Homework 2, Phys 230A, String Theory, Polchinski

Due: Wednesday, Feb. 1, 2012 (5 pm in Ahmed's mailbox, Broida 5207)

1. a) Add a 'cosmological constant'

$$-\mu \int d\tau d\sigma \sqrt{-\gamma}$$

to the Polyakov action. Note that this violates Weyl invariance. Show that there are no nontrivial solutions to the equations of motion.

b) Consider the analog of the Polyakov action for a d-dimensional object (you just change the range of the indices). Show that adding a cosmological constant now makes sense, and that eliminating γ gives an action proportional to the world-volume. (The reason that this is not as nice as the string case is that in $d \geq 3$ the theory is much more complicated, we can't gauge away γ as we do in $d = 2$, and the dynamics seems to be nonrenormalizable).

2. For the X^μ CFT, show that the operators $L'_m = L_m + (m+1)v_\mu \alpha^\mu_m$ also satisfy a Virasoro algebra, and finds its central charge (this should be relatively simple using the known result for the algebra of the L_m's). This has an interpretation in terms of strings moving in a certain nontrivial spacetime.

3. Consider a CFT with the mode algebra

$$\{b_m, c_n\} = \delta_{m+n,0}\,, \quad \{b_m, b_n\} = \{c_m, c_n\} = 0\,,$$

(these are anticommutators). Show that the operators

$$L_m = \sum_{n=-\infty}^{\infty} (2m-n) b_n c_{m-n} \quad (m \neq 0)\,, \quad L_0 = -1 + \sum_{n=1}^{\infty} n(b_{-n}c_n + c_{-n}b_n)\,,$$

satisfy a Virasoro algebra, and determine its central charge. I suggest that, as in class, you pick out the operator (single-commutator) terms first, and get the constant by acting with $[L_m, L_{-m}]$ on some particular state, e.g. the one annihilated by b_n and c_n for $n > 0$ and by c_0 (though any state will give the same answer).