Quantum Entanglement in Carbon Nanotubes

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With the surge of research in quantum information, the issue of producing entangled states has gained prominence. Here, we show that judiciously bringing together two systems of strongly interacting electrons with vastly differing ground states — the gapped BCS superconductor and the Luttinger liquid — can result in quantum entanglement. We propose three sets of measurements involving single-walled metallic carbon nanotubes which have been shown to exhibit Luttinger liquid physics, to test our claim and as nanoscience experiments of interest in and of themselves.

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Entangled pairs are quantum entities consisting of two components sharing a common wave function; a measurement on one component predetermines the state of the other [1]. Such pairs are a basic resource for quantum information processing, and recent years have begun to see many promising approaches to their production in small numbers (e.g., [2]). For large scale implementation of quantum information technology, a realization of entanglement in solid state systems and an appropriate means of transporting the components of the entangled pair over long distances are essential. While the constituent Cooper pairs of a gapped BCS superconductor have been studied as a possible natural source of such pairs [3], the question of separating and transporting them requires further investigation. Here we show how electron-electron interactions in one dimension enable sequential injection of entangled pairs from a superconductor into *two* single-walled metallic carbon nanotubes (SWNT). The SWNTs, in turn, would allow for the transport of entangled states over appreciable distances.

SWNTs, essentially long conducting cylinders of nanoscale diameters and lengths "L" of several microns, are indeed well suited to transport spin states in a coherent fashion. They are extremely pure systems with large Fermi velocities of $v_F \approx 10^6 m/s$, and are known to exhibit ballistic transport over long distances [4]. In particular, at low energies compared to the subband spacing $\epsilon_0 \approx$ 1 eV, transport is characterized by four ballistic modes propagating with linear dispersion. At temperatures $T \lesssim T_\phi = \hbar v_F / k_B L$ (which is of the order of a few kelvin), electrons can thus travel the entire length of the tube without losing coherence due to thermal effects. Moreover, nanotubes are expected to be nearly ideal spin conductors [5], and current experiments indicate that the spin-flip scattering length ℓ_{sf} in *multiwalled* nanotubes is at least 130 nm [6]. In SWNTs, we expect this length to be *longer* than the elastic mean free path, so at least $\ell_{sf} > 1\ \mu$m. While all these features bode well for transport and usage of entangled pairs, as detailed in what follows, the actual injection and separation of these pairs into the two tubes rely on the fact that nanotubes have demonstrated Luttinger liquid behavior characteristic of electrons interacting in 1D [7].

The basic setup we consider consists of two nanotubes A and B, end contacted well within a coherence length of each other to a gapped singlet-paired superconductor as in Fig. 1. Each wire can be described in bosonized language by a four channel Luttinger liquid Hamiltonian [8,9]

$$H_i = \sum_a \int_0^\infty dx\, v_a [g_a^{-1} (\partial_x \theta_a^i)^2 + g_a (\partial_x \phi_a^i)^2], \qquad (1)$$

where $i = A, B$ denotes the wires, and $a = \rho_\pm, \sigma_\pm$ correspond to the four free sectors of the theory where, by linear transformations, we have made a change of basis from the spin-channel indices $(1\uparrow, 1\downarrow, 2\uparrow, 2\downarrow)$. The relation between the bosonic fields $\theta^i_{n\alpha}$, $\phi^i_{n\alpha} (n = 1/2, \alpha = \uparrow/\downarrow)$ and the original chiral right-/left-moving electron fields $\psi^i_{R/Ln\alpha}$ is expressed through the Bosonization procedure via the transformation $\psi^i_{R/Ln\alpha} \sim e^{i(\phi^i_{n\alpha} \pm \theta^i_{n\alpha})}$. The parameter g_a captures the strength of interactions; $g_a = 1$ for

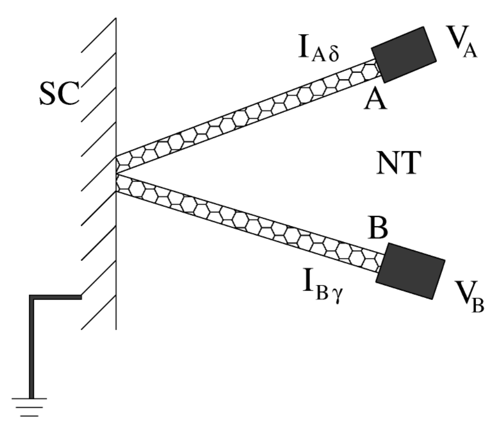

FIG. 1. Setup of two nanotubes A and B end contacted to a superconductor. Voltage drops V_A and V_B may be preferentially applied across tubes A and B, respectively, and currents through each of them may be measured.

the noninteracting channels $a = (\rho-, \sigma\pm)$, while for the charge sector $g_{\rho+}$ has the value $g_{\rho+} = g \approx 0.25$ [7,8]. The velocities of the free modes v_a are given by $v_a = v_F/g_a$.

