Particle-Hole Symmetry and the $\nu = \frac{5}{2}$ Quantum Hall State

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We discuss the implications of approximate particle-hole symmetry in a half-filled Landau level in which a paired quantum Hall state forms. We note that the Pfaffian state is not particle-hole symmetric. Therefore, in the limit of vanishing Landau-level mixing, in which particle-hole transformation is an exact symmetry, the Pfaffian spontaneously breaks this symmetry. There is a particle-hole conjugate state, which we call the anti-Pfaffian, which is degenerate with the Pfaffian in this limit. We observe that strong Landau-level mixing should favor the Pfaffian, but it is an open problem which state is favored for the moderate Landau-level mixing which is present in experiments. We discuss the bulk and edge physics of the anti-Pfaffian. We analyze a simplified model in which transitions between analogs of the two states can be studied in detail. Finally, we discuss experimental implications.

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Introduction.—The nature of the observed quantum Hall plateau at $\sigma_{xy} = \frac{5}{2}\frac{e^2}{h}$ [1,2] is an important unresolved problem. If the Moore-Read Pfaffian state (Pf state) [3–6] were realized at this plateau, then it would be the first non-Abelian topological phase observed in nature. Such a discovery could pave the way towards the realization of a topological quantum computer [7]. However, although several experiments have been proposed which could directly confirm it [7–10], the only evidence which currently suggests that the $\nu = 5/2$ plateau is in the universality class of the Pf state is the numerical diagonalization of the Hamiltonian for systems with a small number of electrons [11,12]. In this Letter, we add a new wrinkle to this discussion. We note that the particle-hole (PH) conjugate of the Pf is a new state which will be exactly degenerate in energy with the Pf in the limit of vanishing Landau-level mixing. Landau-level mixing is a symmetry-breaking perturbation which lifts the degeneracy between the two states. It is an open problem which state is favored at the moderate Landau-level mixing which is present in experiments. Since the effects of Landau-level mixing have not been fully accounted for in numerics, we suggest that the anti-Pfaffian ($\overline{\text{Pf}}$) is, at the very least, a new candidate for the observed $\nu = 5/2$ state.

When Landau-level mixing is neglected, the Hamiltonian for electrons at filling fraction $\nu = N + \frac{p}{q}$ can be related to one for electrons at $\nu = N + 1 - \frac{p}{q}$ by an antiunitary PH transformation, $c_m^\dagger \to c_m$, $c_m \to c_m^\dagger$, with m labeling orbitals within the Landau level. The Hamiltonian transforms according to $H_2 \to \tilde{H}_2 + \text{const}$, where $H_2 = \sum_{klmn} V_{klmn} c_k^\dagger c_m c_l^\dagger c_n - \mu \sum_m c_m^\dagger c_m$ and $\tilde{H}_2 = \sum_{klmn} V_{klmn} c_k^\dagger c_m c_l^\dagger c_n + (\mu - 2\mu_{1/2}) \sum_m c_m^\dagger c_m$. Here, V_{klmn} are the matrix elements of the Coulomb interaction, and $\mu_{1/2} = \sum_n V_{mnmn}$. For the special case of $\frac{p}{q} = \frac{1}{2}$, for which $\mu = \mu_{1/2}$, this is a symmetry of the system.

It is widely believed, on the basis of numerical evidence [11,12], that the experimentally-observed plateau at $\nu = \frac{5}{2}$ is in the universality class of the Moore-Read Pf state, by which it is meant that the lowest Landau level (of both spins) is filled, and the electrons in the first excited Landau level are fully spin-polarized and have a wave function which is in the same universality class as the one given by acting with Landau level raising operators on

$$\Psi_{\text{Pf}}(z_i) = \text{Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^2 e^{-\Sigma |z_i|^2/4\ell_0^2}. \quad (1)$$

However, this state is not invariant under a PH transformation. Let σ_{xy} and κ_{xy} denote the contributions to the electrical and thermal Hall conductivities (in units of e^2/h and $\pi^2 k_B^2 T/3h$, respectively) of the fractional state in the lowest unfilled Landau level, in which the Pf is assumed to form (which, in the case of $\nu = 5/2$, is the first excited Landau level). Under a PH transformation of this Landau level, $\sigma_{xy} \to 1 - \sigma_{xy}$ and $\kappa_{xy} \to 1 - \kappa_{xy}$. The Pf state has $\kappa_{xy} = \frac{3}{2}$, as may be seen most easily from its edge theory [13], which has two modes, a chiral boson ϕ and a chiral Majorana fermion ψ, propagating in the same direction:

$$\mathcal{L}_{\text{Pf}}(\psi, \phi) = \psi(-\partial_t + i v_n \partial_x)\psi + \frac{2}{4\pi} \partial_x \phi (i \partial_t + v_c \partial_x) \phi.$$

