Critical Dynamics of Superconductors in the Charged Regime

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We investigate the finite temperature critical dynamics of three-dimensional superconductors in the charged regime, described by a transverse gauge field coupling to the superconducting order parameter. Assuming relaxational dynamics for both the order parameter and the gauge fields, within a dynamical renormalization group scheme, we find a new dynamic universality class characterized by a finite fixed point ratio between the transport coefficients associated with the order parameter and gauge fields, respectively. We find signatures of this universality class in various measurable physical quantities, and in the existence of a universal amplitude ratio formed by a combination of physical quantities.

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Close to the critical temperature T_c of the normal-superconductor transition, in a regime determined by the Ginzburg criterion [1,2], order parameter fluctuations dictate critical properties. For decades, the effect of the charge of the superconducting order parameter in this regime in three dimensions has formed the topic of keen study. While for strongly type-I materials, the coupling of the order parameter to transverse gauge field fluctuations is expected to render the transition first order [3], it is well established that strongly type-II materials should exhibit a *continuous* phase transition, and that sufficiently close to T_c, the charge of the order parameter field is relevant [4]. While the exact location of the boundary between these two types of behavior is still the subject of investigation [5], the static critical properties of the charged-XY universality class are reasonably well understood. With the discovery of superconducting compounds with large critical temperatures and short coherence lengths the critical regime of this transition is now potentially accessible to experimental investigation.

In this Letter, we investigate the less-addressed issue [6,7] of the dynamics of the three-dimensional normal-superconductor transition in the charged regime. In the well-studied case of superfluid He^4, the coupling of the order parameter to a conserved energy density field has nontrivial effects on critical dynamics [8]. Analogously, we propose that the coupling of the superconducting order parameter to relaxational transverse gauge field fluctuations leads to qualitatively new dynamics characterized by a universal ratio C between the zero wave-vector part of the characteristic frequencies for the dynamics of the order parameter ψ_α and that of the gauge field $\mathbf{A}$ at the critical point:

$$C \equiv \lim_{k\to 0} \frac{\omega_\psi(k)}{\omega_A(k)} = \text{const.} \tag{1}$$

Thus we propose the possibility that the strong coupling of these two fields causes them to relax in the same fashion at the critical point, with a single new dynamic exponent z. In what follows, employing dynamic renormalization group (RG) techniques, we show these features to hold within the context of a particular model. We discuss the universal properties obtained from the model and the behavior of measurable physical quantities.

The model.—As a starting point for modeling dynamics, we employ the standard Ginzburg-Landau free energy used to define the finite temperature static critical properties of a three-dimensional superconductor coupled to a transverse electromagnetic field [9], generalized to N complex species of matter field, given by

$$F = \int d^d x \bigg[\sum_\alpha |(\nabla - i\sqrt{g}\mathbf{A})\psi_\alpha|^2 + r\sum_\alpha |\psi_\alpha|^2 + u\bigg(\sum_\alpha |\psi_\alpha|^2\bigg)^2 + \frac{1}{2}|\nabla \times \mathbf{A}|^2 \bigg], \tag{2}$$

where ψ_α (with $\alpha \in 1, 2, \ldots, N$) is the generalized order parameter and $\mathbf{A}$ is a fluctuating massless gauge field. The effective charge is given by $\sqrt{g} = e^*\sqrt{4\pi}/\hbar c$ and deviations from criticality are measured by $r = 2m^*(T - T_c)/\hbar^2 T_c$ [10]. For $N = 1$ and $d = 3$, this model describes the low-energy excitations of a bulk charged superfluid in the regime where charge-density fluctuations are gapped at a high energy (the plasmon gap), but remain coupled to transverse electromagnetic fluctuations.

As originally obtained in Ref. [3], a one-loop static RG analysis of Eq. (2) in $d = 4 - \epsilon$ dimensions shows that no nontrivial charged fixed point exists for $2N < n_c = 365.9$. However, for a range of parameters, more sophisticated methods indeed find a continuous phase transition for $N = 1$ [4,11,12], describing, presumably, the second order transition found in many materials. Many of the salient features of this "charged-XY" universality class are captured by Eq. (2) with $N > N_c \simeq 183$; for instance, the fact that the anomalous dimension

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of the order parameter field is negative. Given that the N-component model does provide insight on the statics of the charged superconducting transition, here we make the reasonable assumption that when the free energy defined by Eq. (2) is augmented by appropriate equations of motion, the model captures the basic features of the critical dynamics of the transition as well.

