Electrical current carried by neutral quasiparticles

Chetan Nayak,[1] Kirill Shtengel,[2] Dror Orgad,[1] Matthew P. A. Fisher,[3] and S. M. Girvin[3,4]
[1]Department of Physics and Astronomy, University of California at Los Angeles, Los Angeles, California 90095-1547
[2]Department of Physics and Astronomy, University of California at Irvine, Irvine, California 92697-4575
[3]Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030
[4]Department of Physics, Indiana University, Bloomington, Indiana 47405

(Received 25 May 2001; published 28 November 2001)

The current should be proportional to the momentum in a Galilean-invariant system of particles of fixed charge-to-mass ratio, such as an electron liquid in jellium. However, strongly-interacting electron systems can have phases characterized by broken symmetry or fractionalization. Such phases can have neutral excitations which can presumably carry momentum but not current. In this paper, we show that there is no contradiction: "neutral" excitations *do* carry current in a Galilean-invariant system of particles of fixed charge-to-mass ratio. This is explicitly demonstrated in the context of spin waves, the Bogoliubov–de Gennes quasiparticles of a superconductor, the one-dimensional electron gas, and spin-charge separated systems in $2+1$ dimensions. We discuss the implications for more realistic systems, which are not Galilean invariant.

DOI: 10.1103/PhysRevB.64.235113 PACS number(s): 72.10.$-$d, 74.25.$-$q

I. INTRODUCTION

Conventional wisdom holds that, in a Galilean-invariant system of particles of fixed charge-to-mass ratio e/m, the local current density is proportional to the local momentum density, $\mathbf{J}(\mathbf{x})=(e/m)\mathbf{P}(\mathbf{x})$. The conservation of total momentum then implies conservation of the total current, $(d/dt)\mathbf{J}=0$. This is a stronger condition than charge conservation, $(d/dt)\rho+\nabla\cdot\mathbf{J}=0$, since it implies that the real part of the conductivity is given by $\sigma(\omega)=(ne^2/m)\delta(\omega)$, where n is the particle density. One might imagine that this hypothetical situation has some applicability to extremely clean real systems in which the effects of the lattice are unimportant because the Fermi surface is far from any nesting vector and the electron-phonon coupling is very weak. In such a case, one would be tempted to forget about impurities and the lattice of ions altogether and focus on the electrons, which have a fixed charge-to-mass ratio e/m.

On the other hand, we have become accustomed to quantum number fractionalization, particularly in the context of quasi-one-dimensional materials,[1] the fractional quantum Hall effect,[2] and theories of high-temperature superconductivity and frustrated quantum magnets.[3–16] Spin-charge separation is one possible pattern of quantum number fractionalization. It leads to charged, spinless quasiparticles—often called "holons"—and neutral, spin-1/2 quasiparticles—often called "spinons." Conventional wisdom would lead us to expect that the latter, being neutral, would carry no current, even when endowed with nonzero momentum. This is merely the most extreme and exotic case of a general phenomenon: the low-energy quasiparticles of a strongly interacting system need not evince much resemblance to the underlying electron. This is true *a fortiori* if the low-temperature phase of the system exhibits fractionalization or broken symmetry. In particular, there is no reason why the quasiparticle charge-to-mass ratio should be e/m. A more familiar, but no less dramatic example is given by spin waves in a ferromagnet—neutral spin-1 excitations which carry momentum but, presumably, no current.

Clearly, there is some tension, if not an outright contradiction, between these two papers of conventional wisdom. The resolution, which we describe in this paper, is that "neutral" quasiparticles *do* carry current according to $\mathbf{J}(\mathbf{x})=(e/m)\mathbf{P}(\mathbf{x})$ in a Galilean-invariant system. However, even a small explicit breaking of Galilean invariance can have drastic consequences for this relation. As a result, even a small density of impurities or a weak periodic potential can result in a state in which neutral quasiparticles carry momentum but no current and the dc conductivity is zero rather than infinity.

The current carried by neutral quasiparticles can be understood as arising from a Doppler shift interaction between them and the charge carriers. The latter are always gapless in a Galilean-invariant system, and they mediate the coupling between the electromagnetic field and the neutral quasiparticles. We will illustrate our thesis in a number of different contexts: spin waves, the Bogoliubov–de Gennes quasiparticles of a superconductor (which carry momentum but are not charge eigenstates), the one-dimensional electron gas, and spin-charge separated systems in $2+1$ dimensions. Finally, we will comment on our results and their applicability to realistic systems, which do not have Galilean invariance.

II. SPIN WAVES IN AN ELECTRON LIQUID

As mentioned in the Introduction, one might think that the paradox is already manifest in the context of spin waves (or other collective excitations) which can carry momentum but ought not—if we are to think of them as neutral excitations—carry current. Since a spin wave is composed of an electron and a hole, it is, indeed, neutral. At a formal level, the creation operator for an $S_z=1$ spin wave,

$$S_+(\mathbf{x},t)=c^\dagger_\uparrow(\mathbf{x},t),c_\downarrow(\mathbf{x},t), \tag{1}$$

is invariant under a gauge transformation, $c_\alpha(\mathbf{x},t)\to e^{i\phi(\mathbf{x},t)}c_\alpha(\mathbf{x},t)$. Consequently, such an operator does not couple to the electromagnetic field through minimal coupling.