Here, $e^{i\phi}$ creates a charge $e/2$ semion with scaling dimension $1/4$, and v_c, v_n denote the two edge velocities. A chiral boson contributes $\kappa_{xy} = 1$, while a chiral Majorana fermion carries $\kappa_{xy} = \frac{1}{2}$. The PH conjugate of the Pf, which we will call the anti-Pfaffian ($\overline{\text{Pf}}$), with wave function of the form (as dictated by the transformation in [14]):

$$\Psi_{\overline{\text{Pf}}} = \int \prod_\alpha d^2 \eta_\alpha \prod_{i<j} (z_i - z_j) e^{-\Sigma |z_i|^2/4\ell_0^2} \times \prod_{i,\alpha} (z_i - \eta_\alpha) \prod_{\beta<\gamma} (\eta_\beta - \eta_\gamma) e^{-\Sigma |\eta_\alpha|^2/4\ell_0^2} \Psi_{\text{Pf}}(\bar{\eta}_\alpha)$$

must have $\kappa_{xy} = -\frac{1}{2}$, $\sigma_{xy} = \frac{1}{2}$. Therefore, the anti-Pfaffian has counter-propagating edge modes, which will have direct experimental significance, as we discuss later.

Landau-level mixing breaks PH symmetry. If we treat it perturbatively (although it is not particularly small in experiments), then, in addition to renormalizing the Coulomb repulsion, which does not break PH symmetry, it also generates three-body interactions, which do break the symmetry. In fact, the Pf is the exact ground state of the simplest repulsive nonvanishing three-body interaction. The $\overline{\text{Pf}}$ is, therefore, the exact ground state of the PH conjugate of this interaction, which is an attractive three-body interaction together with a repulsive two-body interaction with the same coefficient. When two-body Coulomb interactions and weaker three-body interactions are present, it is unclear which phase occurs. (Although we expect that the specific wave function Ψ_{Pf} is lower in energy than $\Psi_{\overline{\text{Pf}}}$ if the three-body interaction is repulsive, this does not tell us which phase the actual ground state of the Hamiltonian is in.) Fermion Chern-Simons theory, at the mean-field level, which includes nonperturbative mixing between all Landau levels, implies that the Pf is the ground state [4]. However, it is an open question which state is favored by the moderate Landau-level mixing which actually occurs in experiments, where the strength of Landau-level mixing, $(e^2/\epsilon\ell_0)/\hbar\omega_c \sim 1$. On the sphere, which does not have translational symmetry, a PH transformation is not a symmetry of the Hamiltonian; the Pf and $\overline{\text{Pf}}$ states of N particles occur at magnetic fluxes of $N_\Phi = 2N - 3$ and $N_\Phi = 2N + 1$, respectively. This may explain why the $\overline{\text{Pf}}$ has not been identified in numerical studies of finite systems on the sphere [11]. A PH transformation is a symmetry on the torus. However, in a finite system, we expect mixing between the Pf and the $\overline{\text{Pf}}$ so that the symmetric combination is the ground state, consistent with numerics [12].

Edge Excitations of the $\overline{\text{Pf}}$.—Under a PH transformation, the edge between the $\overline{\text{Pf}}$ and the vacuum ($\nu = 0$) is mapped onto the edge between the Pf and a $\nu = 1$ Hall liquid. (At $\nu = 5/2$, there are also the edges of the lowest Landau level of both spins, but they play no role in our discussion, so we drop them for simplicity.) Therefore, we can deduce the former from the properties of the latter, in which $\mathcal{L}_{\text{Pf}}$ is coupled to a counter-propagating free chiral Dirac fermion (or its bosonized equivalent):

$$\mathcal{L} = \frac{1}{4\pi}\partial_x\phi_1(-i\partial_t + v_1\partial_x)\phi_1 + \mathcal{L}_{\text{Pf}}(\psi_1, \phi_2) + \frac{1}{4\pi}2v_{12}\partial_x\phi_1\partial_x\phi_2 + \xi(x)\psi_1 e^{i(\phi_1 - 2\phi_2)} + \text{H.c.} \quad (2)$$

