Problem Set 6 215A- Quantum Mechanics - Fall 2017

Due: Friday, December 8, 2017 by 5pm

Put homework in mailbox labelled 215A on 1st floor of Broida (by elevators).

1.) Problems from Sakurai (2nd edition) Chapter 4: 4.1, 4.2, 4.3, 4.7, 4.8, 4.9, 4.11 and 4.12

2.) VARIATIONAL METHOD FOR 1d OSCILLATOR

To get a feel for how well the variational approximation works, it is useful to try it out on a simple soluble model - here the 1d Harmonic oscillator with Hamiltonian,

$$H = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega^2\hat{X}^2. \tag{1}$$

Using the following variational wave functions:

a.) $\psi_\alpha(x) = e^{-\alpha|x|}$,

b.) $\psi_\alpha(x) = e^{-\alpha x^2}$,

c.) $\psi_\alpha(x) = \alpha - |x|$ for $|x| \leq \alpha$ and $\psi = 0$ otherwise,

estimate the ground state energy of the harmonic oscillator, and *discuss* how your estimate compares with the exact result, $E_0 = \hbar\omega/2$.

A variational bound on the energy of the first excited state, E_1, can be obtained by using an odd parity variational wave function (why?). Estimate E_1 using

d.) $\psi_\alpha(x) = xe^{-\alpha|x|}$,

e.) $\psi_\alpha(x) = xe^{-\alpha x^2}$,

and compare to the exact result.

The quality of your variational estimate for the ground state *wavefunction* can be determined by evaluating the overlap between the (normalized) variational state $|\psi_{var}\rangle$ and the exact (normalized) ground state $|0\rangle$ as

$$\mathcal{O} = \sqrt{|\langle\psi_{var}|0\rangle|^2}. \tag{2}$$

Bonus: Evaluate $\mathcal{O}$ for the three variational states (a), (b) and (c) and discuss.