Critical Spin-liquid Phases in Spin-1/2 Triangular Antiferromagnets

In collaboration with:
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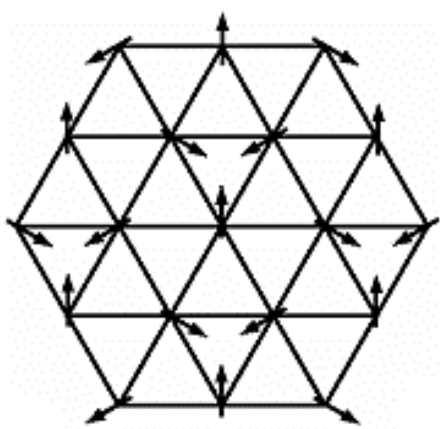
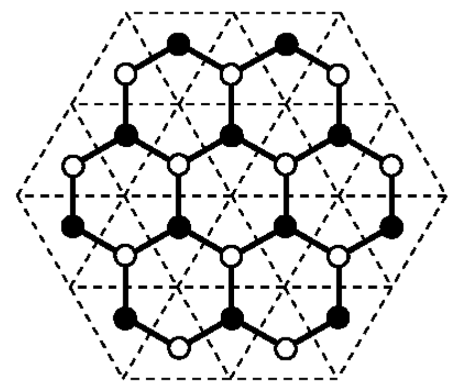




Outline

I. Background

- Avoiding conventional symmetry-breaking in s=1/2 AF
- Topological versus “critical” spin-liquids

II. Experiment

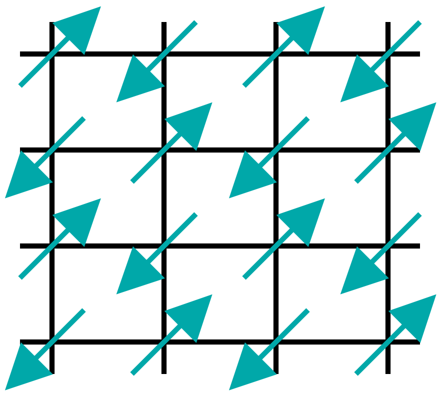
- Cs_2CuCl_4: S=1/2 triangular antiferromagnet
- Evidence of critical spin-liquid physics in neutrons

III. Theory

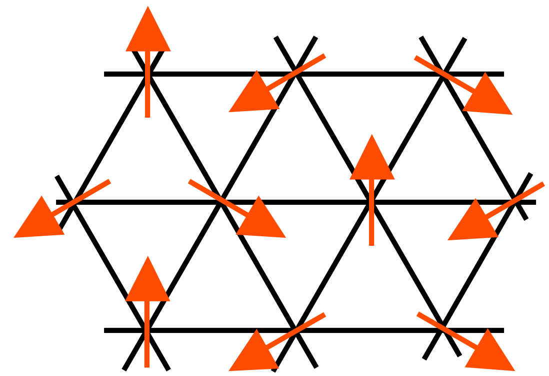
- Fermionized-vortex approach
- Experimental implications

Symmetry-broken ground states in s=1/2 AFM's

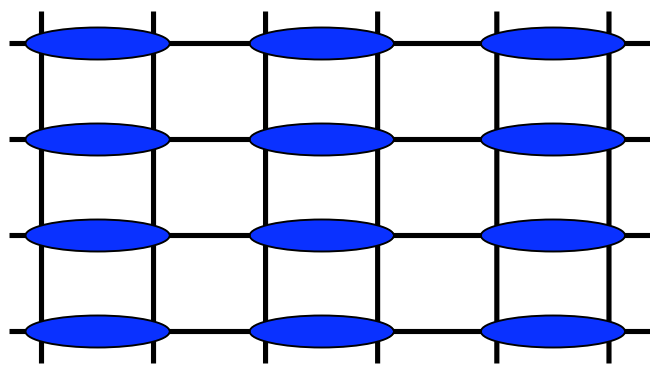
$$H = J\sum_{\langle \boldsymbol{x}\boldsymbol{x}'\rangle} \boldsymbol{S}(\boldsymbol{x})\cdot\boldsymbol{S}(\boldsymbol{x}') + \text{Other spin interactions}$$

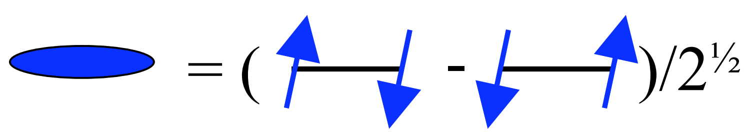


Neel phases
(gapless "Goldstone" modes)



Valence Bond Solid
(fully gapped)

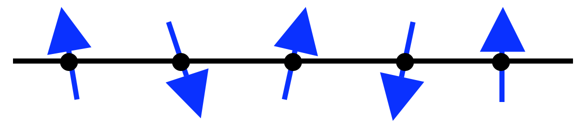




How to suppress order (*i.e.*, symmetry-breaking)?

- Low spin (*i.e.*, *s* = ½)

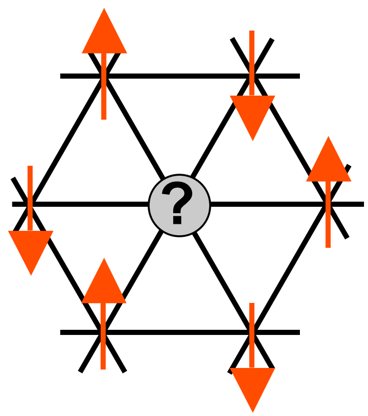
  - Low dimensionality
    - *e.g.*, 1D Heisenberg chain (simplest example of critical phase)
    - Much harder in 2D!



"almost" AFM order:

$$\langle \mathbf{S}(r)\cdot\mathbf{S}(0)\rangle \sim (-1)^{r} / r^{2\Delta}$$

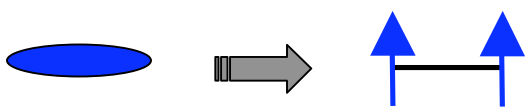
- Geometric Frustration
  - *e.g.*, triangular lattice



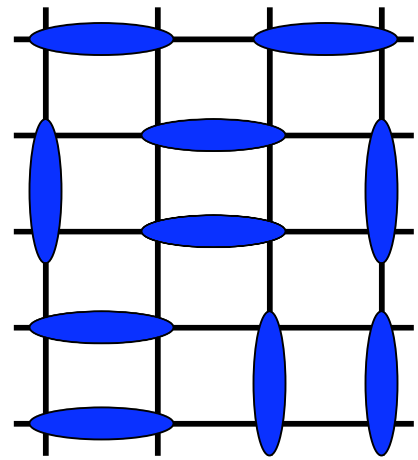
⇒ Good 2D candidate is *s* = ½ triangular antiferromagnet