Nevertheless, a spin wave *does* carry current. When the vector potential **A** vanishes, the current takes the form

$$\mathbf{J}=\sum_{\mathbf{k}}\frac{e}{m}\mathbf{k}c^{\dagger}_{\alpha}(\mathbf{k})c_{\alpha}(\mathbf{k}). \tag{2}$$

The current operator has this form irrespective of the electron-electron interaction terms, so long as they are Galilean invariant—i.e., so long as they are momentum independent and translationally invariant.

Consider the operator which creates a spin wave of momentum **q**:

$$S_{+}(\mathbf{q})=\sum_{\mathbf{k}}c^{\dagger}_{\uparrow}(\mathbf{k}+\mathbf{q})c_{\downarrow}(\mathbf{k}). \tag{3}$$

In so doing, it actually creates current as well, as may be seen by taking its commutator with the current operator

$$[\mathbf{J},S_{+}(\mathbf{q})]=\frac{e}{m}\mathbf{q}S_{+}(\mathbf{q}). \tag{4}$$

Hence spin waves carry current. This is a purely kinematic statement which follows from the form of the current operator (2) which, in turn, follows from Galilean invariance. Our conclusion holds whether or not the electron liquid orders electronically.

However, it may be difficult to see how this electrical current appears in an effective field theory of spin waves in, for instance, the ferromagnetic state. Suppose we take our Galilean-invariant electronic Lagrangian,

$$\mathcal{L}=c^{\dagger}_{\alpha}(i\partial_t-eA_t)c_{\alpha}+\frac{1}{2m}c^{\dagger}_{\alpha}(i\nabla-e\mathbf{A})^2c_{\alpha}+\mathcal{L}_{\text{int}}, \tag{5}$$

and decouple $\mathcal{L}_{\text{int}}$ with a Hubbard-Stratonovich field **S** which couples linearly to $c^{\dagger}_{\alpha}\ \boldsymbol{\sigma}_{\alpha\beta}\ c_{\beta}$. We can integrate out the electrons and expand the resulting action about a ferromagnetic state which is ordered in the $\hat{\mathbf{z}}$ direction. On general grounds, we expect that the resulting effective action will be of the form

$$\mathcal{L}_{\text{eff}}=S_{+}i\partial_t S_{-}-D\nabla S_{+}\cdot\nabla S_{-}+\ \dots\ . \tag{6}$$

As we noted above, $S_{\pm}$ is invariant under a gauge transformation, so it is hard to imagine how it can be coupled to the electromagnetic field **A**. On the other hand, $\mathbf{J}=\partial\mathcal{L}/\partial\mathbf{A}$, so there will be no current carried by $S_{\pm}$ in the absence of such a coupling.

The resolution is that there is a coupling to **A** hidden in the " . . . " in Eq. (6). If it is difficult to guess the form of this term, this is because we would be wrong in assuming that it is local. Since we have integrated out gapless fermionic degrees of freedom in obtaining Eq. (6), we should actually expect nonlocal terms. There are no nonlocal terms in the spin dynamics of Eq. (6) because the up- and down-spin Fermi wave vectors are different as a result of the development of ferromagnetic order; consequently spinful excitations of the Fermi surface have a minimum wave vector. However, the charged excitations extend down to $\mathbf{q}=0$, and the coupling of $S_{\pm}$ to **A** is, indeed, nonlocal. It may be obtained by computing the diagrams of Fig. 1 and takes the form

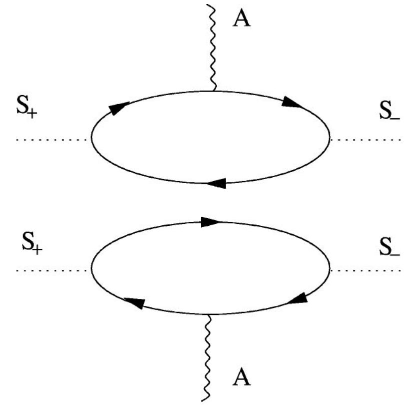

FIG. 1. The diagrams which contribute to the coupling between spin waves and the electromagnetic field.

$$\mathcal{L}_{\text{A}}=\frac{e}{m}\mathbf{A}^{T}\cdot S_{+}i\nabla S_{-}. \tag{7}$$

In this equation, $\mathbf{A}^T$ denotes the transverse part of **A**, which is given in momentum space by

$$\mathbf{A}^{T}(\mathbf{q})=\mathbf{A}(\mathbf{q})-\mathbf{q}\frac{\mathbf{q}\cdot\mathbf{A}(\mathbf{q})}{q^2}. \tag{8}$$

This is both nonlocal and *gauge invariant* since a gauge transformation,

$$\mathbf{A}(\mathbf{q})\rightarrow\mathbf{A}(\mathbf{q})+\mathbf{q}\,\phi(\mathbf{q}), \tag{9}$$

with $\phi(\mathbf{q})$ arbitrary, leaves $\mathbf{A}^{T}(\mathbf{q})$ unchanged. Since $S_{+}\nabla S_{-}$ is also invariant under a gauge transformation, the entire term (7) is gauge-invariant, which is a cause for some relief.

