Physics 221A: HW6 solutions

November 14, 2012

1. Srednicki 14.5. The graph representing the divergent 1-loop contribution in real scalar ϕ^4 theory is the leftmost in figure 31.4 on page 195, plus the counterterm. This calculation is similar to the one in chapter 14, where Srednicki calculates the the *second* diagram of fig 31.4, relevant to theories with a three-point interaction.

   Our calculation is actually easier than in chapter 14 because we only have one loop propagator, thus only one denominator with a loop momentum in it, and so you don't need to use the Feynman trick to combine any denominators. The computation is straightforward:

$$i\Pi(k^2) = -\frac{i\lambda}{2}\int \frac{d^d l}{(2\pi)^d}\frac{1}{i}\Delta(l^2) - i(Ak^2 + Bm^2) + \dots \tag{1}$$

   where the presence of higher terms will always be implied. Analytically continuing to imaginary l^0 gives

$$i\Pi(k^2) = -\frac{i\lambda}{2}\int \frac{d^d \bar{l}}{(2\pi)^d}\Delta(\bar{l}^2) - iBm^2 + \dots \tag{2}$$

   The integral is of the useful form (14.27) with $a = 0$ and $b = 1$, so

$$i\Pi(k^2) = -\frac{i\lambda}{2}\frac{\Gamma(1 - d/2)m^{d-2}}{(4\pi)^{d/2}} - i(Ak^2 + Bm^2) + \dots \tag{3}$$

   Of course the integral is not convergent in $d = 4$ and we need to take $d = 4 - \epsilon$, writing $\lambda \mapsto \lambda\tilde{\mu}^\epsilon$ so that the mass dimension of λ remains the same as before we analytically continued in d. Expand using (14.26) and (14.33) and redefine $\mu \equiv \sqrt{4\pi}e^{-\gamma/2}\tilde{\mu}$ to get

$$i\Pi(k^2) = \frac{i\lambda}{(4\pi)^2}\left[\frac{1}{\epsilon} + \log\frac{\mu}{m} + \frac{1}{2}\right]m^2 - i(Ak^2 + Bm^2) + \dots \tag{4}$$

   from which our renormalization conditions give $A = O(\lambda^2)$ and $B = \frac{\lambda}{(4\pi)^2}\left[\frac{1}{\epsilon} + \log\frac{\mu}{m} + \frac{1}{2}\right]$. The net result is simply $\Pi(k^2) = 0$ to order λ: this is because the momentum k doesn't flow through the propagator at all. At two loops, order λ^2, it is nonzero.

   One remark on our renormalization scheme is in order. Our renormalization conditions are $\Pi(-m^2) = \Pi'(-m^2) = 0$, which enforce that the propagator have a pole at the physical particle mass, with unit residue. We really need the first for LSZ. However if the residue is not 1, LSZ can be modified by dividing out the residue or redefining the normalization of the field.

2. Srednicki 16.1. This is worked out (in the MSbar scheme) in chapter 31 of the book. In our scheme, on-shell renormalization, we want $V_4 = -\lambda$ when $s = 4m^2$, $t = u = 0$. The relevant contributions to the amplitude are given by (31.8)

$$iV_4 = -iZ_\lambda \lambda + \frac{1}{2}(-i\lambda)^2(\frac{1}{i})^2 \left[iF(-s) + iF(-t) + iF(-u)\right] + O(\lambda^3) \tag{5}$$

where iF is given by (31.9)

$$iF(k^2) = \tilde{\mu}^\epsilon \int \frac{d^dl}{(2\pi)^d} \frac{1}{((l+k)^2 + m^2)(l^2 + m^2)} \tag{6}$$

$$= \frac{i}{16\pi^2}\left[\frac{2}{\epsilon} + \int_0^1 dx \log(\mu^2/D)\right] \tag{7}$$

$$\text{with } D = x(1-x)k^2 + m^2 \tag{8}$$

The x-integral is easy to perform for the kinematic arrangement we use to impose our renormalization condition. We're interested in $t = u = 0$, it's immediate to compute $\int_0^1 dx \log(\mu^2/D_{tu}) = 2\log(\mu/m)$. When we plug in $k^2 = -s = -4m^2$ we get $\int_0^1 dx \log(\mu^2/D_s) = 2\log(\mu/m) + 2$. Putting it all together gives

$$iV_4 = -(1+C)i\lambda + \frac{3i\lambda}{(4\pi)^2}\left[\frac{1}{\epsilon} + \log\frac{\mu}{m} + \frac{1}{3}\right] + O(\lambda) \tag{9}$$

and using the renormalization condition $iV_4|_{s=4m^2} = -i\lambda$ we get

$$C = \frac{3\lambda}{(4\pi)^2}\left[\frac{1}{\epsilon} + \log\frac{\mu}{m} + \frac{1}{3}\right] \tag{10}$$

3. Srednicki 16.2. The contributing diagrams are the same as in the previous problem, but the symmetry factors are different. The s-channel has symmetry factor 2 as before (for exchanging the loop lines) but the t and u have loop lines running in different directions so now have symmetry factor 1. This means our $i\Pi(k^2)$ will be given by that of the previous problem except with double the contribution from the t and u channels.

   This gives

$$iV_4 = -(1+C)i\lambda + \frac{5i\lambda}{(4\pi)^2}\left[\frac{1}{\epsilon} + \log\frac{\mu}{m} + \frac{1}{5}\right] + O(\lambda) \tag{11}$$

and using the renormalization condition $iV_4|_{s=4m^2} = -i\lambda$ we get

$$C = \frac{5\lambda}{(4\pi)^2}\left[\frac{1}{\epsilon} + \log\frac{\mu}{m} + \frac{1}{5}\right] \tag{12}$$