Problem Set 6 215A- Quantum Mechanics - Fall 2017

Due: Friday, December 8, 2017 by 5pm
Put homework in mailbox labelled 215A on 1st floor of Broida (by elevators).

1.) Problems from Sakurai (2nd edition) Chapter 4: 4.1, 4.2, 4.3, 4.7, 4.8, 4.9, 4.11 and 4.12

2.) VARIATIONAL METHOD FOR 1d OSCILLATOR

To get a feel for how well the variational approximation works, it is useful to try it out on a simple soluble model - here the 1d Harmonic oscillator with Hamiltonian,

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{X}^2. \tag{1}$$

Using the following variational wave functions:

a.) $$\psi_\alpha(x) = e^{-\alpha|x|},$$

b.) $$\psi_\alpha(x) = e^{-\alpha x^2},$$

c.) $$\psi_\alpha(x) = \alpha - |x| \text{ for } |x| \leq \alpha \text{ and } \psi = 0 \text{ otherwise,}$$

Estimate the ground state energy of the harmonic oscillator, and discuss how your estimate compares with the exact result, $$E_0 = \frac{\hbar \omega}{2}.$$

A variational bound on the energy of the first excited state, $$E_1$$, can be obtained by using an odd parity variational wave function (why?). Estimate $$E_1$$ using

d.) $$\psi_\alpha(x) = xe^{-\alpha |x|},$$
e.) $$\psi_\alpha(x) = xe^{-\alpha x^2},$$

and compare to the exact result.

The quality of your variational estimate for the ground state wavefunction can be determined by evaluating the overlap between the (normalized) variational state $$|\psi_{\text{var}}\rangle$$ and the exact (normalized) ground state $$|0\rangle$$ as

$$O = \sqrt{|\langle \psi_{\text{var}} | 0 \rangle|^2}. \tag{2}$$

Bonus: Evaluate $$O$$ for the three variational states (a), (b) and (c) and discuss.