UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221A **Quantum Field Theory** **Fall 2011**

Prof: Joe Polchinski joep@kitp.ucsb.edu

ASSIGNMENT #1
Due: Weds., Oct. 3.

0. Simple things to check: that

$$|\mathbf{x}_1, \ldots, \mathbf{x}_n\rangle = \hat{\chi}^\dagger(\mathbf{x}_1)\ldots\hat{\chi}^\dagger(\mathbf{x}_n)|0\rangle.$$

Also,

$$\hat{\chi}^\dagger(\mathbf{x})\hat{\chi}(\mathbf{x})|\mathbf{x}_1, \ldots, \mathbf{x}_n\rangle = \left(\sum_{j=1}^n \delta^3(\mathbf{x}-\mathbf{x}_j)\right)|\mathbf{x}_1, \ldots, \mathbf{x}_n\rangle,$$

so that $\hat{\chi}^\dagger(\mathbf{x})\hat{\chi}(\mathbf{x})$ is the particle density.

1. Srednicki 1.2. The n-particle wavefunction $\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ translates into QFT notation as the state

$$|\psi\rangle = \int d^{3n}\mathbf{x}\,\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n)|\mathbf{x}_1, \ldots, \mathbf{x}_n\rangle.$$

Show that $\hat{H}_{\mathrm{QM}}\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ translates into $\hat{H}_{\mathrm{QFT}}|\psi\rangle$, where $\hat{H}_{\mathrm{QM}}$ is the RHS of 1.30 and $\hat{H}_{\mathrm{QFT}}$ is the RHS of 1.32.

2. Srednicki 1.3. We can break this into parts: first calculate the commutator of $\hat{\chi}^\dagger(\mathbf{x})\hat{\chi}(\mathbf{x})$ with $\hat{\chi}(\mathbf{y})$ and with $\hat{\chi}^\dagger(\mathbf{y})$, then of $\hat{N}$ with $\hat{\chi}(\mathbf{y})$ and $\hat{\chi}^\dagger(\mathbf{y})$, and finally of $\hat{N}$ with $\hat{H}$. The following identities are useful:

$$[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]$$

and

$$[\hat{A}, \hat{B}\hat{C}\hat{D}\hat{E}] = [\hat{A}, \hat{B}]\hat{C}\hat{D}\hat{E} + \hat{B}[\hat{A}, \hat{C}]\hat{D}\hat{E} + \hat{B}\hat{C}[\hat{A}, \hat{D}]\hat{E} + \hat{B}\hat{C}\hat{D}[\hat{A}, \hat{E}].$$

You can think of jumping $\hat{A}$ to the right in steps. Show these, by writing out the commutators explicitly.