Note that spin waves were empowered with the ability to carry a current by the gapless charge degrees of freedom with which they interact. In an insulating ferromagnet, spin waves will not carry a current proportional to their momentum. Since insulating behavior will only occur when a system is not translationally invariant, there is no contradiction here.

III. QUASIPARTICLES IN A SUPERCONDUCTOR

The Bogoliubov–de Gennes quasiparticles of a superconductor are coherent superpositions of electrons and holes. Hence they do not have a well-defined charge. As the Fermi surface is approached, a Bogoliubov–de Gennes quasiparticle becomes an equal superposition of electron and hole; thus one might be tempted to assign it zero charge in this limit. This is not an academic question in an unconventional superconductor such as one of $d_{x^2-y^2}$ symmetry—as the high-T_c cuprates are believed to be—since, in the absence of a full gap, quasiparticles will be thermally excited down to zero temperature and their ability to carry current will have an impact on the superfluid density.

For the sake of concreteness, let us consider a two-dimensional $d_{x^2-y^2}$ superconductor and focus on its nodal quasiparticles. We assume that the system is Galilean invariant so that the order parameter spontaneously breaks rotational symmetry when it chooses nodal directions. The effective action for a superconductor is of the form

$$S=\int_{k_y>0}\frac{d^2k}{(2\pi)^2}dt\Psi^\dagger(k,t)[(i\partial_t-\tau^z eA_t)-\tau^z[\epsilon(\mathbf{k}+\tau^z e\mathbf{A})-\mu]-\tau^+\Delta(k)-\tau^-\Delta^\dagger(k)]\Psi(k,t), \tag{10}$$

where we have used the Nambu-Gorkov notation

$$\Psi_{a\alpha}(\vec{k})=\begin{bmatrix} c_{k\uparrow}\\ c^\dagger_{-k\downarrow}\\ c_{k\downarrow}\\ -c^\dagger_{-k\uparrow}\end{bmatrix} \tag{11}$$

and the τ^i are Pauli matrices which act on the particle-hole index a. If we consider the four-component object as composed of two two-component blocks, the upper and lower blocks, then the τ^i mix the components within a block. There are also Pauli matrices σ^i which act on the spin indices α and mix the upper block with the lower block.

We will linearize this action about the nodes of $\Delta(k)=\Delta_0(\cos k_x a-\cos k_y a)$. We must retain two fermion fields, one for each pair of antipodal nodes, but these pairs of nodes are not coupled to each other in the low-energy limit, so we will often focus on just one. By linearizing about the nodes, we are approximating the momentum of an electron by k_F and discarding the deviation from the Fermi surface. Hence we will verify that the relation $\mathbf{J}=(e/m)\mathbf{P}$ is satisfied *to this level of approximation*, which means $\mathbf{J}=(e/m)N_{\mathrm{qp}}\mathbf{k}_F$, where $N_{qp}=\Psi^\dagger\Psi$ is the difference between the number of electrons at one node and the number at the antipodal node. If we kept the full Galilean-invariant expression $\epsilon(k)=k^2/2m$, then we could verify $\mathbf{J}=(e/m)\mathbf{P}$ exactly. We will do this in one dimension, where it is particularly instructive. For now, we will content ourselves with a crude verification.

We align our coordinate system along the nodal dierection and linearize the single-particle dispersion: $\epsilon(k)-\mu\approx(k_F/m)k_x$, where k_x is the momentum perpendicular to the Fermi surface, measured away from the node. A similar expression holds for the other pair of nodes, with k_x replaced by k_y. We also linearize the gap about the nodes,

$$\Delta\tau^+\approx v_\Delta\tau^+e^{ie\varphi/2}(-i\partial_y)e^{ie\varphi/2}, \tag{12}$$

where $e^{ie\varphi}$ is the phase of the superconducting order parameter. Some care was needed in obtaining the correct ordering of derivatives and φ's; for details, see Refs. 13 and 17. Integrating out the electronic states far from the nodes and the fluctuations of the amplitude of the order parameter, we obtain the action

$$S=\int\Psi^\dagger\Big[(i\partial_t-\tau^z eA_t)+\tau^z\frac{k_F}{m}(i\partial_x-\tau^z eA_x)+v_\Delta\tau^s e^{ies\varphi/2}i\partial_y e^{ies\varphi/2}\Big]\Psi+\frac{1}{2}\rho_s\int\Big[\frac{1}{v_c^2}(\partial_t\varphi+2A_t)^2-(\partial_i\varphi+2A_i)^2\Big]+\dots, \tag{13}$$

where $s=\pm$, and ρ_s and v_c are the bare superfluid density and velocity. The " . . . " includes the action for the other pair of nodes and higher-order terms, which we neglect.

