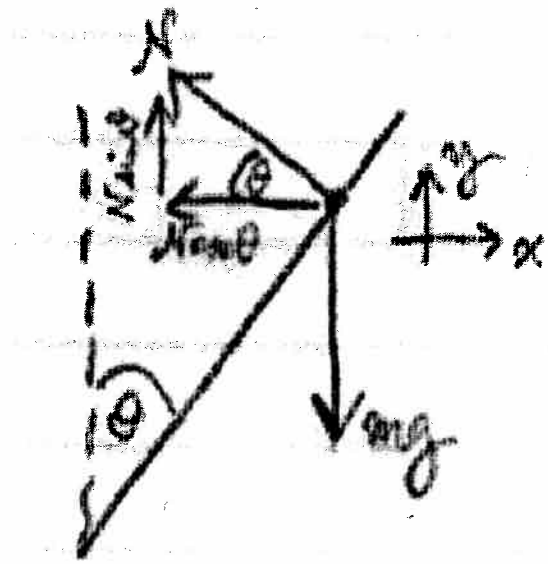
6. KK 2,9



Goal: Find R (the distance from the object to the dotted line).

The sums of forces are:

$$\sum F_x: \; -N\cos\theta = -m\frac{v^2}{R} \implies R = \frac{mv^2}{N\cos\theta}$$
$$\sum F_y: \; N\sin\theta - mg = 0 \implies N = \frac{mg}{\sin\theta}$$

So the radius of circular motion is

$$R = \frac{v^2}{g}\tan\theta\,.$$

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