For completeness, we note that the free energy density is invariant under the following transformations:

$$\psi_\alpha^a \to \delta_{ab}\mathcal{O}_{\alpha\beta}\psi_\beta^b, \tag{3}$$

$$\begin{cases} \psi_\alpha^a \to e^{i[\sqrt{g}\Lambda(\vec{r})+\theta]\sigma_{ab}^y}\delta_{\alpha\beta}\psi_\beta^b, \\ \mathbf{A}(\mathbf{r}) \to \mathbf{A}(\mathbf{r}) + \nabla\Lambda(\mathbf{r}), \end{cases} \tag{4}$$

where $\alpha, \beta \in \{1, \ldots, N\}$, $a, b \in \{1, 2\}$, $\mathcal{O}$ is an $N \times N$ orthogonal matrix, $\theta \in [0, 2\pi]$, and σ_{ab}^y is the usual 2×2 Pauli matrix. ψ_α^1 and ψ_α^2 are the real and imaginary parts, respectively, of the αth complex field. The term in Eq. (2) with coupling constant u is the only quartic matter-matter interaction allowed by these symmetries.

The simplest equations of motion augmenting the statics described by Eq. (2) are relaxational in both ψ_α and $\mathbf{A}$. In the presence of external fields $h_{\psi\alpha}$ and $h_{\mathbf{A}}$ coupling to ψ_α and $\mathbf{A}$, respectively, they take the form

$$\partial_t\psi_\alpha = -\Gamma_\psi\left(\frac{\delta F}{\delta\psi_\alpha^*} - h_{\psi\alpha}\right) + \eta_\alpha, \tag{5}$$

$$\partial_t A_i = -\Gamma_A\left(\frac{\delta F}{\delta A_i} - h_{Ai}\right) + \zeta_i. \tag{6}$$

Here, Γ_ψ and Γ_A are transport coefficients associated with ψ_α and $\mathbf{A}$, respectively. The fields η_α, η_α^*, and $\boldsymbol{\zeta}$ are white noise correlated and ensure that the fluctuation-dissipation theorem is satisfied. Thus, they obey the constraints

$$\langle\eta_\alpha(\mathbf{r}, t)\eta_\beta^*(\mathbf{r}', t')\rangle = 2T\Gamma_\psi\delta_{\alpha\beta}\delta^3(\mathbf{r} - \mathbf{r}')\delta(t - t'), \tag{7}$$

$$\langle\eta_\alpha(\mathbf{r}, t)\rangle = \langle\eta_\alpha(\mathbf{r}, t)\eta_\beta(\mathbf{r}', t')\rangle = 0, \tag{8}$$

$$\langle\zeta_i(\mathbf{r}, t)\zeta_j(\mathbf{r}', t')\rangle = 2T\Gamma_A\delta_{ij}\delta^3(\mathbf{r} - \mathbf{r}')\delta(t - t'), \tag{9}$$

$$\langle\zeta_i(\mathbf{r}, t)\rangle = 0. \tag{10}$$

In fact, if we assume that the normal state of the system is a metal, Eq. (6) can be derived from the low-frequency form of Maxwell's equation

$$\nabla \times \mathbf{B} = 4\pi\mathbf{J}/c + \partial_t\mathbf{E}/c, \tag{11}$$

in the gauge $A_0 = 0$, where the electric and magnetic fields are given by $\mathbf{E} = -\partial_t\mathbf{A}/c$ and $\mathbf{B} = \nabla \times \mathbf{A}$, respectively. The net current $\mathbf{J} = \mathbf{j}_s + \mathbf{j}_n$ has a superfluid component $\mathbf{j}_s = -\frac{\delta}{\delta\mathbf{A}}|(\nabla - i\sqrt{g}\mathbf{A})\psi_\alpha|^2$ and a normal component $\mathbf{j}_n$. The average normal current is given by $\sigma_n\mathbf{E}$, where σ_n is the normal conductivity. Thermal fluctuations of the normal fluid give rise to the noise in Eq. (6). With these assumptions, one can retrieve Eq. (6) from Eq. (11) in the limit $\omega \to 0$ and identify the inverse transport coefficient, Γ_A^{-1}, with the bare conductivity, σ_n.

A complete model for the dynamics requires identification of all conserved quantities and Poisson-bracket relations applicable to the normal-superconductor system in the charged regime [8]. Even in the uncharged regime, one might expect nondissipative coupling of the order parameter to a combination of energy and mass density to exhibit model E dynamics as in superfluid helium [8]. However, hydrodynamic analyses show that in the presence of impurities (which is implicit in the assumption of finite conductivity well within the normal state) this coupling does not survive [13], as indicated by the absence of second sound modes in actual superconducting systems. However, in principle, a conserved energy density mode could couple to the order parameter mode via nonlinear interactions, leading to model C dynamics in the uncharged superconductor [13]. Likewise, in the charged regime, we do not expect any nondissipative coupling of energy-mass density to the order parameter, due to the presence of impurities in real samples of interest. However, the possibility of nonlinear interactions with conserved quantities cannot be completely eliminated. The model we employ consisting of the order parameter and gauge field modes alone, each with relaxational dynamics, is the simplest, but hitherto unexplored, possibility.