Following Ref. 13, we can simplify this action by defining *neutral quasiparticles* χ according to[18]

$$\chi=\exp(-ie\varphi\tau^z/2)\Psi. \tag{14}$$

The action now takes the form

$$S=\int\chi^\dagger\Big[i\partial_t+\tau^z\frac{k_F}{m}i\partial_x+v_\Delta\tau^x i\partial_y\Big]\chi-\frac{1}{2}\int\Big[e\chi^\dagger\tau^z\chi(\partial_t\varphi+2A_t)+e\chi^\dagger\chi\frac{k_F}{m}(\partial_x\varphi+2A_x)\Big]+\frac{1}{2}\rho_s\int\Big[\frac{1}{v_c^2}(\partial_t\varphi+2A_t)^2-(\partial_i\varphi+2A_i)^2\Big]+\dots\ . \tag{15}$$

The quasiparticle annihilation operator χ is gauge invariant since it is neutral, but φ, which is charged, is not. The action (15) is gauge invariant because χ is only coupled to gauge-invariant quantities, such as the superfluid density and current, $\partial_\mu\varphi+2A_\mu$.

These neutral excitations nevertheless carry current. By differentiating the Lagrangian of Eq. (15) with respect to A_x, we find that the current in the x direction is

$$J_x=2\rho_s(\partial_x\varphi+2A_x)+\frac{e}{m}k_F\chi^\dagger\chi. \tag{16}$$

The first term is the supercurrent; it derives from the final line of Eq. (15). The second term comes from the third line of Eq. (15), and it states that the quasiparticles carry a current which is e/m times their momentum k_F:

$$J_x^{\mathrm{qp}}=\frac{e}{m}k_F\chi^\dagger\chi. \tag{17}$$

By differentiating Eq. (15) with respect to A_t, we find that the corresponding charge density is $\rho=-2(\rho_s/v_c^2)(\partial_t\varphi+2A_t)+e\chi^\dagger\tau^z\chi$. The second term is the quasiparticle contribution. Although the quasiparticles are neutral in the sense of being gauge invariant, they contribute to both the charge and current densities.

Suppose that we integrate out the fluctuations of the phase of the superconducting order parameter. What does the coupling between the quasiparticles and the electromagnetic field look like?

To integrate out φ, it is convenient to use the dual representation in which φ is replaced by a dual gauge field a_μ. In this dual representation, Eq. (15) takes the form (for details, see Ref. 13)

$$S=\int \chi^\dagger\left[i\partial_t+\tau^z\frac{k_F}{m}i\partial_x+v_\Delta\tau^x i\partial_y\right]\chi-\int \frac{1}{2\rho_s}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + \left(2A_\mu-\frac{1}{2\rho_s}J_\mu^{\mathrm{qp}}\right)\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda+\ \ldots, \tag{18}$$

where we have chosen units with $v_c=1$ to facilitate the use of ''relativistic'' notation. The dual gauge field a_μ is related to the total current J_μ by

$$J_\mu=\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda. \tag{19}$$

It only enters the action in this transverse combination which is automatically conserved. Furthermore, this means that $\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda$ is only coupled to the transverse parts of A_μ and J_μ^{qp}. Since it appears quadratically, we can now integrate it out, obtaining

$$S=\int \chi^\dagger\left[i\partial_t+\tau^z\frac{k_F}{m}i\partial_x+v_\Delta\tau^x i\partial_y\right]\chi-\int A_\mu^T J_\mu^{T\mathrm{qp}}+\ \ldots\ . \tag{20}$$

The coupling between A_μ and J_μ^{qp} is nonlocal because it only couples their transverse parts, as in Eq. (7). Again, since we have integrated out a_μ which is formally a gapless degree of freedom when A_μ is held fixed, we should not be surprised by the appearance of a nonlocal coupling between A_μ and J_μ^{qp} through which only their gauge-invariant transverse components are coupled. Since a_μ does not couple to the longitudinal parts of A_μ and J_μ^{qp}, it is not possible to generate terms involving them.

Again, the ability of quasiparticles to carry a current depends on their interaction with gapless charged degrees of freedom—in this case, a supercurrent. If the conductivity associated with this supercurrent (i.e., its Drude weight, *not* its superfluid density,[20] see Sec. V) is reduced, e.g., by the localization of some electrons at impurities, then Bogoliubov–DeGennes quasiparticles will carry a reduced current as well.

IV. SPIN-CHARGE SEPARATED ONE-DIMENSIONAL ELECTRON GAS

The one-dimensional electron gas can be described completely in terms of its spin and charge collective modes.[1] The electron itself is a combination of charge and spin carrying solitons—holons and spinons—in these collective modes. Because these collective modes have different velocities, the charge and spin of an electron move apart in time. Both the charge and spin modes can carry momentum, but one might assume that only the charged mode should couple to the electromagnetic field and carry current. Furthermore, the velocities v_c and v_s of these modes depend on the interaction strength; they are, in general, different from k_F/m, which might lead one to expect that even the charged mode will carry a current which is not equal to e/m times its momentum. However, we have come, by now, to distrust such expectations.

