Due: Tuesday, February 5, 2019 by 5pm
Put homework in mailbox labelled 215B on 1st floor of Broida (by elevators).

1.) CORRELATIONS FOR TWO S=1/2’s

Consider the classic EPR set-up, where two spin-1/2 particles, created in a singlet state, are emitted from a source at the origin and detected in two well separated detectors, detector A located at $x = -L$, say, and detector B at $x = L$. Alice and Bob, located at their respective detectors, employ a Stern-Gerlach apparatus to measure the component of the particles’ spins ($\pm 1$ in units of $\hbar/2$) projected along some axes, $\hat{a}$ for Alice and $\hat{b}$ for Bob.

A.) What is the probability that both Alice and Bob will measure the spins of their respective particles parallel to $\hat{a}$ and $\hat{b}$ (ie both spin “up”)? Obtain a general expression for this probability, denoted $P_{++}$, in terms of $\hat{a}$ and $\hat{b}$.

B.) Suppose Alice measures her particle to have spin “down”, ie with value $-1$ along the $\hat{a}$ axis. What is the probability that Bob will find his particle to have spin “up” along his $\hat{b}$ axes?

C.) Suppose, after each measurement, Alice and Bob take their results ($\pm 1$) and multiply them together. Let $D(\hat{a}, \hat{b})$ denote the average of this product taken over many runs. Find an expression for $D(\hat{a}, \hat{b})$.

2.) THREE PHOTONS: BEYOND BELL

Let $|H\rangle$ and $|V\rangle$ represent horizontally and vertically polarized states of a photon, respectively. Suppose we have an experimental optics setup that is able to consistently generate the three-photon state given by

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2|H\rangle_3 + |V\rangle_1|V\rangle_2|V\rangle_3).$$

(1)

In the following, note that left and right circularly polarized light are given by $|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle)$ and $|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle)$. We also consider linear polarization along an axis rotated 45 degrees from the original axis, given by $|H'\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|V'\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$. Note that $\langle H'|V\rangle = 0$ and $\langle L|R\rangle = 0$, so any such pair of orthonormal kets can be used as our basis for measurement.

From here forward, we will define an $\alpha$-measurement to be a measurement in the $|H'\rangle$, $|V'\rangle$ basis, and a $\beta$-measurement to be a measurement in the $|L\rangle$, $|R\rangle$ basis. An $m_1m_2m_3$ experiment is defined to be one which makes an $m_1$-measurement on the first photon, an $m_2$-measurement on the second photon, and an $m_3$-measurement on the third photon. For instance, an $\alpha\alpha\alpha$ experiment measures and records the polarization of each photon in the $|H'\rangle$, $|V'\rangle$ basis.

A.) As a warmup, re-write the state $|\psi\rangle$ such that all three photons are expressed in the $|H'\rangle$, $|V'\rangle$ basis. The result should be an equal weighted superposition of four terms. What is the possibility of each possible outcome of such an $\alpha\alpha\alpha$ experiment? Notice that this experiment has a peculiar feature, namely that knowledge of any two photon measurements completely determines the measurement outcome of the third photon.

B.) Re-write the state $|\psi\rangle$ such that the first two photons are written in the $|L\rangle$, $|R\rangle$ basis and the third photon is written in the $|H'\rangle$, $|V'\rangle$ basis. The result should (again) be an equal weighted superposition of four terms.

C.) Given the equation from the previous part, what is the probability for each possible result of a $\beta\beta\alpha$ experiment? Determine also the probabilities of each possible result of $\beta\alpha\beta$ and $\alpha\beta\beta$ experiments. (In the remaining parts of this problem, we will assume that these three experiments have been performed, and the results have been confirmed to match those predicted by quantum mechanics.)

D.) Suppose that in any given run of the experiment, the “complete” state of the system is characterized by some hidden variable(s) $\lambda$, which specifies the measurement outcome of each photon in every possible basis. The functions $A_i(\lambda)$ take the values $\pm 1$ for measurement outcomes $|H'\rangle$ and $|V'\rangle$ of the $i$th photon, and the functions $B_i(\lambda)$ take the values $\pm 1$ for outcomes $|L\rangle$ and $|R\rangle$ of the $i$th photon. By considering all possible results of the $\beta\beta\alpha$ experiment that
would be consistent with quantum mechanics, determine the product \( B_1(\lambda)B_2(\lambda)A_3(\lambda) \). (Hint: each experimental outcome with nonzero probability should give the same product.) Likewise, what are the products \( B_1(\lambda)A_2(\lambda)B_3(\lambda) \) and \( A_1(\lambda)B_2(\lambda)B_3(\lambda) \)?

E.) Using these results, determine the product \( A_1(\lambda)A_2(\lambda)A_3(\lambda) \) that must result from a hidden variables theory. (Hint: what is \([B_i(\lambda)]^2\)?) List all results of an \( \alpha \alpha \alpha \) experiment that would be consistent with local hidden variables.

F.) Return now to quantum mechanics. Given each possible result (i.e. with nonzero probability) of an \( \alpha \alpha \alpha \) experiment, what would we have expected the product \( A_1A_2A_3 \) to be? (Hint: it may help to reference the result derived in the first part of this problem.) Do these results differ from the experimental outcomes predicted by a local hidden variables theory?

G.) Note that there is no inequality derived in this problem. In what way is the current experiment conceptually different from an experiment demonstrating the violation of Bell’s inequality for a two-photon system?