We have included a density-density interaction between the $\nu = 1$ and $\nu = 1/2$ edge modes, and also an intermode electron tunneling term. With an assumed intermode momentum mismatch and in the presence of impurities, the electron tunneling amplitude $\xi(x)$ can be taken as a random (complex) function with zero mean and short-ranged correlations, $\overline{\xi^*(x)\xi(x')} = W\delta(x - x')$. For large v_{12}, the tunneling term is relevant and can then be conveniently analyzed [15] by introducing a charge/neutral decomposition, $\phi_\rho = \phi_1 - \phi_2$ and $\phi_\sigma = \phi_1 - 2\phi_2$, and then fermionizing the neutral chiral boson: $e^{i\phi_\sigma} \equiv \psi_2 + i\psi_3$. The Lagrangian then takes the form $\mathcal{L} = \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{pert}}$, where

$$\mathcal{L}_{\text{sym}} = \frac{2}{4\pi}\partial_x\phi_\rho(-i\partial_t + v_\rho\partial_x)\phi_\rho + \psi_a(-\partial_t + iv_\sigma\partial_x)\psi_a$$

$$\mathcal{L}_{\text{pert}} = 2i\psi_1(\xi_1\psi_3 + \xi_2\psi_2) + \delta v_1\psi_1 i\partial_x\psi_1 + iv\psi_2\psi_3\partial_x\phi_\rho$$

with ξ_1, ξ_2 the real and imaginary parts of $\xi(x)$. The three Majorana fermions ψ_a, $a = 1, 2, 3$ form an $SU(2)_2$ triplet in the absence of the symmetry-breaking terms $\mathcal{L}_{\text{pert}}$. The first term in $\mathcal{L}_{\text{pert}}$ can be eliminated from the action by an SU(2) rotation, $\psi = O\tilde{\psi}$, with $O(x) = P\exp[-i\int_{-\infty}^{x} dx'(\xi_1(x')T_2 - \xi_2(x')T_3)/v_\sigma]$, where P denotes a path ordering of the integral, and T_a, $a = 1, 2, 3$ are the SU(2) generators in the spin-1 representation. In the transformed variables, the second and third terms in $\mathcal{L}_{\text{pert}}$ will have spatially dependent random coefficients. Integrating out ξ_1, ξ_2 (using the replica method, for instance), results in terms which have scaling dimension (-1) and are, therefore, perturbatively irrelevant. Hence, we obtain $\mathcal{L}_{\text{sym}}$ as the action for the edge between the Pf state and a $\nu = 1$ Hall droplet. The edge theory between the $\overline{\text{Pf}}$ state and the vacuum is obtained by flipping the directions of all the modes in $\mathcal{L}_{\text{sym}}$. There are *three* counter-propagating neutral Majorana fermion edge modes, which yield the expected value for the thermal Hall conductivity, $\kappa_{xy} = -1/2$.

In the $\overline{\text{Pf}}$ edge theory, the minimal dimension electron operator is $e^{i\phi_1} = (\psi_2 - i\psi_3)e^{2i\phi_\rho}$. There are 6 primary fields which are local with respect to this electron operator: 1, $e^{i\phi_\rho}$, ψ_a, $\psi_a e^{i\phi_\rho}$, $\phi^\alpha_{1/2}e^{i\phi_\rho/2}$, $\phi^\alpha_{1/2}e^{3i\phi_\rho/2}$. The spin-1/2 primary fields of $SU(2)_2$, denoted $\phi^\alpha_{1/2}$, can be written in terms of the Ising spin and disorder fields σ_a and μ_a of the three Majorana fermions: $\phi^+_{1/2} = \sigma_1\sigma_2\sigma_3 + i\mu_1\mu_2\sigma_3 + \mu_1\mu_2\mu_3 + i\sigma_1\sigma_2\mu_3$ and $\phi^-_{1/2} = \sigma_1\mu_2\mu_3 - i\mu_1\sigma_2\sigma_3 - i\sigma_1\mu_2\sigma_3 + \mu_1\sigma_2\sigma_3$. The fields $\phi^\alpha_{1/2}$ thus act to switch between periodic and antiperiodic boundary conditions on all three Majorana fermions. Note that $\phi_{1/2}$ is a dimension $3/16$ operator, unlike σ, which has dimension $1/16$. The difference in scaling dimension has consequences for quasiparticle tunneling as we discuss below.