As it is desirable to inject entangled pairs individually, we focus on the case of high resistance contacts where successive Cooper pairs hop sequentially from or into the superconductor. This limit corresponds to almost perfect backscattering at the superconductor-nanotube interface, whence $\psi^i_{nL\alpha}(0) = \psi^i_{nR\alpha}(0)$ where $i = A, B$ denotes the wire and n refers to the channel indices 1 and 2. In the bosonized language, these boundary conditions become $\theta^i_a(0) = 0$ for $i = A, B$ and all $a = \rho_\pm, \sigma_\pm$.

In this setup we analyze perturbatively the effects of a small amount of Cooper pair tunneling between the superconductor and the two wires. The corresponding Hamiltonian H_t for such processes is given by $H_t = \sum_{i=A,B} H^t_{ii} + H^t_{AB}$ where H^t_{ii} describes the tunneling of whole pairs into wire "i," and H^t_{AB} describes processes in which one electron of the pair tunnels into the tube A and the other one into the tube B. Thus

$$\begin{aligned} H^t_{ii}(x) &= v_F T_{ii} \sum_{b,c} \\ &\times [\psi^\dagger_{i\uparrow b}(0)\psi^\dagger_{i\downarrow c}(0) - \psi^\dagger_{i\downarrow b}(0)\psi^\dagger_{i\uparrow c}(0) + \text{(H.c.)}], \\ H^t_{AB}(x) &= v_F T_{AB} \sum_{b,c} \\ &\times [\psi^\dagger_{A\uparrow b}(0)\psi^\dagger_{B\downarrow c}(0) - \psi^\dagger_{A\downarrow b}(0)\psi^\dagger_{B\uparrow c}(0) + \text{(H.c.)}], \end{aligned} \tag{2}$$

where $\psi^\dagger_{i\sigma b}$ are creation operators for electrons in wire "i" with corresponding spin σ and flavor $b = 1R$, $1L$, $2R$, and $2L$. The coefficients T_{ii} and T_{AB} are the bare tunneling amplitudes.

As the common wave function for each Cooper pair in the bulk is peaked at its center of mass, in a noninteracting model the transmission probability for a pair to enter, e.g., wire A, $t_{AA} \propto |T_{AA}|^2$, is higher than the transmission probability for the pair-splitting process, $t_{AB} \propto |T_{AB}|^2$, i.e., $t_{AA} \gg t_{AB}$. To make a quantitative estimate, we presume the two nanotubes abut each other at the contact point to the superconductor, so that their center-to-center distance $d \sim 1$ nm, generally much shorter than the superconducting coherence length. The pair-splitting process is then not exponentially suppressed. A clean-limit s-wave BCS theory for a spherical Fermi surface and *point contacts* gives $t_{AA}/t_{AB} \propto (k_F d)^n$ [10], with $n \sim 1$–2 dependent on the geometry of the contact and the superconductor. We view this only as a rough guide, since nanotube contacts are *not* pointlike on the scale of typical k_F's. Nevertheless, we expect that under optimal conditions for typical superconducting metals $t_{AA}/t_{AB} \gtrsim 10$. Naively, then, one might expect each Cooper pair to tunnel entirely into one tube or the other. However, due to interactions, tunneling of charge into the ends of the nanotubes involves more than the mere overlap of electronic wave functions between the tube and the superconductor. Addition of one extra electron into a tube involves the coherent rearrangement of all electrons in its bulk. As a consequence of this Luttinger liquid physics, we find that the single electron tunneling density of states at a low energy E compared to ϵ_0 goes as $\rho_e(E) \sim \epsilon_0^{-1}(E/\epsilon_0)^{(1/4)[(1/g)-1]}$ [7,11], while the density of states available to tunnel in a Cooper pair is $\rho_{2e}(E) \sim \epsilon_0^{-1}(E/\epsilon_0)^{1/g}$. We can now consider two nanotubes raised to a voltage $eV \ll k_BT$ with respect to the superconductor. Given that the Cooper pair density of states in the superconductor is a delta function at the Fermi energy, Fermi's golden rule reveals that the rate Γ_{AA}, at which entire Cooper pairs tunnel from the superconductor into the end of one tube, is proportional to $eV\rho_{2e}$, and at any given temperature T, has the dependence $\Gamma_{AA} \sim (eV/h)(k_BT/\epsilon_0)^{(1/g)-1}$. However, the rate Γ_{AB}, at which split entangled pairs are injected into both tubes involves the single particle tunneling density of states in each of them, and has the dependence $\Gamma_{AB} \sim (eV/h)(k_BT/\epsilon_0)^{(1/2)[(1/g)-1]}$. Remarkably, this implies that for the energy scales of relevance here, $\Gamma_{AB} \gg \Gamma_{AA}$, and thus almost all charge transfer occurs as split entangled pairs. We now propose three sets of measurements that capture these principles in a concrete manner.

