UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221A **Quantum Field Theory** **Winter 2015**

Prof: Joe Polchinski joep@kitp.ucsb.edu

ASSIGNMENT #4
Due: Mon., Feb. 9, in class.

1. The discrete symmetries act in an odd way on the $2+1$ dimensional massive Dirac theory. Take the basis

$$\beta = \gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \gamma^1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \gamma^2 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}.$$

a) Which of the discrete symmetry conditions

$$\gamma^\mu D(P) = \mathcal{P}^\mu{}_\nu D(P)\gamma^\nu, \quad \gamma^{\mu *} D(T) = -\mathcal{T}^\mu{}_\nu D(T)\gamma^\nu, \quad \gamma^{\mu \mathrm{T}}\mathcal{C} = -\mathcal{C}\gamma^\mu,$$

can be satisfied by some matrix $D(P), D(T)$ or $\mathcal{C}$? Recall that parity flips an odd number of spatial directions.

b) Find a CPT transformation of the form $\Theta\Psi(x)\Theta^{-1} = D(\Theta)\overline{\Psi}^{\mathrm{T}}$ that leaves the action invariant.

c) Quantizing the Dirac action yields two states in the rest frame, a particle with spin $S^{12} = +\frac{1}{2}$ and an antiparticle with spin $S^{12} = -\frac{1}{2}$. Explain your result in terms of this spectrum. *Correction: the antiparticle has spin* $S^{12} = +\frac{1}{2}$*...*

d) Find a parity symmetry of the *massless* Dirac theory. What does this do to the mass term?

p.s. There's something that still confuses me about this problem, I may post an update later.

2. Consider the Dirac theory

$$\mathcal{L} = i\overline{\Psi}\gamma^\mu\partial_\mu\Psi - m\overline{\Psi}\Psi$$

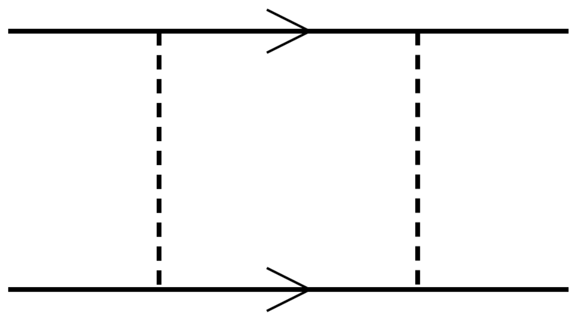
in $1+1$ dimensions. Use the basis

$$\beta = \gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \gamma^1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

a) Find the general plane wave solution to the Dirac equation.

b) Find the canonical anticommutators.

c) Expand the fields in modes and find the anticommutators of the mode operators.



d) From the mode expansions, evaluate $S(x-y)_{\alpha\beta} = i\langle 0|\mathrm{T}\Psi_\alpha(x)\overline{\Psi}_\beta(y)|0\rangle$. For general m you can leave it as a momentum integral. For $m=0$ evaluate it in position space.

3. Srednicki 45.2.

4. Write down the contribution from the graph shown to $\langle 0|\mathrm{T}\Psi_\alpha(x)\overline{\Psi}_\beta(y)\Psi_\gamma(z)\overline{\Psi}_\delta(w)|0\rangle$ (summing over the different ways of attaching the external lines to the fields).