PHYS230A Problem Set 7 Solutions

1. a) As in the dicussion on p. 81 of the notes, an integer-moded boson has zero point sum $-\frac{1}{24}$, an integer-moded fermion $+\frac{1}{24}$, an integer$+\frac{1}{2}$-moded boson $+\frac{1}{48}$, and an integer$+\frac{1}{2}$-moded fermion $-\frac{1}{48}$. The sum of these over the transverse directions, times $1/\alpha'$, is the ground state mass-squared.

From HW 1, ex. 4, you know that a coordinate with both ends N or both ends D is integer moded, while a coordinate with one end D and one end N is integer$+\frac{1}{2}$ moded. Counting the directions $2, 3, \ldots, 9$, the number of integer$+\frac{1}{2}$-moded coordinates in the configuration described is $p-1$, while the number of integer-moded coordinates is $8-(p-1) = 9-p$. In the R sector, the numbers are the same for ψ^i, and in the NS sector reversed. Thus:

$$R: \quad +\frac{p-1}{48} - \frac{9-p}{24} - \frac{p-1}{48} + \frac{9-p}{24} = 0$$

$$NS: \quad +\frac{p-1}{48} - \frac{9-p}{24} + \frac{p-1}{24} - \frac{9-p}{48} = \frac{p-5}{8}$$

In the R sector the ground state mass is zero. This is always true because X^i and ψ^i cancel for each i. In the NS sector we have

$$p = 3,\ M^2 = -\frac{1}{4\alpha'}\,, \quad p = 5,\ M^2 = 0\,, \quad p = 7,\ M^2 = \frac{1}{4\alpha'}\,, \quad p = 9,\ M^2 = \frac{1}{2\alpha'}\,.$$

b) If there are $2k$ fermionic zero modes, we can form k raising-lowering pairs and generate 2^k degenerate ground states. There is a zero mode for each integer-moded ψ. In the R sector $2k = 9-p$, while in the NS sector $2k = p-1$. Thus the degeneracies in the R sector are $8, 4, 2, 1$ for $p = 3, 5, 7, 9$, and in the NS sector they are $2, 4, 8, 16$ respectively.

c) The GSO projection will remove half the states, so halve the above numbers. In particular, a tachyon must remain in the $p = 3$ case. The only ambiguity is the R sector at $p = 9$. Here the result is curious: you get 'half' a state, meaning a massless open string that moves only in one direction along the D1-brane. This is explained on p. 191 of vol. 2 of the Big Book. To see it you have to go to the covariant quantization: the physical state and GSO projections have the same sign for one direction of motion, and opposite signs for the other direction of motion.

d) For D1-D5, the ground state masses (0) and degeracies (2, after the GSO projection) are the same in the bosonic (NS) and fermionic (R) sector. So this looks supersymmetric, and indeed this configuration is. The cases $p = 3$ and $p = 7$ are nonsupersymmetric, but the first has a tachyon and the second does not. The $p = 3$ instability has a simple final state: the D1 dissolves into the D3 and becomes magnetic flux. The case $p = 9$ is also supersymmetric, but I don't expect you to see this. It seems wrong because there are massless fermions but not massless bosons, but the point is that the supersymmetry acts in a funny way on the one-way fermion. In fact, the D1-brane in Type I theory is dual to the heterotic string.

These brane configurations come up in much research, and are discussed further in ch. 13 of the Big Book.

e) What matters is the number of integer-moded and half-integer moded fields, coming respectively from the NN+DD directions and ND+DN. Thus, the following ND+DN=4 configurations, among many others, are essentially the same (actually they are T-dual) and all are supersymmetric:

	0	1	2	3	4	5	6	7	8	9
$D1$	$\times$	$\times$								
$D5$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$				
$D3$	$\times$	$\times$	$\times$	$\times$						
$D7$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		
$D4$	$\times$	$\times$	$\times$	$\times$	$\times$					
$D4$	$\times$	$\times$	$\times$			$\times$	$\times$			

where I have listed in each case the directions in which they are extended. You'll hear a lot about the D3-D7 case, both in braney constructions of the Standard Model and in AdS/CFT dualities with flavor.