Exotic Paired States with Anisotropic Spin-Dependent Fermi Surfaces

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We propose a model for realizing exotic paired states in cold Fermi gases by using a *spin-dependent* optical lattice to engineer mismatched Fermi surfaces for each hyperfine species. The BCS phase diagram shows a stable paired superfluid state with coexisting pockets of momentum space with gapless unpaired carriers, similar to the Sarma state in polarized mixtures, but in our case the system is unpolarized. We propose the possible existence of an exotic "Cooper-pair Bose-metal" phase, which has a gap for single fermion excitations but gapless and uncondensed "Cooper-pair" excitations residing on a "Bose surface" in momentum space.

In recent years we have experienced a renewed interest in some of the basic questions behind the phenomenon of fermionic superfluidity, motivated by technical advances in the control and manipulation of cold atomic gases [1]. In particular, a great deal of attention is being devoted to the quest for unconventional paired states of matter.

Conventional superconductivity (BCS theory [2]) consists of pairing between fermions with opposite spin and equal but opposite momentum. In the presence of (partial) spin polarization more unusual superfluid states are possible. In the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state [3,4] pairing occurs across the mismatched Fermi surfaces so that the Cooper pairs have a finite center-of-mass momentum [5]. The excess unpaired fermions occupy a Fermi sea, so that there are gapless fermionic excitations coexisting with superfluidity. The FFLO state has been observed in solids only recently [6]. An interesting alternative to the FFLO state in partially polarized systems is a state with zero-momentum pairing as suggested by Sarma [7–11]. With disconnected regions of momentum space with pairing, separated by an unpaired polarized sea of fermions, the Sarma state is sometimes referred to as a "breached pair" phase [8,9]. Unusual paired phases can also be realized in rotating trapped fermionic superfluids [12]. Recently, a "pair density wave" or striped superconducting state with a spatially modulated order parameter similar to the FFLO state but without time-reversal symmetry breaking has been proposed to account for experimental observations in $La_{2−x}Ba_xCuO_4$ [13,14].

These unconventional paired states might be accessible in cold atom experiments, with "polarized" Fermi gases in two hyperfine states having different populations [1,15,16]. Additionally, one can explore the effects of differing masses in Fermi mixtures with two atomic species [17]. A mass "imbalance" between the two hyperfine states of a single atomic species can also be achieved in an optical lattice by tuning a *spin-dependent* hopping with light. Indeed, spin-dependent optical lattices have already been demonstrated [18]. A protocol to realize a Hubbard model with spin-dependent hopping has been proposed by Liu *et al.* in Ref. [19], by tuning the lattice lasers between fine structure levels of ^{40}K atoms, and using Feshbach resonances to manipulate the interactions. In reality, heavier atoms, such as Yb may be more suitable for this setup, due to the large fine structure splitting [20].

In this Letter we suggest a different means to access unconventional paired states in ultracold atomic systems. Our idea is to create mismatched Fermi surfaces in an *unpolarized* mixture. Consider an experiment with two hyperfine states (that we label ↑ and ↓ hereafter) with equal population that are moving, for simplicity, on a two-dimensional square lattice. Imagine tuning the hopping so that one spin state hops preferentially along the *x* axis and the other preferentially along the *y* axis. We are interested in the situation where the Fermi surfaces of the two spin states are rotated by 90° with respect to one another, and focus on a near-neighbor hopping Hamiltonian with single particle dispersions,

$$\epsilon_\uparrow(k_x, k_y) = -2t_a \cos(k_x) - 2t_b \cos(k_y) - \mu, \quad (1)$$

$$\epsilon_\downarrow(k_x, k_y) = -2t_b \cos(k_x) - 2t_a \cos(k_y) - \mu, \quad (2)$$

with chemical potential μ. The ratio $t_b/t_a \leq 1$ determines the eccentricity of the two Fermi surfaces (see Fig. 1). A similar Fermi surface geometry has been proposed for the ruthenate materials, [21,22] arising in a multiband model which possesses spin nematic order [23,24]. The nematic behavior originates from hybridization of the bands and is a spontaneous symmetry breaking, while our Hamiltonian explicitly breaks time-reversal and rotational symmetry.

