UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221B **Quantum Field Theory** **Winter 2015**

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ASSIGNMENT #8
Due: Mon., March 9, in class.

1. Srednicki 28.1: the RG for $d = 4$ $\lambda\phi^4$.

2. Consider massless $\lambda\phi^4$ in in $d = 4$.

a) Show that the action is invariant under a scaling symmetry $x' = sx$. That is, $S[\phi'] = S[\phi]$, where $\phi'(x) = s^a\phi(sx)$ for some a.

b) Find the Noether current for this symmetry, by looking at the infinitesimal $s = 1 + \epsilon$.

c) Show that *if* the energy momentum is traceless, $T^\mu{}_\mu = 0$, then $j^\mu = T^{\mu\nu}x_\nu$ is conserved.

d) Is the Noether $T^{\mu\nu}$ for this theory traceless? You can use the equation of motion here if needed.?

e) Show that $I_{\mu\nu} = (\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)\phi^2$ is always conserved, no matter what it the equation of motion is for ϕ.

f) Show that $T'^{\mu\nu} = T^{\mu\nu} + cI^{\mu\nu}$ is traceless for some c. Compare your scale current to $T'^{\mu\nu}x_\nu$.

g) Show that the conserved quantities $P^\mu = \int d^3x\, T^{\mu 0}$ are the same for T and T' (you can assume that the fields fall off fast at infinity). So $I^{\mu\nu}$ is sort of trivial, and either T or T' can be used as the energy momentum tensor (similar to 36.4 and last quarter's HW 9, #5).

h) Consider the current $T'^{\mu\nu}v_\nu(x)$. What is the condition on $v_\nu(x)$ for this to be conserved? The extra solutions to this equation, beyond $v_\mu(x) = x_\mu$, are known as conformal transformations.

It is a somewhat open question whether every scale invariant theory has a traceless T' (and therefore the larger conformal symmetry). There are some proofs under specific conditions, and some exotic counterexamples.

Something to think about: gravity couples to $T_{\mu\nu}$. In what way does the theory change if we add some multiple of I to T?

3. Srednicki 52.3: studying a two-dimensional flow.

4. a) Calculate β_1 and β_2 for a theory with two massless scalars ϕ, χ and interaction

$$\frac{\lambda_1}{24}(\phi^4 + \chi^4) + \frac{\lambda_2}{4}\phi^2\chi^2\,.$$

Sketch the RG flow.

b) Now look at the theory in $4 - \epsilon$ dimensions, so $\hat\beta_i$ picks up a term $-\epsilon\lambda_i$. Find the fixed points, and classify each as IR fixed point, UV fixed point, or saddle point. Sketch the flow.