The simplest experimental signature of the splitting of Cooper pairs may be obtained from the transconductance measured for two nanotubes as shown in Fig. 1. In response to a voltage difference between the nanotubes and the superconductor, we compute the resulting currents flowing into the two wires. We start from the Hamiltonian of Eq. (1) with the appropriate boundary conditions and, along the lines of Ref. [12], we use a nonequilibrium Keldysh technique [13] perturbative in the amount of Cooper pair tunneling described by Eq. (2). To bring out the physics of the Cooper pair splitting, we first consider the specific case of applying a voltage drop V_A across tube A and none across tube B under the condition $k_BT \ll eV_A \lesssim \epsilon_0, \Delta$, where Δ is the superconducting gap. For low T_c superconductors, with $\Delta \approx 1$ meV we estimate probing at voltages of a few tenths of an meV and at temperatures of 10–100 mK. In tube A, the applied voltage would produce a current with two components—one due to entire pairs tunneling in A, $I_{AA} \sim t_{AA}[(2e)^2/h](eV_A/\epsilon_0)^{2\alpha}V_A$, and another due to the splitting of pairs into the two tubes, $I_{AB} \sim t_{AB}(e^2/h)(eV_A/\epsilon_0)^{\alpha}V_A$, where $\alpha = (1/g - 1)/2 \approx 1.5$. Strikingly, the current I_{AB} runs equally in both tubes, in spite of the absence of a voltage drop across tube B. The nonlinear behavior of current is a reflection of the power laws in the density of states, and despite the fact that $t_{AB} \ll t_{AA}$, the contribution from the pair split process clearly dominates at low voltages.

More generally, at finite temperature T, and when voltage drops V_A and V_B are present across both tubes, one can define an associated conductance matrix $G_{ij} = \partial I_i/\partial V_j$, where i, j stand for the tubes A and B. We find it to have the form

$$G_{AA} = t_{AA}\frac{(2e)^2}{h}\left[\frac{k_BT}{\epsilon_0}\right]^{2\alpha}\mathcal{F}_{2\alpha}\left[\frac{2eV_A}{k_BT}\right] + t_{AB}\frac{e^2}{h}\left[\frac{k_BT}{\epsilon_0}\right]^{\alpha}\mathcal{F}_{\alpha}\left[\frac{e(V_A+V_B)}{k_BT}\right], \quad (3)$$

$$G_{AB} = t_{AB}\frac{e^2}{h}\left[\frac{k_BT}{\epsilon_0}\right]^{\alpha}\mathcal{F}_{\alpha}\left[\frac{e(V_A+V_B)}{k_BT}\right], \quad (4)$$

with G_{BB} obtained from G_{AA} by interchanging A and B. Here, the scaling function $\mathcal{F}_\alpha[x] = \partial_x[2\sinh(x/2)|\Gamma(1+\alpha/2+ix/2\pi)|^2]$ [where $\Gamma(z)$ is the gamma function] and has the limits $\mathcal{F}_\alpha(x) \to_{x\to 0} |\Gamma(1+\alpha/2)|^2$, $\mathcal{F}_\alpha(x) \to_{x\to\infty} (1+\alpha)(x/2\pi)^\alpha$ [11]. The dominance of split Cooper pair injection in charge transfer is directly seen in Eqs. (3) and (4). In fact, for probing energy scales of a few tenths of an meV, the ratio of the unsplit to split process contributions is as small as $(G_{AA} - G_{AB})/G_{AB} \approx 10^{-4}$.

The transconductance experiment directly measures the fact that charge is simultaneously injected into both nanotubes. It is not, however, sensitive to the spin state of these electrons. We next consider a Josephson current measurement which verifies that spin 1/2 is added to each wire. Here, as in Fig. 2, we consider two nanotubes A and B, meeting at points X and Y separated by distance "L" along the tubes.

