UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221A **Quantum Field Theory** **Winter 2015**

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ASSIGNMENT #3
Due: Mon., Feb. 2, in class.

1. Consider

$$\mathcal{L}_1 = i\overline{\Psi}\gamma^\mu\partial_\mu\Psi - m_1\overline{\Psi}\Psi - m_2\overline{\Psi}i\gamma_5\Psi\,.$$

a) Verify that the last term is Hermitian (for real m_2), and Lorentz invariant.

b) Verify that the last term is not invariant under the usual parity transformation.

c) Show that for m_2 nonzero there is a field redefinition that makes the Lagrangian parity invariant.

Add a real scalar field,

$$\mathcal{L}_2 = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + i\overline{\Psi}\gamma^\mu\partial_\mu\Psi - m_1\overline{\Psi}\Psi - m_2\overline{\Psi}i\gamma_5\Psi - g_1\phi\overline{\Psi}\Psi - g_2\phi\overline{\Psi}i\gamma_5\Psi\,.$$

The last two terms are known as Yukawa interactions.

d) Show that for general values of m_i and g_i your field redefinition does not lead to a parity-invariant Lagrangian.

e) For the case that P is violated, what about C, T, and PCT?

2. In class, I asserted that any two sets of gamma matrices are related by a change of basis. To show this, define

$$b_1^\pm = (\gamma^1 \pm \gamma^0)/2\,, \quad b_2^\pm = (\gamma^2 \pm i\gamma^3)/2\,.$$

a) Find the anticommutators of all pairs of $b^\pm$'s.

b) Show that there must be a spinor u_0 such that $b_1^- u_0 = b_2^- u_0 = 0$.

Define the spinor basis to be

$$u_0 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \quad b_1^+u_0 = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \quad b_2^+u_0 = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \quad b_1^+b_2^+u_0 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}.$$

c) In this basis find the γ^μ and γ_5.

So whatever basis we started with, we can always get to this one (the spinor u_0 and the $b_i^\pm$ will depend on the original basis), and vice versa.