"Exotic" *spin-liquids* with *no* broken symmetries

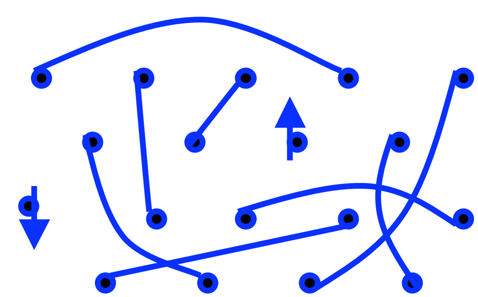
- Topological States
  - Topological degeneracy
    - Desirable for quantum computing
  - Short-range correlations
  - Gapped local excitations
  - ***Deconfined*** spinons



RVB state (Anderson)

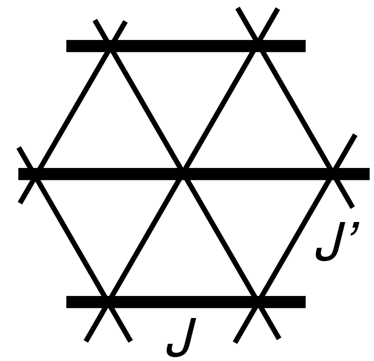


- Critical Phase ("Algebraic Spin-liquids")
  - Gapless bulk excitations
  - Power-law correlations with *no* fine tuning
  - Valence bonds on many length scales

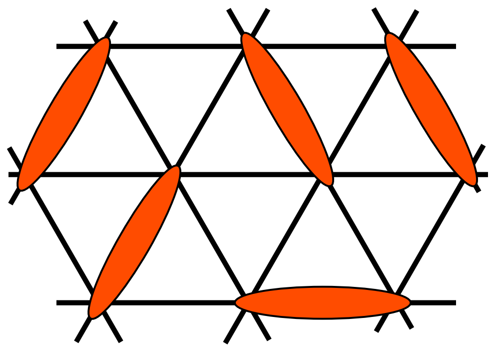


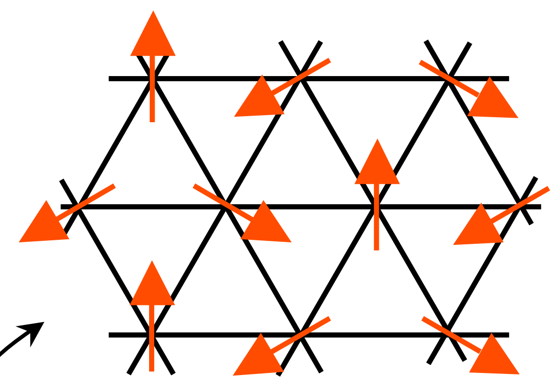
Triangular Lattice s=1/2 AFM

$$\mathcal{H} = \sum_{\langle ij \rangle} J_{ij} \mathbf{S_i} \cdot \mathbf{S_j}$$



- In 1973, Anderson proposed an RVB ground state
- Numerics: true ground state has long-range order





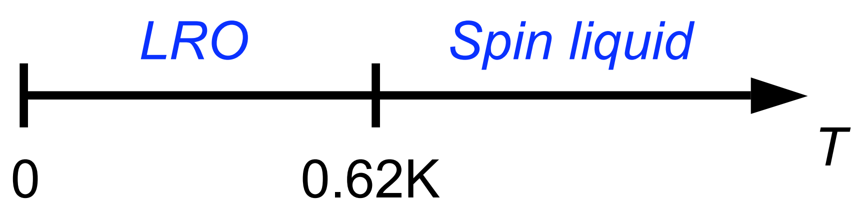
Classical ground state, *with sharply reduced magnetization due to quantum fluctuations*

⇒ Perhaps spin-liquid is not too far away…
(recent experiments support this notion)

Quasi-2D Spin-½ Δ AFMs

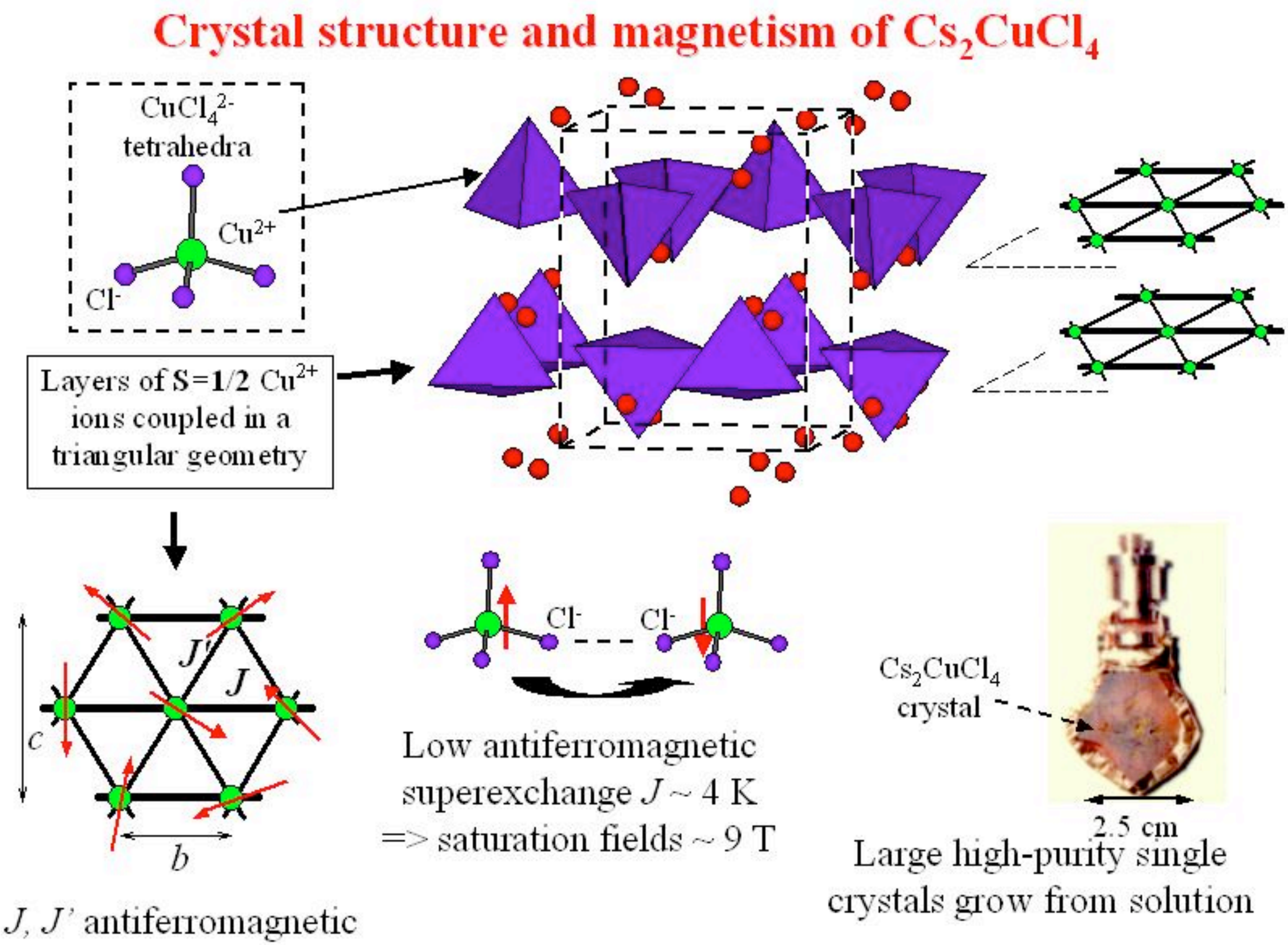
- Organic Mott Insulator, κ-$(ET)_2Cu_2(CN)_3$:
  - Nearly isotropic, large exchange energy ($J \sim 250K$)
  - No LRO detected down to 32mK
  - *Spin-liquid ground state*?