RG analysis.—The equations of motion Eqs. (5) and (6), allow a dynamical RG analysis which we now detail (also see, for example, Ref. [8]). The effective charge $\sqrt{g}$ and the coupling constant u of Eq. (2) are treated perturbatively, as is the deviation from four dimensions, $\epsilon = 4 - d$, in order to avoid infrared divergences [14]. Because the characteristic electron speeds are small compared to the speed of light, the low-energy theory of the system need not be relativistically invariant. This leads to the residual gauge symmetry contained in Eq. (4). (This has been pointed out previously in, for instance, Ref. [15].) To avoid divergences in functional integrals resulting from the fact that multiple choices of gauge lead to the same magnetic field configuration, we perform the Fadeev-Popov procedure and add a term $1/(2\lambda)(\nabla \cdot \mathbf{A})^2$ to the free energy [Eq. (2)] [16]. Fixing the value of λ corresponds to making a choice of gauge, and for the remainder of this Letter we work with $\lambda = 1$(analogous to the Feynman gauge in quantum electrodynamics). Regarding the dynamics, since our focus is on the manner in which the relaxational rates of the fields ψ and $\mathbf{A}$ affect one another, we rescale the theory and write the equations of motion in terms of the ratio of transport coefficients $\Gamma \equiv \Gamma_\psi/\Gamma_A$.

We perform the standard RG procedure, integrating out modes in a momentum shell $\Lambda/b < |\mathbf{k}| < \Lambda$, where Λ is a high momentum cutoff, and all frequencies, followed by a rescaling of space and time: $\mathbf{r} \to b\mathbf{r}$ and

$t \to b^z t$. To one loop, and $\mathcal{O}(\epsilon)$, the β functions for u and g are (as in Ref. [3])

$$\frac{du}{d\ln b} = \epsilon u - 2(N+4)u^2 - 3g^2 + 6gu, \qquad (12)$$

$$\frac{dg}{d\ln b} = \epsilon g - \frac{N}{3}g^2 = g(\epsilon - \eta_A), \qquad (13)$$

yielding Gaussian ($u^* = g^* = 0$), XY ($u^* \neq 0$, $g^* = 0$), and charged-XY ($u^* \neq 0$, $g^* \neq 0$) fixed points for $N \geq 183$. At the charged-XY fixed point of interest, $g^* = 3\epsilon/N$ to leading order in ϵ. We find that the anomalous dimensions of the order parameter and gauge field at this order (and in our choice of gauge) are

$$\eta_\psi = -2g; \qquad \eta_A = Ng/3, \qquad (14)$$

and note that $\eta_A = \epsilon = 4 - d$ at the critical point, as required by gauge invariance [12].

We fix the exponent z by requiring that the time derivative term in the equation of motion for $\mathbf{A}$ return to its "bare" form. The same feat in the equation of motion for ψ is accomplished by allowing the ratio Γ to flow. We find that

$$1 = b^{2-\eta_A - z}\left(1 + g\frac{N}{2\Gamma}\ln b\right), \qquad (15)$$

$$\Gamma' = b^{z-2+\eta_\psi}\Gamma\left(1 - g\frac{\Gamma}{\Gamma+1}\ln b\right), \qquad (16)$$

giving a one-loop β function for Γ:

$$\frac{d\Gamma}{d\ln b} = g\Gamma\left(\frac{N}{2\Gamma} - \frac{\Gamma}{\Gamma+1} - 2 - \frac{N}{3}\right). \qquad (17)$$

At the $\mathcal{O}(\epsilon)$ charged fixed point, Eq. (17) has a stable fixed point solution:

$$\Gamma^* = \frac{(N-12) + \sqrt{(N-12)^2 + 24N(N+9)}}{4(N+9)}, \qquad (18)$$

which in general obeys $0 \leq \Gamma^* < 1.5$ for all $N \geq 0$. We note that $\Gamma = 0$ ($\Gamma_\psi \to 0$ with Γ_A fixed, for instance) is not a fixed point and that $\Gamma^{-1} = 0$ ($\Gamma_A \to 0$ with Γ_ψ fixed, for instance) is an unstable fixed point, at this order. Thus, we are led to conclude that, at least within a one-loop RG analysis, *the critical dynamics of the charged superconductor is governed by a nontrivial fixed point* wherein Γ has a finite ratio, reflecting the fact that the dynamics of the order parameter and those of the gauge field are strongly coupled. At the charged-XY fixed point, we find that $z = 2 + \epsilon[3/(2\Gamma^*) - 1]$ to leading order in ϵ, implying that $z > 2$. While an accurate value of the dynamic exponent would require employing more sophisticated treatments, our results certainly suggest that near criticality, the system relaxes slower than expected for diffusive dynamics.

