**34.2** The all-spatial term was the messiest, lots of ways to organize it. Probably simplest is just to write the M commutator in terms of J’s and K’s, 33.11,.12,.13. Otherwise the space-space parts involve a mess of $\epsilon$-tensor manipulations, for example 
\[ \epsilon_{ijm} \epsilon_{kln} = \delta_{ik} \delta_{jl} \delta_{mn} + \delta_{jk} \delta_{ml} \delta_{in} - \delta_{ik} \delta_{ml} \delta_{jn} - \delta_{jk} \delta_{il} \delta_{mn} - \delta_{mk} \delta_{il} \delta_{jn} \] (all permutations of $ijm$, with minus sign for anti-cyclic). Or take shortcuts by showing that e.g. \([S^{12}, S^{13}]\) is the only nontrivial one, up to permutations.

**35.4** Various ways to organize this. I’d write the LHS as 
\[ \text{Tr}(\epsilon \sigma^\nu \epsilon^T \sigma^{\nu T}) \]
Now, easy to check that 
\[ \epsilon \sigma^\nu = \pm \sigma^{\nu T} \epsilon, \] with a + sign for $\nu = 0$ and a $-$ sign for $\nu$ spacelike. Also, $\epsilon \epsilon^T = I$. So the trace becomes 
\[ \pm \text{Tr}(\sigma^{\nu T} \sigma^{\nu T}) = -2g^{\mu \nu} \]
a is easily checked. Some people derived the trace of 35.5 from 35.4. You would also need to argue (from Lorentz invariance) that the RHS is proportional to $g^{\mu \nu}$, and then 35.4 gives the normalization.

**36.5** a) The kinetic term is invariant for $U^*_j U^*_l = \delta_{kl}$, or $U^* U = I$, $U$ unitary. But the mass term requires $U^*_j U^*_l = \delta_{kl}$, or $U^T U = I$, $U$ orthogonal. Together these mean that $U$ is orthogonal and real, $O(N)$.

d) The obvious symmetry is $U(N)$, but we know that $N$ massless Dirac $= 2N$ massless Weyl, so it must be $U(2N)!$ Not so obvious in Dirac notation, the easiest way to show it is to write it in terms of Weyl fermions. Mark was throwing you a curve ball here! By the way, there is a ‘chiral’ $U(N) \times U(N) \subset U(2N)$ that does come up a lot:
\[ (1 + \gamma_5) \Psi_j \rightarrow (1 + \gamma_5) U^* j \Psi k, \quad (1 - \gamma_5) \Psi_j \rightarrow (1 - \gamma_5) V^*_j \Psi k \]
This is what you will get (later in the course) for a massless Dirac fermion coupled to a gauge field.

e) By the same logic as part d, $U(N)$ is obvious but $O(2N)$ is actually there.

6. Various ways to organize. I’d probably use $\gamma^\mu \gamma^\nu = -\gamma^\nu \gamma^\mu$ for $\mu \neq \nu$ and $g^{\mu \nu}$ for $\mu = \nu$ (from the basic anticommutation relation), and then commute through.

7. The adjoint of the Dirac equation $i \gamma^\mu \partial_\mu \Psi = m \Psi$ is $-i \partial_\mu \Psi^\dagger \gamma^\mu = m \Psi^\dagger$. Multiplying by $\beta$ on the right you get 
\[ -i \partial_\mu \Psi^\dagger \gamma^\mu \beta = -i \partial_\mu \Psi^\dagger \beta \gamma^\mu = -i \partial_\mu \Psi \gamma^\mu = m \Psi^\dagger \beta = m \Psi. \]
With this you can show conservation. (Some people just applied Noether’s theorem, I had wanted you to practice the manipulations above).