- Cs_2CuCl_4:
  - Anisotropic, low exchange energy ($J \sim 1\text{-}4K$)

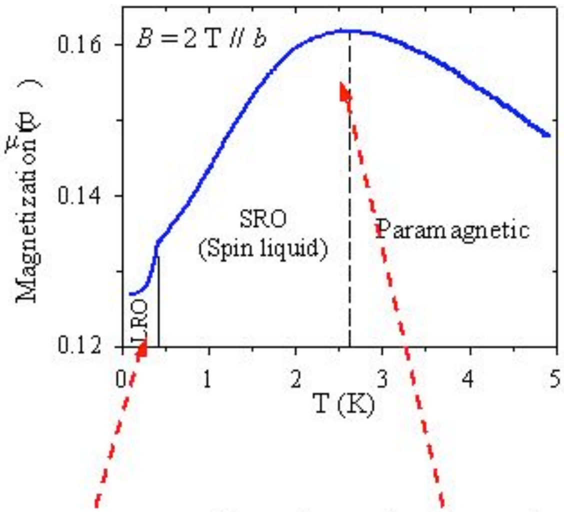


R. Coldea *et al*. PRL **86,** 1335 (2001)
Y. Shimizu *et al*. PRL **91**, 107001 (2003)

Neutron Scattering from Cs_2CuCl_4 - Radu Coldea et.al.



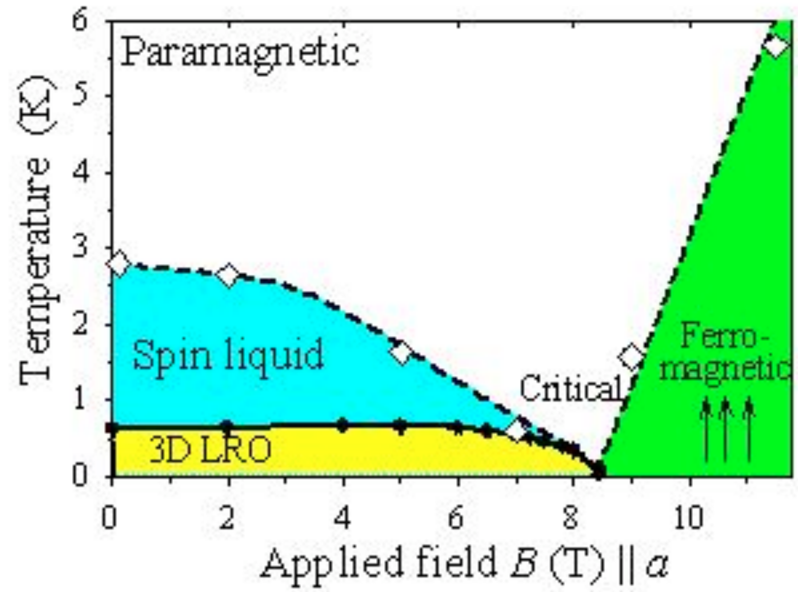
Low-field magnetization vs T

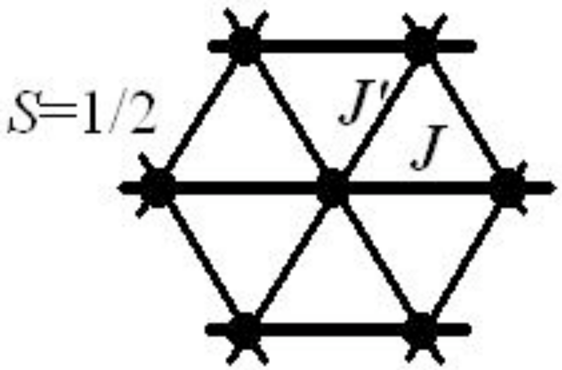


Magnetic order

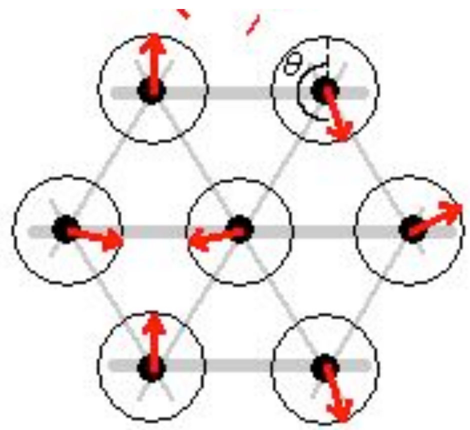
Broad peak characteristic of short-range antiferromagnetic correlations