The Hamiltonian density is often written in the Bosonized form

$$\mathcal{H}=\frac{1}{2}v_c\left[K_c(\partial_x\varphi_c)^2+\frac{1}{K_c}(\partial_x\theta_c)^2\right] +\frac{1}{2}v_s\left[K_s(\partial_x\varphi_s)^2+\frac{1}{K_s}(\partial_x\theta_s)^2\right]+v_F k_F\sqrt{\frac{2}{\pi}}\partial_x\theta_c. \tag{21}$$

The final term is the Fermi energy (for a perfectly linear spectrum) multiplied by the electron number. This term is cancelled by the chemical potential, but we have retained it for purposes of comparison with the corresponding expression for the momentum density. If the system respects $SU(2)$ spin-rotational symmetry, then $K_s=1$. If $K_c=1$ as well, then the Hamiltonian describes free fermions. As K_c is shifted away from 1 by the interactions, the charge of the fundamental charged soliton is also shifted away from e. φ_c and θ_c are dual variables, $v_c\partial_x\theta_c=K_c\partial_t\varphi_c$, as are φ_s and θ_s. They are symmetric and antisymmetric combinations of left- and right-moving fields, $\theta_c=\phi_{cR}-\phi_{cL}$, $\varphi_c=\phi_{cR}+\phi_{cL}$. The charge and spin modes are symmetric and antisymmetric combinations of up- and down-spin modes, $\theta_c=(\theta_\uparrow+\theta_\downarrow)/\sqrt{2}$, $\theta_s=(\theta_\uparrow-\theta_\downarrow)/\sqrt{2}$, etc.

This Hamiltonian describes the physics of interacting fermions with a spectrum which is linearized about the Fermi surface, $\pm k_F$. The annihilation operator for a right-moving spin-up electron is

$$\psi_{R\uparrow}=\frac{1}{\sqrt{2\pi a}}e^{-i\sqrt{\pi/2}(\varphi_c+\theta_c)}e^{-i\sqrt{\pi/2}(\varphi_s+\theta_s)}, \tag{22}$$

where a is a short-distance cutoff. Similar relations hold for down-spin, right-moving electrons and left-moving electrons of both spins. The right- and left-moving charge densities are

$$\rho_{R,L}=\frac{1}{\sqrt{2\pi}}\partial_x(\theta_c\pm\varphi_c). \tag{23}$$

The right- and left-moving S_z densities are given by a similar expression with θ_c,φ_c replaced by θ_s,φ_s.

The momentum can be obtained from the energy-momentum tensor $T_{\mu\nu}$. While the Hamiltonian density is the tt component, $\mathcal{H}=T_{tt}$, the momentum density is given by $P=T_{tx}$.

$$P=k_F\sqrt{\frac{2}{\pi}}\partial_x\varphi_c+[(\partial_x\varphi_c)(\partial_x\theta_c)+(\partial_x\varphi_s)(\partial_x\theta_s)]. \tag{24}$$

Note that this takes a somewhat different form than is usual for relativistic scalar fields since excitations about the ground state are centered at $\pm k_F$; the first term would not be present in an ordinary relativistic system at zero density, where low-energy excitations are centered about $k=0$. It is the counterpart to the final term in Eq. (21); it assigns momentum $\pm k_F$ to each right or left mover. The second term accounts for possible changes in the local value of k_F.

In order to determine the current operator, we modify the Hamiltonian via minimal coupling, which replaces $\partial_x\varphi_c$ with $\partial_x\varphi_c - e\sqrt{2/\pi}A_x$. We now differentiate with respect to A_x to obtain $J_x = -\partial\mathcal{H}/\partial A_x$. This coupling is dictated by the fact that $\varphi_c \to \varphi_c - e\sqrt{2/\pi}\chi$ when $\psi_{R,L\alpha} \to e^{ie\chi}\psi_{R,L\alpha}$, $A_x \to A_x - \partial_x\chi$. Since $v_c\partial_x\theta_c = K_c\partial_t\varphi_c$, it does not couple to A_x.

However, before we do this, we need to exercise some care with regards to Galilean invariance. We would like to consider only momentum-independent interactions. Hence the interaction terms cannot have independent coefficients λ_{RR} and λ_{RL} for the $\rho_R\rho_R + \rho_L\rho_L$ interaction and the $\rho_R\rho_L$ interaction. The only allowed local interaction between charge densities is a simple density-density interaction of the form

$$\lambda\rho\rho = \lambda(\rho_R + \rho_L)^2 = \lambda(\rho_R\rho_R + \rho_L\rho_L) + 2\lambda\rho_R\rho_L = \frac{2\lambda}{\pi}(\partial_x\theta_c)^2, \tag{25}$$

i.e., $\lambda_{RL} = 2\lambda_{RR}$. If $\lambda_{RL} \neq 2\lambda_{RR}$, the Hamiltonian will contain a term of the form $(\rho_R - \rho_L)^2$, which is proportional to the total momentum squared, in which case the Hamiltonian is not Galilean invariant. This is the case for the edge states of a quantum Hall bar or quantum Hall line junction.[19] Hence when we look at the charged sector of the Hamiltonian (21), which arises by combining the free and interaction terms,

$$\mathcal{H}_{\text{charge}} = \frac{1}{2}\frac{k_F}{m}[(\partial_x\varphi_c)^2 + (\partial_x\theta_c)^2] + \frac{2\lambda}{\pi}(\partial_x\theta_c)^2 = \frac{1}{2}v_c\left[K_c(\partial_x\varphi_c)^2 + \frac{1}{K_c}(\partial_x\theta_c)^2\right], \tag{26}$$

we see that $v_c K_c = k_F/m$. In other words, in a Galilean-invariant system, the change in the charge velocity is precisely compensated by the change in the soliton charge so that their product, which will determine the current, is the same as the free fermion value, k_F/m.

