Homework 3, Phys 230A, String Theory, Polchinski

Due Weds. 2/15/12, 5pm.

1. Evaluate the OPE

$$T_{zz}(z) : e_{\mu\nu}\partial_z X^\mu \partial_{\bar{z}} X^\nu e^{ik\cdot X(0,0)} :,$$

where $e_{\mu\nu}$ is a constant 'polarization' tensor. From the OPE, determine the weight of the operator, and the conditions on $e_{\mu\nu}$ for it to be a tensor.

2. Work out the covariant quantization of the open string at the second level (one α_{-2} or two α_{-1}'s), for general D and with A fixed to -1. (Apologies, in class I flipped the sign of A, relative to the text and notes). Verify the assertions made on page 30 of the notes.

3. Show that the residue of the pole in the Virasoro-Shapiro amplitude at $M^2 = 4(N-1)/\alpha'$ for integer N is a polynomial in $t - u \propto \cos\theta$. Compare the order of the polynomial with the maximum spin of a string state at that level (e.g. the maximum eigenvalue of the rotation J_{12}). If you plot spin versus mass-squared, what is the slope?

4. a) There are three terms in the Veneziano amplitude; focus on the one that has poles in s and t. Identify the residue of the pole at $t = 0$. Show that this residue is the same as you would get in field theory from exchange of a photon between two charged scalars, and use this to determine the gauge coupling in terms of g_o.

b) Show that the term that has poles in u and t makes an equal and opposite contribution to this residue.

c) So far this is without Chan-Paton factors; show that the cancelation is no longer present with these, and find the result (you won't have the information to do this part until after Monday's class).