Phase diagram with B perpundicular to planes

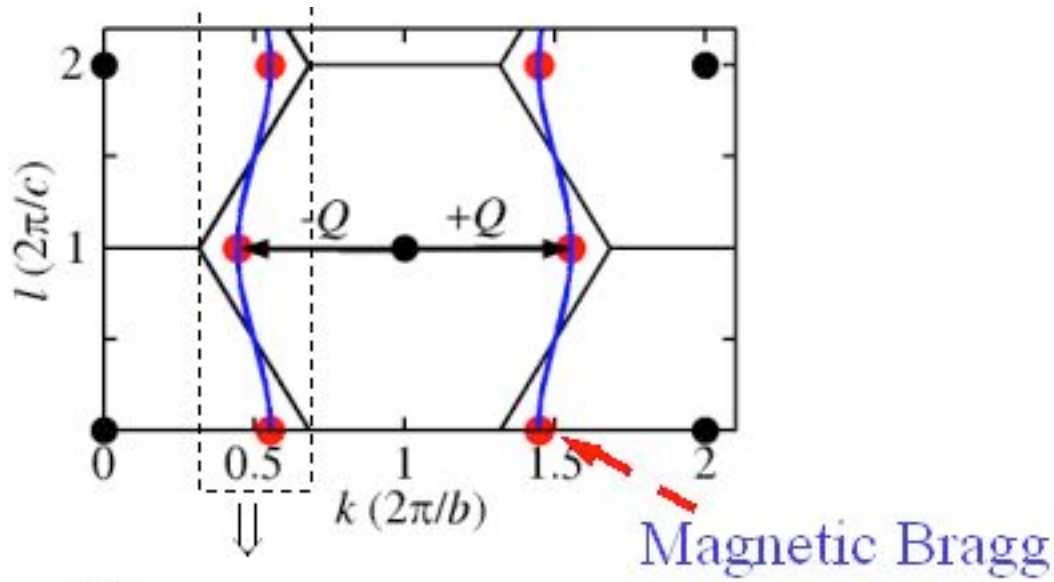




Spiral LRO at low T



- Magnetic Bragg peaks at Q



- Continuum Inelastic Scattering at Q

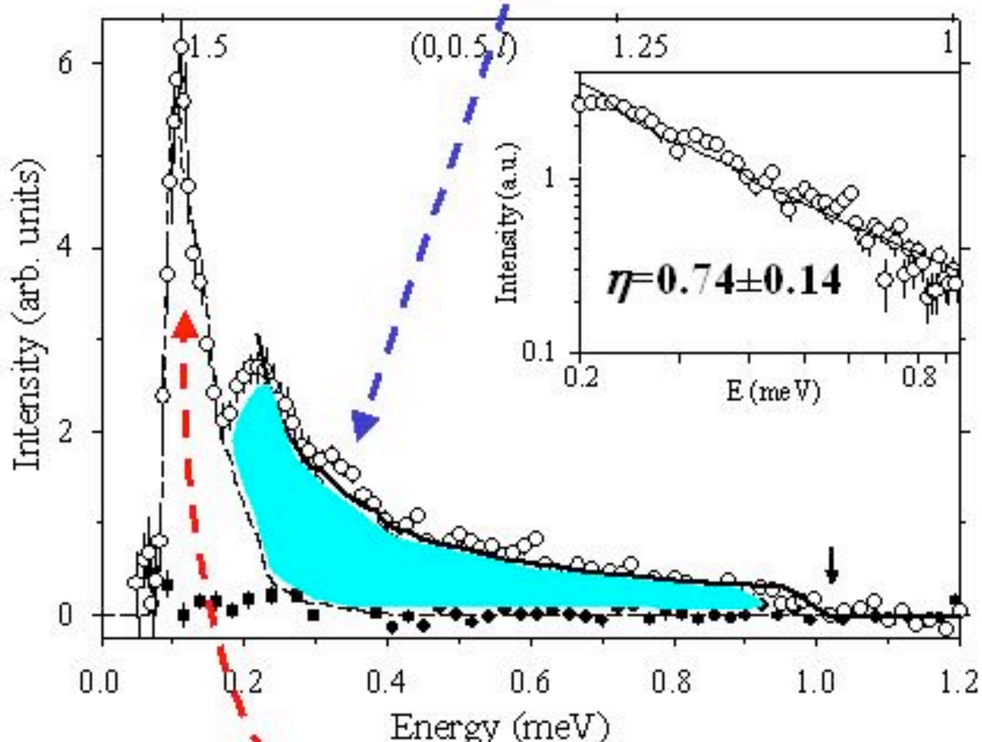
$$\langle S^+(\mathbf{Q},\omega)S^-(-\mathbf{Q},-\omega)\rangle \sim \frac{A}{\omega^{2-\eta}} \qquad \eta \approx 0.75$$

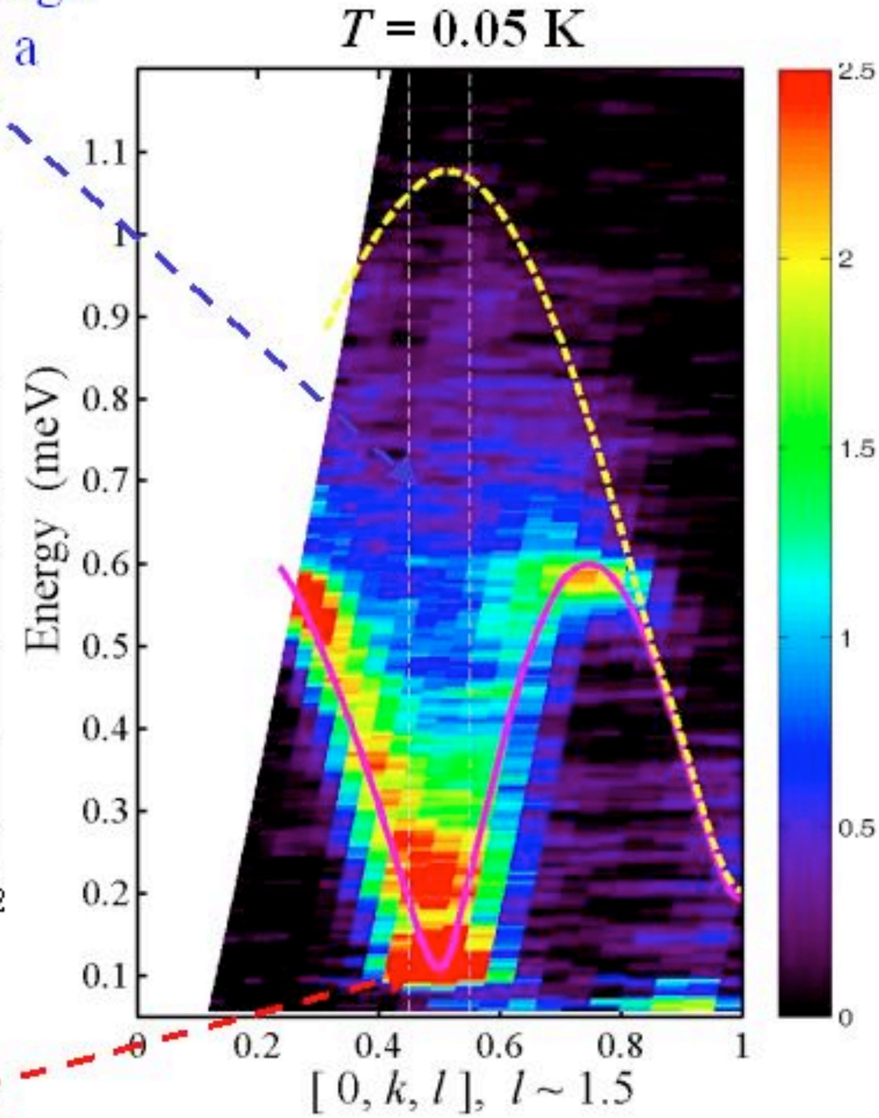
Power law scaling. ?? Algebraic spin liquid ??