Topological properties of the $\overline{\text{Pf}}$.—The 6 primary fields of the conformal field theory for the $\overline{\text{Pf}}$ correspond to its sixfold degeneracy on the torus. A $2d$ (bulk) effective field theory for the $\overline{\text{Pf}}$ state which encodes this degeneracy as well as the other bulk topological properties, can be deduced from consistency with the edge theory or, alternatively, in the following way. We begin by bosonizing the action for electrons at $\nu = 1/2$ employing a Chern-Simons gauge field c_μ:

$$\mathcal{L} = \Phi^*(i\partial_0 + c_0 + A_0)\Phi + \frac{1}{2m}|(i\partial_i + c_i + A_i)\Phi|^2 + V(\Phi) + \frac{1}{4\pi}c_\mu\epsilon_{\mu\nu\lambda}\partial_\nu c_\lambda, \quad (3)$$

where A_μ is the electromagnetic field. At the mean-field level, this is now a system of bosons at $\nu = 1$. We next use boson-vortex duality [16] to transform to a system of vortices, also at $\nu_{\text{eff}} = 1$, minimally coupled to a dual gauge field b_μ. Integrating out c_μ induces a Chern-Simons term for the b_μ:

$$\mathcal{L} = \Phi_v^*(i\partial_0 + b_0)\Phi_v + \frac{1}{2m_v}|(i\partial_i + b_i)\Phi_v|^2 + U(\Phi_v) - \frac{1}{4\pi}b_\mu\epsilon_{\mu\nu\lambda}\partial_\nu b_\lambda - \frac{1}{2\pi}b_\mu\epsilon_{\mu\nu\lambda}\partial_\nu A_\mu. \quad (4)$$

We now shift $b_\mu \to b_\mu - A_\mu$ and refermionize Φ_v since b_μ attaches one flux quantum to it:

$$\mathcal{L} = \bar{\chi}(i\partial_0 - A_0 - \mu)\chi + \frac{1}{2m_f}|(i\partial_i - A_i)\chi|^2 + \frac{1}{4\pi}A_\mu\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda + \ldots \quad (5)$$

The fermion field $\chi^\dagger$ creates holes which carry electrical charge opposite to that of the electron. Therefore, when we attach flux to these fermions, pair them, and condense the pairs [4,17], we will obtain a *negative* contribution to the Hall conductivity. Furthermore, if the pairing is due to gauge field fluctuations, as in [4], the pairs will have a $p - ip$ pairing symmetry. The resulting effective action can be written

$$\mathcal{L} = \bar{\eta}i\partial\!\!\!/\eta - m\bar{\eta}\eta - \frac{2}{4\pi}a^\rho_\mu\epsilon_{\mu\nu\lambda}\partial_\nu a^\rho_\lambda + \frac{1}{4\pi}a^\sigma_\mu\epsilon_{\mu\nu\lambda}\partial_\nu a^\sigma_\lambda - \frac{1}{2\pi}A_\mu\epsilon_{\mu\nu\lambda}\partial_\nu a^\rho_\mu. \quad (6)$$

The negative sign for the mass of the $2d$ Majorana fermion η is a consequence of the reversed pairing symmetry. The last term in (5) has been rewritten by introducing an auxiliary gauge field $\tilde{a}_\mu$, and then a_μ and $\tilde{a}_\mu$ have been replaced by $a^\rho_\mu = a_\mu - \tilde{a}_\mu$ and $a^\sigma_\mu = 2a_\mu - \tilde{a}_\mu$. The action (6) corresponds closely to the edge effective action (3). The most salient topological feature is the reversed chirality of the neutral fermion sector. As a result, the braid matrices are conjugated as compared to the Pf state [5,18]: $T_{ij} = e^{(\pi i/4)}e^{(\pi/4)\gamma_i\gamma_j}$.