To be concrete we assume the particles interact through a short-range *s*-wave potential, that we represent using the attractive Hubbard model:

$$H = \sum_{k,\sigma} \epsilon_\sigma(k) c^\dagger_{k,\sigma} c_{k,\sigma} + U \sum_i n_{i,\uparrow} n_{i,\downarrow}, \quad (3)$$

where $c^\dagger_{k,\sigma}$ creates a fermion with spin $\sigma = \uparrow, \downarrow$ at momen-
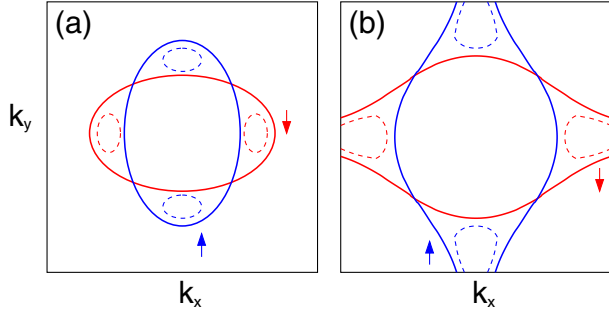
FIG. 1 (color online). Schematic Fermi surfaces for the dispersions $\epsilon_\sigma(k_x, k_y)$ described by Eq. (2): (a) closed Fermi surfaces for small density; (b) open Fermi surfaces for higher density/large anisotropy. Arrows label two hyperfine states of the same atomic species. Dashed lines delimit the pockets of the momentum distribution in the gapless G-SF solution.

tum k, $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$ is the local on-site density, and U is the interaction strength that we will take to be negative (attractive).

By simple inspection of the spin-dependent Fermi surfaces in Fig. 1, we can anticipate several different pairing possibilities within a BCS treatment. For a very strong attractive interaction, $|U| \gg t_a$, t_b, a state with zero-momentum pairing and a fully gapped Fermi surface is expected. For smaller U a zero-momentum paired state with gapless single fermion excitations analogous to the Sarma state in polarized mixtures could occur. Alternatively, pairing could occur across the two mismatched Fermi surfaces with a finite momentum condensate, an unpolarized analog of the FFLO state.

We now implement a self-consistent BCS approximation to obtain the mean-field phase diagram and explore the properties of the resulting paired superfluid states. We consider an on-site pairing of the form $-U/V\langle c_{j\downarrow}c_{j\uparrow}\rangle = \Delta \exp(iQj)$ with gap parameter Δ. Here $V = L^2$ is the number of sites in the system and j denotes the center-of-mass position of a pair in real space. The resulting quadratic Hamiltonian can be readily diagonalized, yielding quasiparticle excitations with dispersion:

$$E_\pm(k) = \sqrt{\epsilon_+(k)^2 + \Delta^2} \pm \epsilon_-(k), \qquad (4)$$

where we have defined

$$\epsilon_\pm(k) = \left[\epsilon_\uparrow\left(-k + \frac{Q}{2}\right) \pm \epsilon_\downarrow\left(k + \frac{Q}{2}\right)\right]/2. \qquad (5)$$

Consider first the case of zero pairing momentum $Q = 0$. Besides the normal phase with $\Delta = 0$, we expect a fully gapped BCS-like solution when $E_\pm(k) > 0$ for all momenta in the Brillouin zone (BCS state). This occurs when the attractive interaction is sufficiently strong. For smaller Δ a gapless superfluid phase (G-SF) is also possible [8,11]. Gapless fermion excitations occur at momenta that satisfy $E_+(k) = 0$ or $E_-(k) = 0$, or equivalently,

$$\epsilon_\uparrow(k)\epsilon_\downarrow(k) = -\Delta^2. \qquad (6)$$

The resulting closed curves in momentum space are sketched schematically as dashed lines in Fig. 1. In the case of closed Fermi surfaces as shown in Fig. 1(a) there are four small pockets near the Fermi points $(\pm k_{Fx}^\downarrow, 0)$ and $(0, \pm k_{Fy}^\uparrow)$. The solution with open Fermi surfaces [see Fig. 1(b)] has two large pockets that wrap around the Brillouin zone.