Analysis of the dominant continuum scattering

Continuum scatering at high energies described by a power-law form

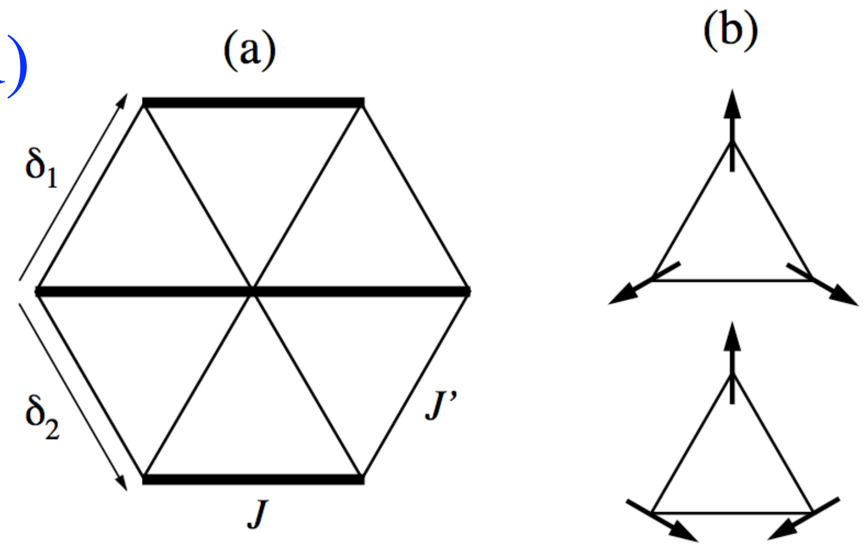
$$S(k,\omega) = I_{k} \frac{\theta(\omega - \omega_{k})\theta(\omega_{k}^{U} - \omega)}{[\omega^2 - \omega_{k}^2]^{1-\eta/2}}$$





At very low energies see a sharp S=1 magnon

Hamiltonian is known (measured)



For a Single Layer of Cs_2CuCl_4:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{DM}$$

$$\mathcal{H}_0 = \sum_{\mathbf{r}} [J\mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}+\delta_1+\delta_2} + J'(\mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}+\delta_1} + \mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}+\delta_2})]$$

$$\mathcal{H}_{DM} = -\sum_{\mathbf{r}} \mathbf{D} \cdot \mathbf{S}_{\mathbf{r}} \times (\mathbf{S}_{\mathbf{r}+\delta_1} + \mathbf{S}_{\mathbf{r}+\delta_2}) \qquad (\mathbf{D} = D\hat{\mathbf{z}})$$

$$J = 4.3K, \quad J'/J = 0.34, \quad D/J = 0.053$$

Dzyaloshinski-Moriya Interaction breaks symmetries:

- SU(2) down to U(1) spin symmetry (and S_z to $-S_z$) - “Easy-Plane”
- Inversion symmetry: x to -x

Easy-Plane Spin Model $(J > J^z)$

with Jason Alicea and Lesik Motrunich

$$\mathcal{H}_{xy} = \sum_{\langle ij \rangle} [J_{ij}(S_i^x S_j^x + S_i^y S_j^y) + J_{ij}^z S_i^z S_j^z]$$

- DM interaction gives (not so weak!) exchange anisotropy

  With classical spins ($S^2=3/4$) have Kosterlitz-Thouless transition into spiral phase at $T_{KT} = 0.84$ K, while experimentally $T_N = 0.62$K.

- Importance of Vortices!
- Ignore inversion symmetry breaking (for now)

Theoretical Approach:

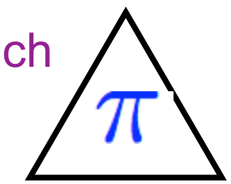
- Duality: Spins (xy) ⟶ Vortices
- Chern-Simons: Vortices ⟶ Fermionized vortices

Easy-plane AF equivalent to hard-core bosons hopping in "magnetic field"

$$\mathcal{H}_{xy} = -\sum_{ij}[JS_i^+ S_j^- e^{iA_{ij}^0} + h.c. + J^z S_i^z S_j^z]$$

boson hopping
on triangular lattice

pi flux thru each
triangle

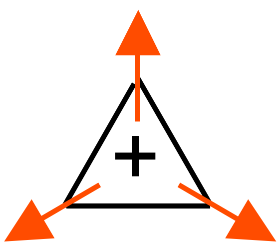


boson interactions

Focus on vortices

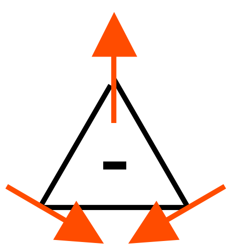
$$S^+ = S^x + iS^y \sim e^{i\phi}$$

$$\int \vec{\nabla}\phi \cdot d\vec{\ell} = 2\pi$$



"Vortex"

Vortex number N=1



"Anti-vortex"

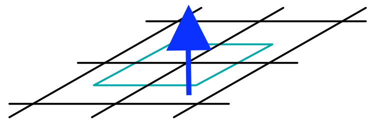
Vortex number N=0

Due to frustration,
the dual vortices
are at "half-filling"

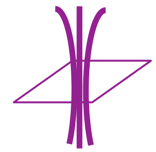
Boson-Vortex Duality

- Exact mapping from boson to vortex variables.

$$s_z + \frac{1}{2} = \frac{1}{2\pi} \vec{\nabla} \times \vec{a}$$



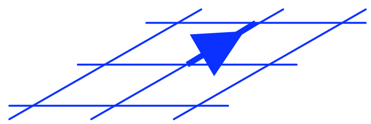
$$s_z = 1/2$$



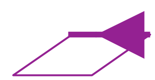
$$\int d^2x \;\; b = 2\pi$$

Dual "magnetic" field

$$\vec{\nabla}\phi = 2\pi \hat{z} \times \vec{e}$$



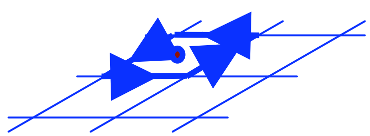
$$\vec{\nabla}\phi$$



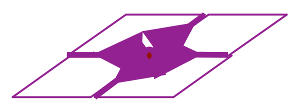
$$\vec{e}$$

Dual "electric" field

$$\vec{\nabla} \cdot \vec{e} = N$$



$$\int \vec{\nabla}\phi \cdot d\vec{\ell} = 2\pi$$



$$N = 1$$

Vortex number

Vortex carries dual gauge charge

- All non-locality is accounted for by dual U(1) gauge force

Vortices experience average dual magnetic field

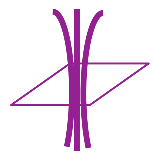
With:

