Problem Set 4 223B Condensed Matter Physics, Winter 2017

Due: Monday, March 13, 2017 by 11pm

Put homework in mailbox labelled 223B on 1st floor of Broida (by elevators).

1.) LONDON THEORY

Ashcroft and Mermin Chapter 34 problem 2.

2.) BOUNDARY BETWEEN TYPE I AND TYPE II SUPERCONDUCTIVITY

Within Landau theory for a superconductor in a magnetic field, the Gibbs free energy ($G = F - (1/4\pi)\int BH$) for the Meissner phase equals that for the normal phase when $H = H_c$. Using Landau theory, show that the free energy per unit area of an interface separating these two co-existing phases can be expressed as:

$$\gamma = \int_{-\infty}^{+\infty} dx[(-b/2)|\psi|^4 + \frac{1}{8\pi}(B - H_c)^2] \tag{1}$$

with the coordinate x running normal to the interface. Using the Ginzburg-Landau differential equations, show that when $\kappa = \lambda/\xi = 1/\sqrt{2}$, the interfacial energy γ is identically zero. Explain why this value of κ de-markates the boundary between type I and type II superconductors. [Hint: Take the magnetic field along the z-direction, and choose a gauge with $\vec{A} = A(x)\hat{y}$. The two Ginzburg-Landau equations are then differential equations for (real) $\psi(x)$ and $A(x)$. Assuming the integrand in (1) vanishes, gives one a third differential equation for the *two* functions, ψ and A. Show that these three differential equations can be simultaneously satisfied when $\kappa = 1/\sqrt{2}$. It is *not* necessary to obtain explicit solutions for the equations.]

3.) ISOLATED VORTEX and H_{c1}

At the lower critical field H_{c1}, the Gibbs energy of a Type II superconductor with one isolated vortex line equals the Gibbs energy with no lines present.

A.) Use this to show that

$$H_{c1} = 4\pi\epsilon_1/\Phi_0, \tag{2}$$

where ϵ_1 is the free energy per length of the vortex line.

B.) In the extreme Type II limit, ϵ_1 is dominated by the region outside of the vortex core (of size ξ), where London theory is valid (since ρ_s is a constant). Due to the singular structure at the vortex core, London's equation in the presence of a single vortex line should be modified:

$$(4\pi\lambda^2/c)\nabla \times \vec{J} + \vec{B} = \Phi_0\hat{z}\delta^2(x). \tag{3}$$

Show that the magentic field distribution around a vortex satisfies

$$B(r) \approx (\Phi_0/2\pi\lambda^2)ln(\lambda/r), \tag{4}$$

for distances $\xi << r << \lambda$ from the vortex, and $B(r) \approx exp(-r/\lambda)$ for $r >> \lambda$.

C.) The vortex line energy is a sum of field and kinetic energy contributions,

$$\epsilon_1 = \frac{1}{8\pi}\int d^2x\vec{B}^2 + \frac{1}{2}m\rho_s\int d^2x\vec{v}_s^2, \tag{5}$$

where $\vec{J} = \rho_s e^*\vec{v}_s$. Using the modified London equation (3), show that this can be expressed as

$$\epsilon_1 = \frac{1}{8\pi}\Phi_0\int d^2xB\delta^2(x) \approx \frac{1}{8\pi}\Phi_0 B(r = \xi), \tag{6}$$

where in the second expression have approximated the delta function with value at core radius. Combining Eqn's 2,4 and 6 gives an approximate expression for H_{c1} in terms of penetration and coherence lengths. Evaluate H_{c1} for $\lambda \approx 1500$ Angstroms and $\xi \approx 15$ Angstroms - values typical for the High-Tc cuprates.