Homework 6, Phys 230A, String Theory, Polchinski

Due 3/7/12.

1. a) Start with the gauge-fixed Polyakov action for a single period coordinate $X \cong X + 2\pi R$ with closed string boundary conditions $\sigma \cong \sigma + 2\pi$. Take a Lorenztian world-sheet because it's more familiar:

$$S_P = \frac{1}{4\pi\alpha'} \int d\tau d\sigma \left(\partial_\tau X \partial_\tau X - \partial_\sigma X \partial_\sigma X\right).$$

Now reduce the problem to the zero (non-oscillator) modes,

$$X(\tau, \sigma) = x(\tau) + wR\sigma\,.$$

Inserting this to the action, obtain the Hamiltonian. Periodicity of X requires the canonical momenta to be quantized; from this, obtain the spectrum and verify that it gives the zero mode part of $L_0 + \tilde{L}_0$.

b) For two spacetime dimensions $X^{1,2}$, each of periodicity $2\pi R$, you just get two copies of the above. Now here's the first new ingredient: the two dimensions need not be the same radius, and need not be orthogonal. You can introduce this by making the periodicity more complicated

$$X^1 \equiv X^1 + 2\pi R_1 m_1 + 2\pi R_2' m_2 \quad X^2 \equiv X^2 + 2\pi R_2 m_2$$

for any integers m_1, m_2 (it might help to draw a picture of this). Repeat the above to find the spectrum (there are four integers, $w_{1,2}$ and $n_{1,2}$). The momentum quantization is obtained by requiring the wavefunction to be invariant under the spacetime periodicity above, and the winding is similarly with respect to the above periodicity.

c) Here's the next second new ingredient: introduce a constant background B_{12} into the action, and repeat the above steps.

d) If the dimensions are orthogonal and both at the self-dual radius with $B_{12} = 0$, you get gauge group $SU(2) \times SU(2) \times SU(2) \times SU(2)$, a total of 12 gauge bosons (4 from reduction of G and B, and 8 extra at the self dual radius). Show that for a specific choice of periodicity and B_{12} you can get 12 extra massless gauge bosons. The gauge group is $SU(3) \times SU(3)$; this can be seen from the root lattice, the pattern of charges.

This was the undoable problem on last year's final: I forgot to include the B-field!

A hint/check: in parts b and c, the quantized values of $p^m_{L,R}$ are always such that $(\alpha'/2)(p^m_L p^m_R - p^m_R p^m_R)$ is even.