$s_z = +1/2 \Rightarrow b = 2\pi$

$s_z = -1/2 \Rightarrow b = 0$

Thus, on average have non-zero dual "magnetic" flux:

$$\langle s_z \rangle = 0 \quad \rightarrow \langle b \rangle = \pi$$

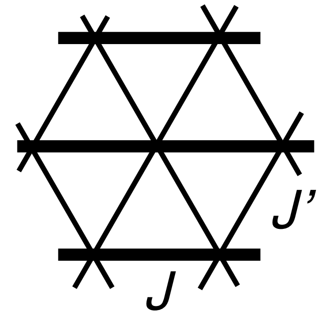


$$\int d^2x \; \vec{\nabla} \times \vec{a}^0 = \pi$$

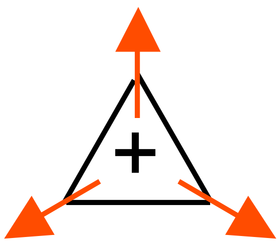
Vortices hop on the sites of the dual lattice
and "see"pi flux per plaquette

Duality for easy-plane triangular AF

$$\mathcal{H}_{xy} = \sum_{\langle ij \rangle} [J_{ij}(S_i^x S_j^x + S_i^y S_j^y) + J_{ij}^z S_i^z S_j^z]$$



Frustrated spins



"Vortex"

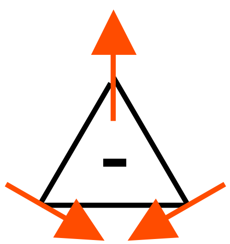
vortex creation/annihilation ops: $b_i^\dagger, b_i$

$$H = \sum_{\langle ij \rangle} J_{ij} e_{ij}^2 + U \sum_i (\Delta \times a)_i^2$$

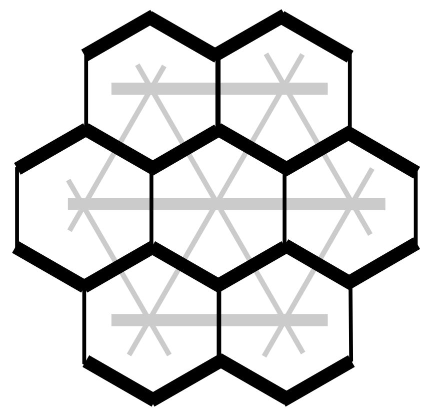
$$- \sum_{\langle ij \rangle} t_{ij} b_i^\dagger b_j e^{i(a_{ij} + a_{ij}^0)} + h.c.$$

vortex hopping

Half-filled bosonic vortices w/ "electromagnetic" interactions



"Anti-vortex"

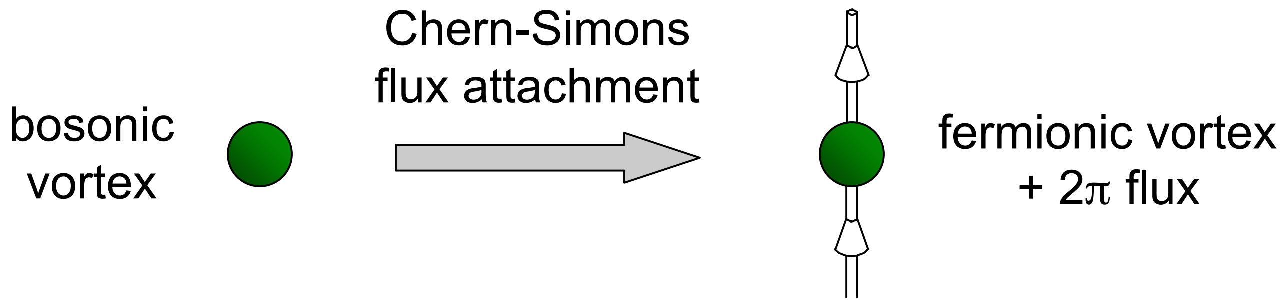


$$\vec{\nabla} \times \vec{a}^0 = \pi$$

Vortices see pi flux thru each hexagon

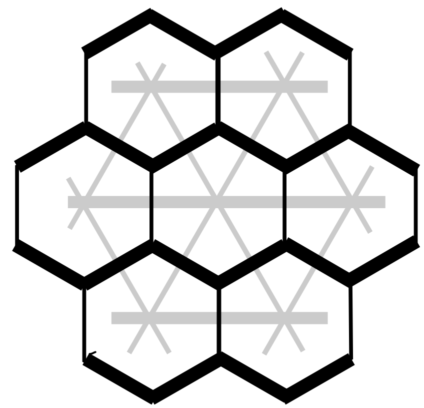
Chern-Simons Flux Attachment: Fermionic Vortices

- Difficult to work with half-filled bosonic vortices → ***fermionize!***

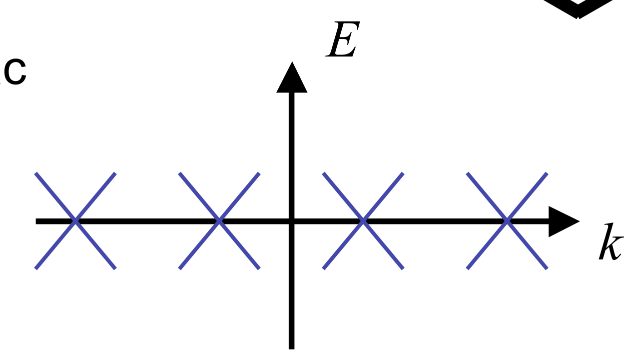


- "Flux-smearing" mean-field: Half-filled fermions on honeycomb with pi-flux

$$H_{MF} = -\sum_{\langle ij \rangle} \tilde{t}_{ij} f_i^{\dagger} f_j + h.c.$$



- Band structure: 4 Dirac points



Low Energy Dual Theory - (expand about the Dirac nodes)

$N = 4$ flavors

$$\mathcal{L} = \overline{\psi}_a \gamma^\mu (\partial_\mu - i a_\mu - i A_\mu) \psi_a + \frac{1}{2e^2} (\nabla \times a)^2 + \frac{i}{4\pi} A \cdot (\nabla \times A) + \mathcal{L}_{4\mathrm{f}}$$

With log vortex interactions can ***eliminate*** Chern-Simons term $(\tilde{a}_\mu = a_\mu + A_\mu)$