It is instructive to compute the momentum distribution function for the two spin species, $n_\sigma(k) \equiv \langle c_{k,\sigma}^\dagger c_{k,\sigma}\rangle$. There are three distinct regions in momentum space determined by the conditions: (i) $E_-(k) > 0$ and $E_+(k) < 0$ which occurs inside the $\uparrow$ pockets; (ii) $E_+(k) > 0$ and $E_-(k) < 0$ which occurs inside the $\downarrow$ pockets; or (iii) both $E_\pm(k) > 0$ which occurs over the rest of the Brillouin zone. For momentum where (i) is satisfied we obtain $n_\uparrow(k) = 1$, $n_\downarrow(k) = 0$, whereas in region (ii) one has $n_\uparrow(k) = 0$, $n_\downarrow(k) = 1$. Thus, inside these pockets the fermions are completely unpaired, and there is no mixing between the two spin species. Finally, in region (iii) the fermions are completely paired and one obtains

$$n_\uparrow(k) = n_\downarrow(k) = \frac{1}{2}\left(1 - \frac{\epsilon_+(k)}{\sqrt{\epsilon_+(k)^2 + \Delta^2}}\right). \qquad (7)$$

While possessing regions in momentum space with complete pairing and other regions with no pairing whatsoever is reminiscent of the Sarma phase, this novel *unpolarized* G-SF phase has unpaired fermion pockets of *both* spin species.

We now analyze the (mean-field) stability of the normal, BCS and G-SF phases, by solving the BCS saddle point (gap) equations numerically on the square lattice. We work in the canonical ensemble, with fixed density, which is the relevant situation for cold fermionic gases, also allowing for finite momentum pairing with momenta Q either along the x and y axes or along the diagonals in the Brillouin
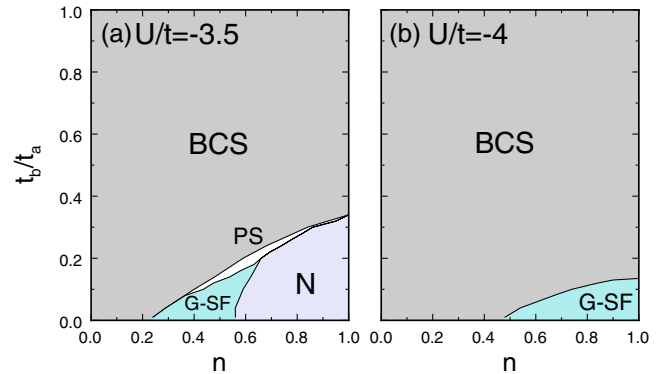

FIG. 2 (color online). Mean-field phase diagram of the attractive Hubbard model as a function of the anisotropy t_b/t_a and fermion density n, for values of the interaction (a) $U = -3.5t_a$ and (b) $U = -4t_a$. We find regions occupied by a fully paired solution with $Q = 0$ (BCS), normal (N), gapless superfluid (G-SF), and phase separation (PS).

zone. The Helmholtz free energy in the canonical ensemble is computed and minimized in systems sizes up to 300×300 sites following the prescription detailed in Ref. [25].