At each junction, a superconducting lead makes a point contact with both tubes. To probe the spin state of the electrons, tube A (tube B) is subjected to a magnetic field $\vec{B} = B_A\hat{x}(\vec{B} = B_B\hat{x})$. It may be convenient to choose the field axis $\hat{x}$ parallel to the junction so as to minimize orbital effects. For this particular setup, the Hamiltonian for each wire is similar to the one described in Eq. (1), with the limits of integration extending over the entire range in the position space, as opposed to the imposition of hard boundary conditions of the previous setup. We compute the free energy of the system perturbatively in the tunneling of Cooper pairs at points X and Y described by a tunneling term similar (but not identical) to Eq. (2). We find that at temperatures $T < T_\phi = \hbar v_F/k_BL$, the nanotubes act as a Josephson weak link with associated critical current

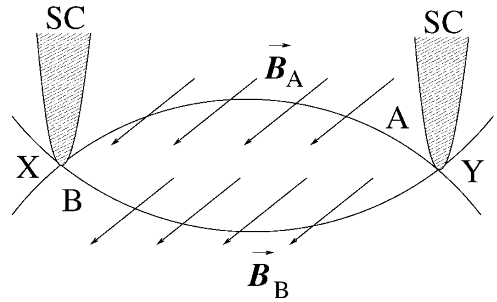

FIG. 2. Setup of two infinite nanotubes A and B crossing at points X and Y separated by a distance "L" along each tube. Superconducting point contacts at junctions X and Y allow for Cooper pair tunneling into the tubes.

$$I_c = \frac{ev_F}{L}\left\{\sum_{i=A,B}\left[\tilde{t}_{ii}^{(1)}\left(\frac{d}{L}\right)^{2\beta} + \tilde{t}_{ii}^{(2)}\left(\frac{d}{L}\right)^{\alpha}\cos\left(\frac{\mathbf{g}\mu_B B_i L}{\hbar v_F}\right)\right] + \left(\frac{d}{L}\right)^{\beta}\left[\tilde{t}_{AB}^{(1)}\cos\left(\frac{\mathbf{g}\mu_B \delta B L}{2\hbar v_F}\right) + \tilde{t}_{AB}^{(2)}\cos\left(\frac{\mathbf{g}\mu_B B_T L}{2\hbar v_F}\right)\right]\right\}, \quad (5)$$

where $\delta B = B_A - B_B$, $B_T = B_A + B_B$ and $\beta = (g + 1/g)/4 - 1/2$. Also, d is the diameter of the tube and is of the order of a few nanometers, $\mathbf{g}$ is the Landé factor, and μ_B is the Bohr magneton. The dimensionless constants $\tilde{t}_{ii}^{(1/2)}$, $\tilde{t}_{AB}^{(1/2)}$ are proportional to the square of the (small) bare transmission probabilities of Cooper pairs from the superconductors to the nanotubes in the unsplit and split processes, respectively. The index "1" refers to the injection of pairs of electrons with the same chirality (two right movers or two left movers) into the wires, while the index "2" refers to the injection of pairs consisting of a left-moving and a right-moving electron. Notice the highly anomalous length dependence (with markedly different power laws for the split and unsplit processes) compared to the case of noninteracting wires, where the Josephson current is inversely proportional to the separation length L. Strikingly, as a function of either one of the applied magnetic fields B_A and B_B, contributions from Cooper pair split processes oscillate with *half* the frequency of those generated by unsplit pairs ($\tilde{t}_{ii}^{(2)}$). We estimate the period of these magnetic field oscillations to be in the Tesla range. In an actual experiment, in which some flux between the nanotubes is inevitable, the above critical current will give the *envelope* for much faster Aharanov-Bohm oscillations (on the scale of a few Gauss) in the critical current, but the two types of oscillations can be easily distinguished by their very different periodicities.

While the proposed measurements establish that charge enters from the superconductor in the form of separated electrons with spin 1/2 each, they do not establish that these electrons are actually in the (maximally) entangled singlet state $[|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B]/\sqrt{2}$. This entanglement is encoded in correlations between the injected spins. As a simple but revealing example, consider the joint probability $P_{\uparrow\uparrow}$ that both electrons in a given pair have spin "up" along a selected axis $\hat{\mathbf{z}}$. In the singlet case, illustrating the EPR "paradox," once spin up is measured in tube A, spin down is automatically selected in tube B, and the probability is zero. Contrast this with the measurement of an unentangled up-down pair $|\uparrow\rangle_{A\hat{\mathbf{m}}}|\downarrow\rangle_{B\hat{\mathbf{m}}}$ along an arbitrary direction $\hat{\mathbf{m}}$. If one attempts to preserve spin-rotational invariance on average by choosing the axis $\hat{\mathbf{m}}$ with uniform probability on the unit sphere, the probability $P_{\uparrow\uparrow} > 0$ is nonvanishing, due to pairs that are not oriented along $\hat{\mathbf{z}}$.