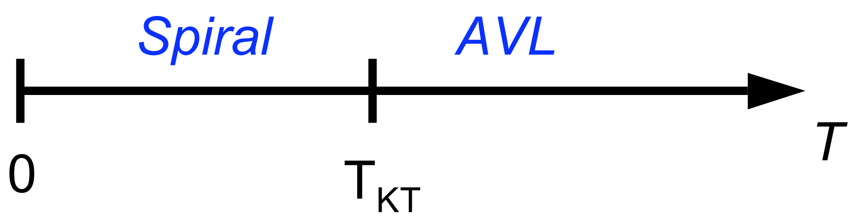
$$\mathcal{L}_{QED3} = \overline{\psi}_a \gamma^\mu (\partial_\mu - i \tilde{a}_\mu) \psi_a + \frac{1}{2e^2} (\nabla \times \tilde{a})^2 + \mathcal{L}_{4f}$$

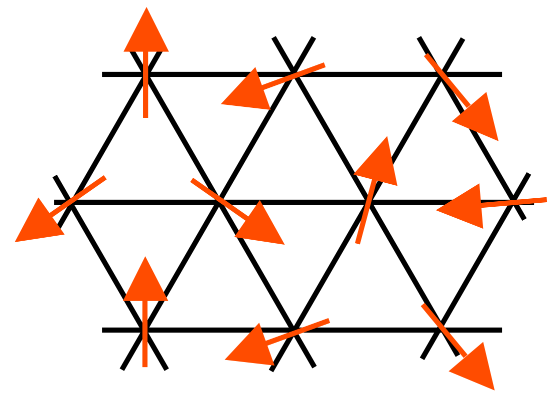
Irrelevant at "large-*N*"
Postulate *N* = 4 is large enough

- ***"Algebraic vortex liquid"***
  - "Critical Phase" with no free particle description
  - No broken symmetries - rather an emergent global SU(4) symmetry
  - Power-law correlations
  - Stable gapless spin-liquid (no fine tuning)

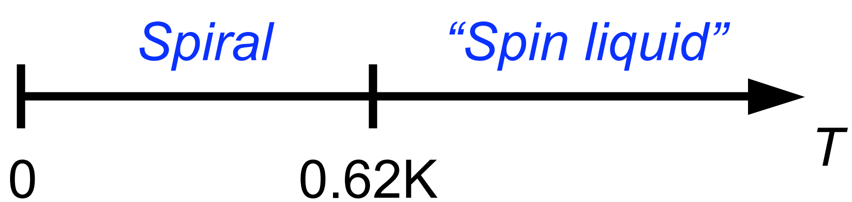
What about *inversion* symmetry breaking by DM?

- With no inversion symmetry can add one (and only one) Dirac mass term
- Gives staggered "chemical potential" for the vortices
- Drives system into incommensurate spiral at low energies





- Consistent with Cs_2CuCl_4!

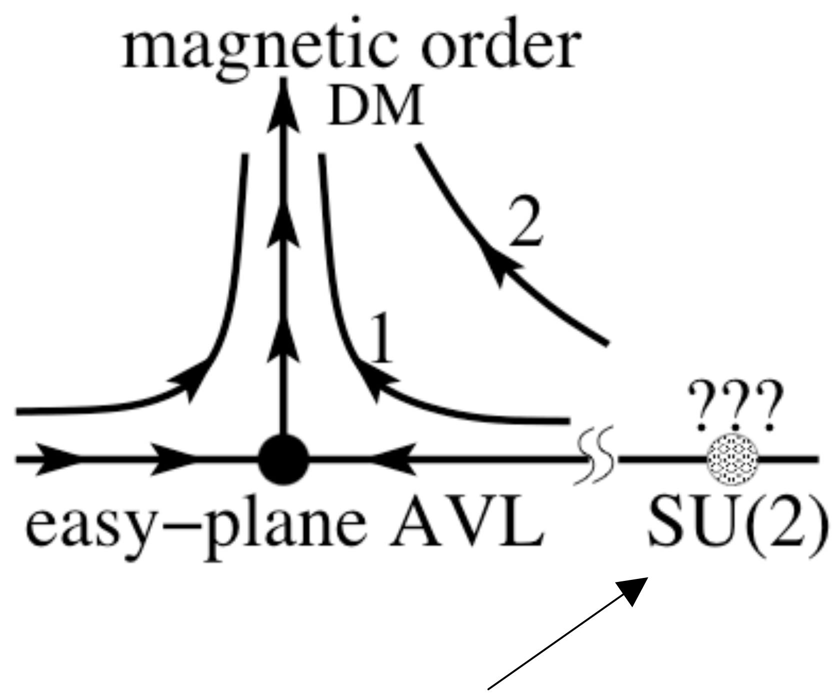


Relevance to Cs_2CuCl_4?

• Schematic RG flows near the Algebraic Vortex Liquid (AVL) fixed point

• Horizontal axes: Easy-plane spin model

• DM interaction breaking inversion symmetry



Putative SU(2) invariant "Algebraic Spin Liquid" for anisotropic triangular lattice

a) RG Flow #1: AVL fixed point controls the behavior over intermediate energy scales

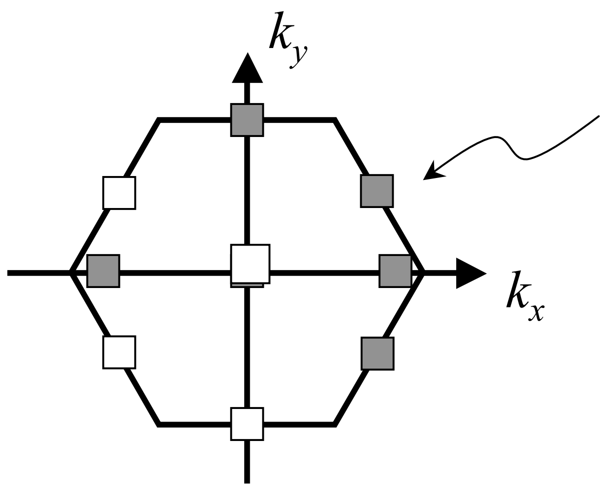
b) RG Flow #2: Putative SU(2) invariant fixed point controls the intermediate scales

Which flow is appropriate for Cs_2CuCl_4?