Toy model.—We next describe a simple *lattice* model of spinless fermions which has similar physics to the neutral sectors of the Pf and $\overline{\text{Pf}}$ phases. The fermions hop only between near neighbor sites of a square lattice. At half filling, the model is invariant under the antiunitary symmetry $c_i \to (-1)^i c_i^\dagger$, $c_i^\dagger \to (-1)^i c_i$. With interactions present, we suppose that the fermions can develop the following order parameters: Δ_{ij}, which is a $\Delta(p) = \sin p_x + i\sin p_y$ superconducting order parameter which spontaneously breaks U(1); φ, which spontaneously breaks PH symmetry by enabling next-nearest-neighbor hopping; and θ, which spontaneously breaks $\pi/2$ rotational symmetry. The mean-field Hamiltonian with these order parameters is

$$H = \sum_{\langle i,j\rangle}(-tc_i^\dagger c_j + \Delta_{ij}c_i^\dagger c_j^\dagger + \text{H.c.}) - \mu\sum_i c_i^\dagger c_i - \sum_{\langle\langle i,j\rangle\rangle}\varphi c_i^\dagger c_j + \text{H.c.} + \sum_i \theta(c_i^\dagger c_{i+a\hat{x}} - c_i^\dagger c_{i+a\hat{y}}) + \text{H.c.}$$

When $\mu = \varphi = 0$, this Hamiltonian is PH symmetric. When $\varphi = \theta = \mu = 0$, there are gapless excitations at two "nodes," with momenta $\mathbf{k} = (\pi, 0), (0, \pi)$. Such gapless excitations would not be present for an off-lattice $p + ip$ superconductor. The low-energy excitations in the vicinity of each nodes is a (two-component) Majorana fermion, which can be combined into a single Dirac fermion ψ. Nonzero φ and θ are, respectively, Dirac and Majorana mass terms for ψ. The former breaks PH symmetry, but not the O(2) which rotates one Majorana fermion into the other [a $\pi/2$ lattice rotation is π rotation within this O(2)]; the latter does not break PH symmetry but breaks O(2).

Let us suppose that φ orders. Then μ must be adjusted to maintain half-filling. If $\varphi > 0$, we need $\mu > 0$, and the electron Fermi surface (when $\Delta = 0$) is closed and the hole Fermi surface is open. The system is adiabatically connect to electrons in the continuum, which essentially form a continuum $p + ip$ superconductor. In particular, it supports a gapless chiral Majorana fermion edge mode [17] with $\kappa_{xy} = \frac{1}{2}$. The long-distance form of the pair wave function is $g(\mathbf{r}) \sim 1/z$. Hence, we identify this phase with the neutral sector of the Pf. For $\varphi < 0$, the masses of the two nodal Majorana fermions change sign, giving $\kappa_{xy} = -\frac{1}{2}$. In this case, $\mu < 0$, and the holes form a closed Fermi surface rather than the electrons. The $\sin p_x + i\sin p_y$ pairs of electrons are, in this phase, better interpreted as $\sin p_x - i\sin p_y$ pairs of holes. This is consistent with our effective field theory of the $\overline{\text{Pf}}$ (5), which has $p - ip$ pairs of composite fermions obtained by attaching flux to holes. Moreover, $g(\mathbf{r}) \sim 1/z - (-1)^x/\bar{z} - (-1)^y/\bar{z}$. We identify this phase with the neutral sector of the $\overline{\text{Pf}}$.

At the transition between these two phases, $\varphi = 0$, and there are gapless excitations described by a single massless Dirac fermion: $\mathcal{L} = \bar{\psi}i\partial\!\!\!/\psi - g(\bar{\psi}\psi)^2$, a $2d$ analogue of the Gross-Neveu model. For $g < g_c$, the $\mathbb{Z}_2$ symmetry $\psi(t, x, y) \to \gamma^1\psi(t, -x, y)$, $\bar{\psi}(t, x, y) \to \bar{\psi}(t, -x, y)\gamma^1$ is unbroken (γ^1 is a purely imaginary γ matrix), and the nodal fermions are gapless. However, for $g > g_c$, PH symmetry can spontaneously break $\varphi \propto \langle\bar{\psi}\psi\rangle \neq 0$, and by varying explicit symmetry-breaking terms, such as μ or a second-neighbor hopping t', the system can be driven through a first-order transition between the two phases. At $g = g_c$, the system is critical. Exponents are known in the large-N limit, where N is the number of flavors of fermions, e.g., $\nu = 1 + 8/(3\pi^2 N)$ [19]. As the critical point is approached from within the symmetry-broken phase, the velocity of the gapless chiral Majorana fermion edge mode vanishes. The small value of the velocity of the neutral Majorana fermion edge mode in numerical studies of the Pf state may indicate proximity to such a critical point [20]. Note that with $\theta \neq 0$, there is also an intermediate phase with $\kappa_{xy} = 0$ which breaks rotational symmetry (perhaps slightly reminiscent of the nematic phase at $\nu = 5/2$ in a tilted field [21]).