Physical consequences.—The most striking new feature of this fixed point (in contrast with the uncharged dynamics) is the existence of the universal fixed ratio, Γ^*. Physically, its existence requires that the order parameter and gauge fields relax in the same fashion. In fact, using scaling arguments [8], one can show that at the critical point, the ratio of the characteristic frequencies at $k = 0$ of the two fields is exactly the universal amplitude ratio, identifying the constant C of Eq. (1) with Γ^*; i.e.,

$$C \equiv \lim_{k\to 0}\frac{\omega_\psi(k)}{\omega_A(k)} = \Gamma^*. \qquad (19)$$

Here, the characteristic frequency of a field Q is defined by

$$\omega_Q^{-1}(k) = i\chi_Q(k, \omega = 0)\frac{\partial \chi_Q^{-1}(k,\omega)}{\partial \omega}\bigg|_{\omega=0}, \qquad (20)$$

where χ_Q is the dynamic linear response function, $\langle Q\rangle = \chi_Q h_Q$. In our case, close to criticality, the characteristic frequency has the scaling behavior $\omega_Q = \xi^{-z}\Omega_Q(k\xi)$ for both fields ψ and $\mathbf{A}$, where ξ is the divergent correlation length associated with the order parameter, and Ω_Q is a universal function. In principle, the characteristic frequency for each field can be obtained by measuring the static susceptibility and dynamic linear response function associated with the field.

The dynamic linear response functions themselves carry valuable information on critical dynamics. Close to T_c, they have the scaling form

$$\chi_Q(k,\omega) = \xi^{2-\eta_Q} f_Q(\omega\xi^z, k\xi), \qquad (21)$$

where, associated with each field ψ_α and $\mathbf{A}$, η_Q and f_Q are the anomalous dimension and a scaling function, respectively.

Each of these functions is manifest in measurable quantities. The order parameter response function, $\chi_\psi(k,\omega)$, is the pair susceptibility appearing in Josephson tunneling experiments [17]. In principle, the scaling behavior of χ_ψ in Eq. (21) could be used to extract the dynamic exponent, z. Accessing $\chi_A(k = 0, \omega)$ should be relatively straightforward, since it is related to the resistivity in linear response. The external field that couples to $\mathbf{A}$ is an applied current $\mathbf{j}_{\text{ext}}$, so that $\langle \mathbf{A}\rangle = \chi_A \mathbf{j}_{\text{ext}}$, with $\sigma(\mathbf{k} = 0, \omega) = -i\chi_A^{-1}(\mathbf{k} = 0, \omega)/\omega$. Close to criticality and above T_c, we expect this to provide the dominant contribution to the net conductivity and, using Eq. (21), to diverge as $\sigma(k = 0, \omega) = \xi^{z-2+\eta_A}\mathcal{G}(\omega\xi^z) \sim \xi^{z+2-d}$, where $\mathcal{G}$ is a scaling function. This is a consequence of the Josephson scaling relation [12,18] which holds in both charged and uncharged regimes, as a result of gauge invariance [12]. Notably, in the charged regime, the exponent z has a different value than in the uncharged case.

We see that various features of the dynamic universality class of the charged superconductor appear in measurable quantities. The definitive signature of this universality class would be in the extraction of the universal amplitude ratio, Eq. (19).

Concluding remarks.—The charged superconductor-normal transition is a continuing source of rich physics. Research on the static transition of the charged Ginzburg-Landau model indicates differing behavior for type-I and type-II superconductors, as well as the relevance (sufficiently close to the continuous transition) of the charge of magnetic field fluctuations. Numerical and analytical work in Ref. [6] on the dynamics of this transition also seem to reveal distinctly different dynamic behavior between strongly and weakly screened superconductors. In Ref. [7], Monte Carlo studies of superconductor dynamics in the vortex representation find that for strong magnetic screening, $z \approx 2.7$, in qualitative agreement with our result that the order parameter dynamics in the charged regime is subdiffusive. Here, as our key point, we suggest that for materials with continuous transitions from normal metal to superconductor, the dynamics in the charged regime will be governed by a new universality class. Further analyses of all these issues are well in order.

In experiments, the charged regime of the superconductor-metal transition is not easily accessible. The Ginzburg criterion indicates that materials with high critical temperature, large anisotropy and extreme type-II behavior should manifest large regimes of fluctuations. However, within this fluctuation regime, the region close to T_c where the system crosses over to the regime of *charged* fluctuations is often too narrow to access [2]. For instance, high T_c materials such as $YBa_2Cu_3O_{7-d}$, while possessing large regimes of critical fluctuations, are too strongly type-II to observe charged critical fluctuations. However, weakly type-II materials with high T_c's or granular texture, and moderate anisotropies could open up a window into this new regime. We are hopeful that an investigation of such materials will yield an understanding of the effect of charge on the critical dynamics of this transition.

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