Homework 5, Phys 230A, String Theory, Polchinski

Due 2/29/12.

1. We obtained the annulus/cylinder amplitude as a one-loop open string graph, tracing over the open string spectrum. Now we will obtain it from a closed string calculation. We represented this world-sheet as a strip of width π and periodic length $2\pi t$. By rescaling (and interchanging the two directions) we can think of it as a cylinder of circumference 2π and height π/t: at the boundaries at either end a closed string appears or disappears into the vacuum. Then the X path integral should have a canonical representation as

$$\langle B|e^{-\pi H/t}|B\rangle,$$

where $|B\rangle$ is the state in which the closed string appears.

Determine $|B\rangle$ by writing the boundary condition on X in terms of the α_n and $\tilde{\alpha}_n$, and imposing this as a constraint on $|B\rangle$. Now solve for $|B\rangle$. (Hint: it can be written as an exponential of something quadratic in the modes). Calculate the matrix element above. Guess the gauge-fixing determinant, and use a modular transformation to verify that you get the same answer as from the open string calculation, up to normalization. (The closed string calculation does not give you an easy way to get the overall normalization, whereas the open string calculation does.)