Transitions and intermediate phases.—Informed by the preceding discussion, we can write down an effective field theory for the Pf and $\overline{\text{Pf}}$ states and the transition between them:

$$\mathcal{L} = \frac{-2}{4\pi} a_\mu^\rho \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda^\rho - \frac{1}{2\pi} A_\mu \epsilon_{\mu\nu\lambda} \partial_\nu a_\mu^\rho + \frac{1}{4\pi} a_\mu^\sigma \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda^\sigma + \Phi^*(i\partial_0 + a_0^\sigma)\Phi + \frac{1}{2m^*}|(i\partial_i + a_i^\sigma)\Phi|^2 + U(\Phi) + \bar{\Psi} i \not{\partial} \Psi - g(\bar{\Psi}\Psi)^2. \tag{7}$$

The phase in which $\langle\bar{\Psi}\Psi\rangle < 0$ and $\langle\Phi\rangle \neq 0$ corresponds to the Pf state: a_μ^σ is gapped by the Anderson-Higgs mechanism and the sign of the fermion Ψ's mass gives a right-handed Majorana fermion at the edge. The phase in which $\langle\bar{\Psi}\Psi\rangle > 0$ and $\langle\Phi\rangle = 0$ corresponds to the $\overline{\text{Pf}}$ state: there is a left-handed Dirac fermion at the edge associated with a_μ^σ and a left-handed Majorana fermion due to the sign of the mass of Ψ. At the critical point, both Φ and Ψ are critical. If Φ is fermionized using a_μ^σ, then the critical theory has two massless interacting Dirac fermions, in contrast to the toy model which possessed only one, having no bosonic edge modes.

However, one can imagine a scenario in which only one of these fields becomes critical. Then, the system will go into a phase with $\kappa_{xy} = \frac{1}{2}$, such as the phase in which $\langle\bar{\Psi}\Psi\rangle > 0$ and $\langle\Phi\rangle \neq 0$, in which there is only a single neutral Majorana fermion, but it is left-moving. In the phase with $\langle\bar{\Psi}\Psi\rangle < 0$ and $\langle\Phi\rangle = 0$, which also has $\kappa_{xy} = \frac{1}{2}$, there are three neutral Majorana fermions; two are right-moving and one is left-moving. Although these intermediate phases are logical possibilities, they are not related to the Pf or $\overline{\text{Pf}}$ phases by PH symmetry, and may be much higher in energy.

Discussion.—Our investigations open the door to a number of interesting questions. In the toy model, the difference between the two phases can be understood in terms of Fermi surface topology. Can the difference between the Pf and $\overline{\text{Pf}}$ states be understood in terms of similar momentum space structure in the lowest Landau level? If the Pf and $\overline{\text{Pf}}$ states are analogous to the two ordered states of the Ising model at low temperatures (spin-up and spin-down), then what is the analogue of the high-temperature phase? In the toy model, it is a critical superconducting phase with two gapless $(2 + 1) - d$ Majorana fermions. In the quantum Hall context, is it an analogous phase with bulk gapless neutral excitations yet with a quantized Hall conductance? Finally, there is the question of which state (if any of these) is seen in experiments. Since the main arguments for the Pf state derive from numerical studies which do not account for the effects of Landau-level mixing, we suggest that they be reconsidered in light of the $\overline{\text{Pf}}$ state. One signature of the Pf and $\overline{\text{Pf}}$ phases are their differing thermal conductivities, $\kappa_{xy} = 3/2$ and $\kappa_{xy} = -1/2$, respectively. Electrical transport measurements through a point contact will also differ in the two phases. As described in Ref. [22], weak tunneling of the charge $e/4$ non-Abelian quasiparticles between the edges of a Pf Hall bar leads to $R_{xx} \sim T^{-3/2}$. For the $\overline{\text{Pf}}$, one obtains $R_{xx} \sim T^{-1}$, different due to the extra edge modes present. (See Ref. [23] for experiments in this direction.) Interestingly, weak interedge tunneling of the charge $e/2$ Laughlin quasiparticle also gives $R_{xx} \sim T^{-1}$, in both the Pf and $\overline{\text{Pf}}$ states. The existence of counter-propagating neutral modes in the $\overline{\text{Pf}}$ state might also be detectable, and would have implications for interferometry experiments [7–10] if it proves to be realized at $\nu = 5/2$.

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Note added.—Upon completion of this work, we became aware of similar predictions made by Levin *et al.*, arXiv:0707.0483.

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