1. Consider

\[ \mathcal{L}_1 = i \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m_1 \bar{\Psi} \Psi - m_2 \bar{\Psi} i\gamma_5 \Psi. \]

a) Verify that the last term is Hermitian (for real \( m_2 \)), and Lorentz invariant.

b) Verify that the last term is not invariant under the usual parity transformation.

c) Show that for \( m_2 \) nonzero there is a field redefinition that makes the Lagrangian parity invariant.

Add a real scalar field,

\[ \mathcal{L}_2 = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + i \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m_1 \bar{\Psi} \Psi - m_2 \bar{\Psi} i\gamma_5 \Psi - g_1 \phi \bar{\Psi} \Psi - g_2 \phi \bar{\Psi} i\gamma_5 \Psi. \]

The last two terms are known as Yukawa interactions.

d) Show that for general values of \( m_i \) and \( g_i \) your field redefinition does not lead to a parity-invariant Lagrangian.

e) For the case that \( P \) is violated, what about \( C \), \( T \), and \( PCT \)?

2. In class, I asserted that any two sets of gamma matrices are related by a change of basis. To show this, define

\[ b_1^\pm = (\gamma^1 \pm \gamma^0)/2, \quad b_2^\pm = (\gamma^2 \pm i \gamma^3)/2. \]

a) Find the anticommutators of all pairs of \( b_i^\pm \)’s.

b) Show that there must be a spinor \( u_0 \) such that \( b_1^- u_0 = b_2^- u_0 = 0 \).

Define the spinor basis to be

\[ u_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad b_1^+ u_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad b_2^+ u_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad b_1^+ b_2^+ u_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \]

c) In this basis find the \( \gamma^\mu \) and \( \gamma_5 \).

So whatever basis we started with, we can always get to this one (the spinor \( u_0 \) and the \( b_i^\pm \) will depend on the original basis), and vice versa.