For small values of the attraction $|U|/t_a$ we find that the system is in the normal state, independently of the total fermion density, n, provided that the anisotropy is not too close to $t_b/t_a = 1$, while for large values of the interaction strength, the system is in a fully gapped superfluid state with $Q = 0$ (BCS). The interesting regime is for values of $2t \lesssim |U| \lesssim 4t_a$, where we find a competition of phases. In Fig. 2 we show the phase diagram for two different values of $|U|/t_a$, as a function of the fermion density and the anisotropy t_b/t_a. At $|U|/t_a = 3.5$, the normal phase occupies a region at high densities and high anisotropy, while the superconducting phase (BCS) dominates at intermediate and small anisotropy. Separating these two phases is a window where the G-SF state is realized. There is also a tiny sliver where phase separation takes place, that coincides with the onset of the instability toward the G-SF state. We determined the boundaries of this region by identifying densities with negative compressibility $dn/d\mu$, and using the Maxwell construction. Contrary to the Sarma phase in polarized mixtures, the G-SF state is stable in a wide region of parameters [10,11]. At larger values of the interaction the normal phase becomes energetically unfavorable, leaving a region of the G-SF phase that shrinks with increasing interaction. In all cases the G-SF phase only occurred in the with open Fermi surfaces/large pockets [see Fig. 1(b)]. We also looked for energy minima at finite momentum, but we have not found solutions for the system sizes and in the parameter space that we studied.

We now turn to the possibility of paired states which are *not superfluids*. This is motivated by recent work [26] which introduced and studied a hard-core boson model with ring exchange, and made strong arguments for the existence of a novel Bose-metal phase, a *D*-wave Bose liquid. Here we will refer to this phase as a *D*-wave Bose-metal (DBM). The proposed Bose-metal phase is not a superfluid, having zero condensate and superfluid densities, but possesses gapless excitations which reside on surfaces in momentum space. In contrast to a conventional Fermi liquid, in the DBM phase the gapless excitations cannot be described in terms of weakly interacting quasiparticles. The low energy excitations are intrinsically strongly interacting fluctuations, and a single particle description is incorrect. In the DBM phase pairs of *bosons* possess *D*-wave correlations, but the phase is *not* a paired-boson condensate.

A connection between our model of spin-full fermions with spin-dependent spatially anisotropic Fermi surfaces and the Bose-metal phase explored in Ref. [26] can be made as follows. First we define an on-site "Cooper-pair" operator, $b_i^\dagger = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger$. This operator is a "hard-core" boson, commuting at different sites, $[b_i, b_j^\dagger] = 0$ for $i \neq j$, yet satisfying $b_i^\dagger b_i^\dagger = 0$. Now consider the regime with $|U| \gg t_a, t_b$ where all of the fermions are tightly bound

into on-site Cooper pairs. The goal is to derive an *effective* Hamiltonian by considering a perturbation expansion in powers of $t_a/|U|$, $t_b/|U|$. At second order one obtains a boson hopping term H_J with strength $J \sim t_a t_b/|U|$,

$$H_J = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + \text{H.c.} \qquad (8)$$

The hopping J is proportional to a product of t_a and t_b since it is necessary to hop both the $\uparrow$-spin and $\downarrow$-spin fermion in order to move the hard-core boson. Notice that in the extreme anisotropic limit with $t_b = 0$ and $t_a \neq 0$ it is actually impossible for a single boson to move, $J = 0$.

But consider the boson ring-exchange term as depicted in Fig. 3, which involves two bosons on opposite corners of an elementary square plaquette rotating by ± 90 degrees,

$$H_{\text{ring}} = K_{\text{ring}} \sum_{\text{plaquettes}} b_1^\dagger b_2 b_3^\dagger b_4 + \text{H.c.}, \qquad (9)$$

with $i = 1, 2, 3, 4$ labeling sites taken clockwise around the square plaquette. The ring-exchange coupling strength $K_{\text{ring}} > 0$ can be computed at fourth order in $t_{a,b}/|U|$. In the highly anisotropic limit with $t_a \gg t_b$, the process depicted in Fig. 3 dominates, giving $K_{\text{ring}} \sim t_a^4/|U|^3$. Remarkably, while the single boson hopping term vanishes in the extreme anisotropic limit, $t_b/t_a \to 0$, the ring term which hops pairs of bosons is nonzero. Thus, with increasing anisotropy, t_a/t_b, the ratio $K_{\text{ring}}/J \sim (t_a/t_b)^4$ increases, and the ring term becomes increasingly important in the Hamiltonian, $H_b = H_J + H_{\text{ring}}$.