We now propose a specific measurement to test the presence of entanglement through current correlations, i.e., noise. Specifically, in the transconductance setup of Fig. 1, what is required is an experimental measurement of the currents $I_{i\hat{\mathbf{n}}}$ of electrons with a given spin orientation (along $\hat{\mathbf{n}}$) in each nanotube, $i = A, B$. Experimentally, this could be accomplished by a variety of spin-filtering techniques, e.g., by attaching two oppositely polarized half-metallic ferromagnets via *ideal adiabatic contacts* to each nanotube (many other schemes, e.g., Ref. [14], are possible.) Consider measuring spin-filtered currents along the $\hat{\mathbf{z}}$ axis in tube A and along an axis $\hat{\mathbf{n}}$ oriented at an angle θ with respect to $\hat{\mathbf{z}}$ in tube B. When a finite voltage drop V_A is applied across tube A, the most revealing measurements are those with no voltage drop across tube B, since then all the current in it is due to pair splitting processes.

In a manner similar to the one used to compute the conductance matrix for the system described in Fig. 1 we make use of a perturbative Keldysh approach [12,13] to derive forms for the spin-filtered currents and current-current correlations. For the case of singlet Cooper pair tunneling, the spin-filtered current correlations at zero temperature are found to be

$$\begin{aligned} \langle I_{A\pm\hat{\mathbf{z}}} I_{B\pm\hat{\mathbf{n}}} \rangle &= e \sin^2 \tfrac{\theta}{2} \langle I_{B\hat{\mathbf{n}}} \rangle , \\ \langle I_{A\pm\hat{\mathbf{z}}} I_{B\mp\hat{\mathbf{n}}} \rangle &= e \cos^2 \tfrac{\theta}{2} \langle I_{B\hat{\mathbf{n}}} \rangle , \\ \langle I_{B\hat{\mathbf{n}}} \rangle &= \langle I_{B-\hat{\mathbf{n}}} \rangle , \end{aligned} \tag{6}$$

where θ is the angle between the $\hat{\mathbf{n}}$ and $\hat{\mathbf{z}}$ axes. From Eq. (6) we see that when both measurements are made along the $\hat{\mathbf{z}}$ axis ($\theta = 0$), as expected, correlations between like spin currents in the two wires vanish. By contrast, in the case considered above of tunneling of classically random unentangled spin-up spin-down pairs, one would expect zero temperature correlations of the form

$$\begin{aligned} \langle I_{A\pm\hat{\mathbf{z}}} I_{B\pm\hat{\mathbf{n}}} \rangle &= \tfrac{e}{3}[1 + \sin^2 \tfrac{\theta}{2}] \langle I_{B\hat{\mathbf{n}}} \rangle , \\ \langle I_{A\pm\hat{\mathbf{z}}} I_{B\mp\hat{\mathbf{n}}} \rangle &= \tfrac{e}{3}[1 + \cos^2 \tfrac{\theta}{2}] \langle I_{B\hat{\mathbf{n}}} \rangle . \end{aligned} \tag{7}$$

Though the correlations show an angular dependence on θ, their form is very different from the entangled case of Eq. (6). Specifically, as anticipated for measurements along the $\hat{\mathbf{z}}$ axis in both tubes ($\theta = 0$), we have $\langle I_{A\pm\hat{\mathbf{z}}} I_{B\pm\hat{\mathbf{z}}} \rangle = e/3 \langle I_{B\hat{\mathbf{z}}} \rangle$, which is nonzero, in stark contrast to the entangled case.

To conclude, in the vast search for physical realizations of entanglement, we have described one method of extracting singlet pairs from a superconducting source. If employing nanotubes for this purpose indeed proves tractable, the next stage in the realm of quantum information would involve new challenges such as probing information at the single electron level and building arrays of coupled logic gates. In the fields of nanoscience and Luttinger physics, attention, both theoretical [15] and experimental [16], has fallen on bringing effectively one-dimensional systems into contact with superconductors. Here we have hoped to provide more food for thought in these fields by describing two nanotubes in contact with a gapped BCS superconductor.

While writing up this work we became aware of a related independent proposal by Recher and Loss [10]. The analysis of [10] is based on a setup similar to the one we describe here, yielding similar results with the ones we derive in the first part of our paper.

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