Algebraic Vortex Liquid: Experimental predictions

- Easy-plane symmetry ⇒ $\langle S^+(\mathbf{q}, \omega)\, S^-(-\mathbf{q}, -\omega)\rangle$ should dominate structure factor
  - Check with polarized neutrons?
- In-plane spin correlations:

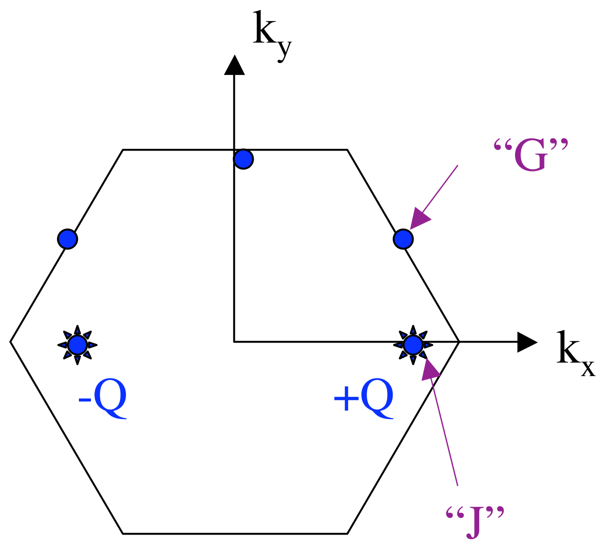
S^+ adds $S^z = 1$ ⟺ $M^\dagger$ adds $(\Delta \times a) = 2\pi$ flux *(Monopole)*



5 momenta Q_j carried by critical monopole operators, which have the ***same scaling dimension*** η

$$\langle S^+(\mathbf{Q}_j, \omega) S^-(-\mathbf{Q}_j, -\omega)\rangle \sim \frac{A_j}{\omega^{2-\eta}}$$

Comparison with Cs_2CuCl_4



1st B-zone of triangular lattice

Key prediction:

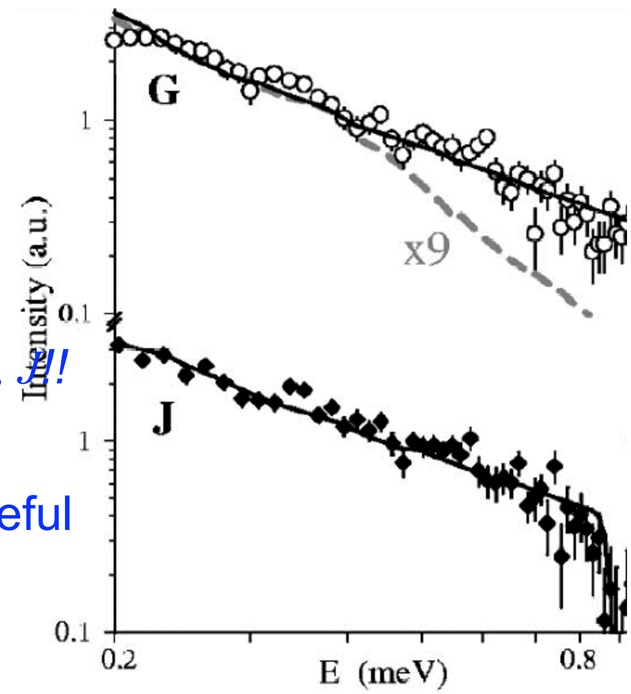
$$\langle S^+(\mathbf{Q}_j,\omega)S^-(-\mathbf{Q}_j,-\omega)\rangle \sim \frac{A_j}{\omega^{2-\eta}}$$

Same η for all $\mathbf{Q}_i; \quad i = 1, 2, ..., 5$

Inelastic Neutron Scattering

Experiments find same scaling exponent at G, J!

- but, not much scattering along $k_x = 0$
- closer look between J and G would be useful



R. Coldea, D. A. Tennant, and Z. Tylczynski
Phys. Rev. B **68**, 134424 (2003)

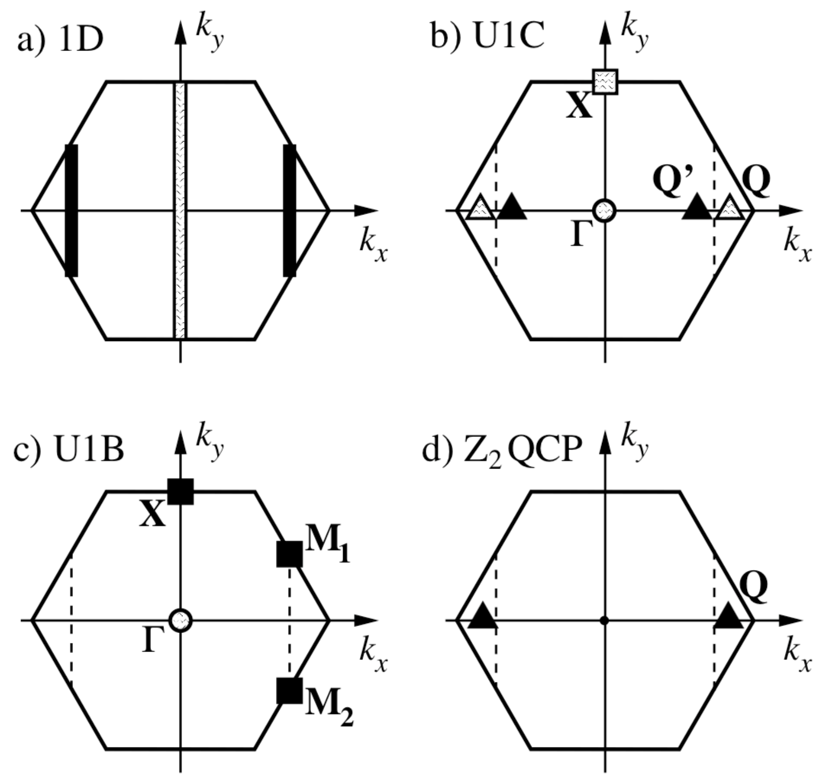
Some other theories for Cs_2CuCl_4

a) "One-dimensional": Decoupled chains and perturb in J' (Tsvelik et. al.)

b) Algebraic Spin Liquid with fermionic spinons and U(1) compact gauge field - pi flux thru half the triangles - U1C (X.G. Wen)

c) Another ASL with different mean field flux pattern - U1B (X.G. Wen)

d) Quantum critical point between a Z_2 spin liquid with gapped bosonic spinons and a spiral ordered phase (Isakov, Senthil and Kim)



Location of gapless s=1 excitations, leading to power law structure functions

$$\mathcal{S}(\mathbf{Q},\omega) \sim 1/\omega^{2-\eta}$$

Summary & Conclusions

- Mott insulators with one electron/unit cell are intrinsically strongly interacting, and a good place to look for paradigm shifting experiments

- Due to frustration spin-1/2 triangular antiferromagnets are good candidate systems for exotic spin liquids (possible realization of a "critical" spin liquid in Cs_2CuCl_4)

- Duality transformations (eg. boson-vortex) are extremely useful in exposing new physics and accessing novel quantum phases

- Much future work:
  - Characterize "critical" spin liquids?
  - Exotic (non-Fermi liquid) phases with itinerant electrons?
  - Unambiguously establish exotica in experiments??
  - Learn to engineer topological phases for fault tolerant quantum computing?