The second point which requires some care is the linearization of the Hamiltonian. By linearizing our Hamiltonian about the Fermi surface, we are approximating our system by a "relativistic" one. In a relativistic system, the current density and momentum density cannot be proportional to each other since the former is the spatial component of a vector J_μ and the other is a component of a tensor $T_{\mu\nu}$ (the total momentum *is* the spatial component of vector, but this is obtained by integrating the momentum density over the entire system); a relation of the form $J_x = (e/m)T_{tx}$ would break relativistic invariance. Hence we need to retain the terms which break relativistic invariance and contain the information about Galilean invariance. While the linearized terms in the Hamiltonian are of the form $v_F(k-k_F)$, the terms which "know" about Galilean invariance are of the form $(k-k_F)^2/2m$. These terms actually couple the spin and charge modes, thereby resulting in an electrical current carried by spinons.

To see this, consider a term in the Hamiltonian which gives a quadratic spectrum, $(k-k_F)^2/2m$, and its bosonized form,

$$\psi^\dagger_{R\uparrow}\frac{1}{2m}(i\partial_x)^2\psi_{R\uparrow} = \frac{1}{3}\frac{1}{2\pi}\frac{1}{2m}\left[\partial_x\sqrt{\frac{\pi}{2}}(\varphi_c + \theta_c + \varphi_s + \theta_s)\right]^3 + \text{total derivative terms.} \tag{27}$$

Hence, summing over both spins and over right and left movers, we have

$$\psi^\dagger_{R\alpha}\frac{1}{2m}\partial_x^2\psi_{R\alpha} + \psi^\dagger_{L\alpha}\frac{1}{2m}\partial_x^2\psi_{L\alpha} = \frac{1}{2m}\sqrt{\frac{\pi}{2}}[(\partial_x\varphi_c)^2(\partial_x\theta_c) + 2(\partial_x\varphi_c)(\partial_x\varphi_s)(\partial_x\theta_s)] + \text{terms which do not contain } \varphi_c. \tag{28}$$

In a Galilean-invariant system, with single-particle kinetic energy $k^2/2m$, these are the only other terms which we must add.

Hence, going beyond linearization about the Fermi points and retaining the quadratic single-particle spectrum of a Galilean-invariant system, we have the following Hamiltonian:

$$\mathcal{H} = \frac{1}{2}\left[\frac{k_F}{m}(\partial_x\varphi_c)^2 + \frac{v_c}{K_c}(\partial_x\theta_c)^2\right] + \frac{1}{2}v_s\left[K_s(\partial_x\varphi_s)^2 + \frac{1}{K_s}(\partial_x\theta_s)^2\right] + \frac{1}{2m}\sqrt{\frac{\pi}{2}}[(\partial_x\varphi_c)^2(\partial_x\theta_c) + 2(\partial_x\varphi_c)(\partial_x\varphi_s)(\partial_x\theta_s)] + \text{terms which do not contain } \varphi_c. \tag{29}$$

If we now apply minimal coupling, $\partial_x\varphi_c \to \partial_x\varphi_c - e\sqrt{2/\pi}A_x$ and differentiate with respect to A_x to obtain $J_x = -\partial\mathcal{H}/\partial A_x$, we find the current operator:

$$J_x = e\frac{k_F}{m}\sqrt{\frac{2}{\pi}}\partial_x\varphi_c + \frac{e}{m}[(\partial_x\varphi_c)(\partial_x\theta_c) + (\partial_x\varphi_s)(\partial_x\theta_s)]. \tag{30}$$

Comparing this expression with Eq. (24), we see that it satisfies the relation $J = (e/m)P$.

The charged field $\partial_x\varphi_c$ carries large momentum k_F. The spin field $\partial_x\varphi_s$ only carries the small momentum of deviations from the Fermi points (as do $\partial_x\theta_c$ and $\partial_x\theta_s$). Hence the latter can get lost in the shuffle if we only keep the leading terms in a gradient expansion about the Fermi points. To see the relation between current and momentum, we must keep the quadratic terms in both.

Note that the condition $\lambda_{RL} = 2\lambda_{RR}$ as well as the constraints on the cubic terms in the Hamiltonian both followed from Galilean invariance. Even a mild breaking of this invariance such as that caused by a lattice which is far from any nesting condition could lead to a violation of these conditions and hence of the relation between the current and the momentum.

V. SPIN-CHARGE SEPARATION IN 2+1 DIMENSIONS

We will describe a spin-charge separated state as a quantum disordered superconducting state[13,14] which has the advantage, in the current context, of allowing us to take as our starting point the discussion in Sec. III of Bogoliubov–deGennes quasiparticles in a superconductor.

