Homework 1, Phys 230A, String Theory, Polchinski

Due: Wednesday, Jan. 25, 2012

(Equation numbers refer to the notes).

0. It goes without saying that you should work through the intermediate steps in the notes wherever needed, for example in deriving the mode expansions 2.29-2.32 and (from 3.1) the commutators 3.2 (for both open and closed strings).

1. a) Consider a relativistic point particle moving in a curved spacetime with metric $G_{\mu\nu}(X)$. Using this metric in the point particle action,

$$S = -m \int d\tau \sqrt{-G_{\mu\nu}(X)\partial_\tau X^\mu \partial_\tau X^\nu}\,,$$

derive the equation of motion for $X^\mu(\tau)$ and show that it is the geodesic equation.

b) For a relativistic point particle moving in flat spacetime coupled to an electromagnetic field, the action is

$$S = -m \int d\tau \sqrt{-\partial_\tau X^\mu \partial_\tau X_\mu} + q \int d\tau A_\mu(X)\partial_\tau X^\mu\,.$$

Show that this gives the expected equation of motion for a charged particle.

We will use the same idea to discuss strings moving in curved spaces.

2. Consider a long string running roughly in the x^1 direction. (Suppose for simplicity that the string is infinite in length). A convenient gauge is then simply to set $\tau = X^0$ and $\sigma = X^1$, so that $X^i(\tau,\sigma)$ for $i = 2,\ldots$ are the independent variable in this 'static' gauge.

a) Write the Nambu-Goto action in static gauge.

b) Expand the action through second order in X^i. Explain why this is valid precisely when the motion of the string is nonrelaivistic.

c) From this, what is the rest mass per unit length?

d) What is the mass per unit length as read from the kinetic energy for nonrelativistic motion?

3. Generalize the sum in 3.10 to

$$\sum_{n=1}^{\infty} k_n$$

where $k_n = n - \lambda$ for some constant λ. Use a convergence factor $e^{-\epsilon k_n}$. You will need this for problem 5.

4. In the notes I give the mode expansion for an open string with both ends on a Dp-brane. [This will be in Tuesday's update of the notes]. Now consider two parallel Dp-branes separated by a distance y in the 25-direction: one is at $x^{p+1} = \ldots = x^{25} = 0$ and the other is at $x^{p+1} = \ldots = x^{24} = 0$ and $x^{25} = y$. Give the mode expansion for an open string with

one end on each Dp-brane. What is the condition for the ground state of this open string to have a non-negative M^2?

5. a) Consider an open string coordinate X^μ that has a Dirichlet condition at $\sigma = 0$ and a Neumann condition at $\sigma = \pi$. Give the corresponding mode expansion.

b) Consider a Dp-brane which has a Neumann condition for $\mu = 0, \ldots, p$ and a Dirichlet condition $X^\mu = 0$ for $\mu > p$, and a Dp'-brane which has a Neumann condition for $\mu = 0, \ldots, p'$ and the Dirichlet condition $X^\mu = 0$ for $\mu > p'$: for $p > p'$ the latter lies within the former. There are three kinds of open string: those with both ends on the Dp, those with both ends on the Dp', and those with one end on each. For this last kind, give the mode expansion for each X^μ. What is the mass-squared of the lightest state? What are the conditions on p and p' such that this is non-negative?

Configurations of D-branes of different orientations and dimensions play important roles in gauge/gravity duality and in string compactification, and have a rich dynamics and interplay with supersymmetry.