Consider the phase diagram for the boson Hamiltonian H_b, when the bosons are at some generic density, $n_b \equiv \langle b_i^\dagger b_i \rangle$. For $K_{\text{ring}} \ll J$ one expects a Bose-condensed superfluid phase. In Ref. [26] variational wave function studies found that the *D*-wave Bose-metal (DBM) was stabilized for $K_{\text{ring}}/J \gtrsim 2$ and for boson densities larger than $n_b \gtrsim 0.4$ (or, under particle-hole symmetry, smaller than 0.6). At other densities in this large K_{ring} regime, phase separation into a (probable CDW) phase at half-filling and a zero density state was found.
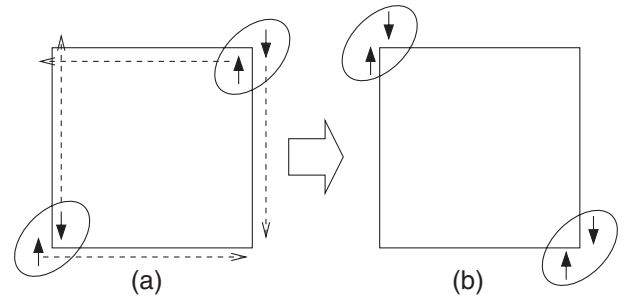

FIG. 3. Processes that would give origin to an effective ring-exchange of pairs. Fermions of different hyperfine species (with opposite spin) prefer to move along perpendicular directions (left).

The boson superfluid state corresponds to a fully gapped (conventional) BCS state of the fermions. However, the DBM phase of the bosons would correspond to a truly exotic nonsuperfluid paired state of fermions, with a full gap for the single fermion excitations, but with uncondensed Cooper pairs. The Cooper pairs would form a collective state with gapless excitations along surfaces in momentum space—a "Cooper-pair Bose-metal" (CPBM).

Since the fermion density, $n = n_\uparrow + n_\downarrow = 2n_b$, this would suggest that the CPBM phase might be present in the phase diagram in the regime of intermediate U/t_a, strong anisotropy with t_b/t_a small and for densities near half-filling, $n \gtrsim 0.8$. As one can see from Fig. 3(b), this is precisely the regime where the BCS mean-field treatment finds the stable G-SF state. The G-SF and CPBM phases are dramatically different. The G-SF state is a superfluid with gapless fermion excitations, while in the CPBM phase the fermions are fully gapped but the state is *not* a superfluid. Wether these states are realized by the original attractive Hubbard model is unknown, and certainly worthy of future investigation. Recent studies of the boson ring model on the two-leg ladder have found compelling evidence for a quasi-1D Bose metal, a ladder descendant of the 2D DBM state [27].

To summarize, we have proposed a model that allows for realizing exotic paired phases of unpolarized fermion mixtures using spin-dependent optical lattices. The main ingredient is the existence of mismatched Fermi surfaces between both hyperfine states. We explored the BCS mean-field phase diagram for two anisotropic Fermi surfaces rotated by 90° respect to one another, checking for the relative stability of the BCS, FFLO, gapless superfluid and normal metal phases. Other mean-field states such as a nematic superfluid [28], or multimodal FFLO states [29,30] could also be considered. Even though the weak-coupling BCS treatment used in this work has limited applicability in 2D, our results demonstrate the rich physics that emerges once one allows for spin-dependent Fermi surfaces. Other possibilities can also be explored, such as a combination of circular and elliptical Fermi surfaces, with the only requirement that both Fermi surfaces enclose the same area. We also argued that anisotropic Fermi surfaces plus attractive interactions leads to an effective model of Cooper pairs with a ring-exchange term, that may allow to realize a paired but nonsuperfluid Bose-metal phase. Because of the strong coupling nature of this Cooper-pair Bose-metal phase, determining its existence in the attractive Hubbard model would require a numerical approach and will be explored in future work.

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