A quantum-disordered d-wave superconductor can be described by an extension of Eq. (18) to include vortices:

$$S=\int \chi^{\dagger}\left[i\partial_t+\tau^z\frac{k_F}{m}i\partial_x+v_\Delta\tau^x i\partial_y\right]\chi-\int \frac{1}{2\rho_s}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 +\left(2A_\mu-\frac{1}{2\rho_s}J_\mu^{\mathrm{qp}}\right)\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda+\int |(i\partial_\mu+2a_\mu)\Phi_{hc/e}|^2 -V(\Phi_{hc/e}). \quad (31)$$

The last line of Eq. (31) implements the Magnus force interaction between vortices and the supercurrent. Here, $\Phi_{hc/e}$ is the annihilation operator for a flux hc/e vortex, which we assume is the lightest vortex near the quantum-disordered state. If $hc/2e$ vortices condense instead, then spin and charge are confined.[14] When superconductivity is destroyed through the condensation of flux hc/e vortices, the resulting state supports holons, which are spinless, charge-e solitons in the vortex condensate, and nodons or spinons χ, which are neutral spin-1/2 excitations.

When vortices condense, superconductivity is destroyed because magnetic flux is no longer expelled. In other words, the superfluid density vanishes. In the dual description offered in Eq. (31), charges $\epsilon_{ij}\partial_i a_j$ enter the vortex condensate in a lattice—a "holon Wigner crystal."[13,14] However, in a Galilean-invariant system, the Wigner crystal can slide. Hence, even though the superconductivity is destroyed with the disappearance of the Meissner effect, the system is still a perfect conductor. Thus when we integrate out a_μ, we will still obtain a coupling between A_μ and J_μ^{spinon} which is of the form

$$\int A_\mu\langle \epsilon_{\mu\alpha\beta}\partial_\alpha a_\beta \epsilon_{\nu\gamma\delta}\partial_\gamma a_\delta\rangle J_\nu^{\mathrm{spinon}}. \quad (32)$$

In the limit of $\mathbf{q}=0$, $\omega\to 0$, this is determined by the conductivity— or Drude weight—which is the same as in the superconducting case. (It is not determined by the Meissner or diamagnetic response, which vanishes.) Hence, upon integrating out a_μ and $\Phi_{hc/e}$, we obtain the same induced coupling between spinons and the electromagnetic field $A_\mu^T J_\mu^{T\mathrm{spinon}}$ that we obtained for Bogoliubov–de Gennes quasiparticles.

However, even *infinitesimal* translational symmetry breaking, such as that caused by a small density of impurities, will pin the holon Wigner crystal. Consequently, the system will be an insulator and a_μ will be gapped. The coupling between spinons and the electromagnetic field will now be of the form[13]

$$S_{\mathrm{coupling}}=\int A_\mu(\partial^2 J_\mu^{\mathrm{spinon}}-\partial_\mu\partial_\nu J_\nu^{\mathrm{spinon}}). \quad (33)$$

In other words, spinons will be truly neutral since they do not carry a current proportional to their momentum density, in contrast to the merely neutral spinons that do.

Note that holons are not necessarily bosonic. A bosonic holon can form a bound state with an uncondensed $hc/2e$ vortex, or "vison," thereby becoming Fermionic.[21] In this case, the holon Wigner crystal is not the only possible nonsuperconducting ground state because the holons could form a perfectly conducting Fermi liquid. If the spinons pair and form a spin gap, then a spin-gapped metallic state can result, in which spinons carry a current proportional to their momentum density.

VI. DISCUSSION

The basic form of the interaction between neutral and charged quasiparticles is $J_\mu^{\mathrm{neutral}}J_\mu^{\mathrm{charged}}$. It can be interpreted as a Doppler shift by which the motion of the neutral quasiparticles brings the charged ones along for the ride. As a result, the relation $\mathbf{J}(\mathbf{x})=(e/m)\mathbf{P}(\mathbf{x})$ is satisfied even in a system with formally neutral quasiparticles. The facility with which the charge carriers can move along with the neutral quasiparticles follows as a consequence of Galilean invariance. However, even in the absence of Galilean invariance, the charge carriers can remain gapless; in such a case, the neutral quasiparticles can carry a current, but it will not be fixed to the value $\mathbf{J}(\mathbf{x})=(e/m)\mathbf{P}(\mathbf{x})$.

On the other hand, it is possible for a mild violation of Galilean invariance to have dramatic consequences for the relationship between current and momentum and hence for the conductivity. In the case of spin-charge separation in $2+1$ dimensions, we saw that infinitesimal translation symmetry breaking can make a perfect conductor into an insulator; as a consquence, neutral quasiparticles which carry a current proportional to their momentum become truly neutral quasiparticles carrying no current. Similarly, spin waves in a Galilean-invariant electron system carry current, but spin waves in an insulating ferromagnet on a lattice do not carry current. Thus the lattice has a large effect on the electrical properties of spin waves, even though it does not seem to be particularly important for the magnetic properties of the ferromagnet phase. Even in $1+1$ dimensions, in those situations in which the effects of the ionic lattice are otherwise mild because the Fermi surface is far from nested, the relation between current and momentum can be strongly violated as a result of the effect of the lattice on interaction parameters and "small" corrections to the band dispersion.

ACKNOWLEDGMENTS

We wish to thank S. Kivelson and T. Senthil for helpful discussions. C.N. was supported by the National Science Foundation under Grant No. DMR-9983544 and the A.P. Sloan Foundation. K.S. was supported by the Department of Energy under Grant No. DE-FG03-00ER45843. S.M.G. was supported by the National Science Foundation under Grant Nos. DMR-0087133 and PHY9907949. M.P.A.F. was supported by the National Science Foundation under Grant Nos. DMR9704005 and PHY9907949.

[1] For reviews, see, e.g., V. Emery, in *Highly-Conducting One-Dimensional Solids*, edited by J. T. Devreese, R. P. Evrard, and V. E. van Doren (Plenum, New York, 1979); A. O. Gogolin, A. A. Nersesyan, and A.M. Tsvelik, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, England, 1998); E. Fradkin, *Field Theories of Condensed Matter Systems* (Addison-Wesley, New York, 1991).

[2] See, e.g., *The Quantum Hall Effect*, edited by R. Prange and S. M. Girvin (Springer-Verlag, New York, 1987); *Perspectives in Quantum Hall Effects: Novel Quantum Liquids in Low-Dimensional Semiconductor Structures*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997), and references therein.

[3] P. W. Anderson, *The Theory of Superconductivity in the High-*T_c *Cuprates* (Princeton University Press, Princeton, NJ, 1997) and references therein.

[4] S. A. Kivelson, D. S. Rokhsar, and J. P. Sethna, Phys. Rev. B **35**, 8865 (1987).

[5] D. S. Rokhsar and S. Kivelson, Phys. Rev. Lett. **61**, 2376 (1988).

[6] X. G. Wen and Q. Niu, Phys. Rev. B **41**, 9377 (1990); X. G. Wen, *ibid.* **44**, 2664 (1991).

[7] E. Fradkin and S. A. Kivelson, Mod. Phys. Lett. B **4**, 225 (1990).

[8] N. Read and S. Sachdev, Phys. Rev. Lett. **66**, 1773 (1991); S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).

[9] R. Jalabert and S. Sachdev, Phys. Rev. B **44**, 686 (1991).

[10] X. G. Wen, Phys. Rev. B **44**, 2664 (1991).

[11] G. Baskaran, Z. Zou, and P. W. Anderson, Solid State Commun. **63**, 973 (1987); J. B. Marston and I. Affleck, Phys. Rev. B **39**, 11 538 (1989); G. Baskaran and P. W. Anderson, *ibid.* **37**, 580 (1988); G. Kotliar and Liu Jialin, *ibid.* **38**, 5142 (1988); N. Read and B. Chakraborty, *ibid.* **40**, 7133 (1989); L. B. Ioffe and A. I. Larkin, *ibid.* **39**, 8988 (1989); P. A. Lee and N. Nagaosa, *ibid.* **46**, 5621 (1992); A. M. Tikofsky and R. B. Laughlin, *ibid.* **50**, 10 165 (1994); B. L. Altshuler, L. B. Ioffe, and A. J. Millis, *ibid.* **53**, 415 (1996); P. A. Lee and X. G. Wen, J. Phys. Chem. Solids **59**, 1723 (1998).

[12] V. J. Emery, S. A. Kivlson, and O. Zachar, Phys. Rev. B **56**, 6120 (1997).

[13] L. Balents, M. P. A. Fisher, and C. Nayak, Int. J. Mod. Phys. B **12**, 1033 (1998).

[14] L. Balents, M. P. A. Fisher, and C. Nayak, Phys. Rev. B **60**, 1654 (1999); **61**, 6307 (2000).

[15] T. Senthil and M. P. A. Fisher, Phys. Rev. B **62**, 7850 (2000); cond-mat/0008082 (unpublished); cond-mat/0006481 (unpublished).

[16] R. Moessner, S. L. Sondhi, and E. Fradkin, cond-mat/0103396 (unpublished).

[17] X. Yang and C. Nayak (unpublished).

[18] The field χ is multivalued in the presence of a flux $hc/2e$ vortex, about which φ winds by 2π. This can be incorporated by introducing Chern-Simons gauge fields which mediate the interaction between vortices and quasiparticles (Ref. 14). Since vortices also interact with the total electrical current through the Magnus force, they can mediate an interaction between the χ's and the electrical current. For simplicity, we will assume that there are no vortices in our superconductor. When we consider spin-charge separation, we will assume that flux $hc/2e$ vortices remain gapped and assume that hc/e vortices condense. The field χ is single valued in the presence of flux hc/e vortices, so there is no problem.

[19] A. Mitra and S. M. Girvin, Phys. Rev. B **64**, 041309 (2001).

[20] D. Scalapino, S. White, and S. Zhang, Phys. Rev. Lett. **68**, 2830 (1992).

[21] N. Read and B. Chakraborty, Phys. Rev. B **40**, 7133 (1989); S. A. Kivelson, *ibid.* **39**, 259 (1989); For a more recent discussion, see E. Demler, C. Nayak, H.-Y. Kee, Y.-B. Kim, and T. Senthil (unpublished), and references therein.