

## Measurement of the Intrinsic Subgap Dissipation in Josephson Junctions

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We show that the subgap dissipation in sufficiently high-quality Josephson junctions, as measured directly from the current at which the junctions switch from the voltage state into the zero-voltage state, is controlled by thermally excited quasiparticle tunneling. The dissipation is in excellent agreement with a recent theory of Chen, Fisher, and Leggett.

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Dissipation plays a crucial role in the dynamic properties of many physical systems. One such system is the Josephson junction, which has been used to study macroscopic thermal activation<sup>1,2</sup> and quantum tunneling<sup>3-7</sup> out of the zero-voltage state, and has been suggested as a good system for the study of macroscopic quantum coherence.<sup>8</sup> All of these studies require an understanding of the magnitude, temperature dependence, and frequency dependence of the dissipation. In particular, macroscopic quantum coherence effects have been predicted to be observable only in a system with very low subgap dissipation.<sup>9</sup> Nevertheless, workers in the field have disagreed as to the source of dissipation in Josephson junctions. Voss and Webb<sup>3</sup> and Washburn *et al.*,<sup>4</sup> in analyses of macroscopic quantum tunneling data, used the normal-state tunneling resistance ( $R_n$ ) to calculate an effective subgap damping parameter. In contrast, Silvestrini *et al.*<sup>2</sup> compared their effective resistances inferred from thermal activation data to their measured subgap resistances, but found a discrepancy of a factor of 20 and therefore attributed their dissipation to a non-tunneling process involving thermally activated quasiparticles. Devoret, Martinis, and Clarke<sup>6</sup> found that external circuit loading significantly influenced the effective dissipation in their tunnel junctions.

Until now, estimates of the subgap dissipation in Josephson junctions have been made by fitting data describing switching out of the zero-voltage state by thermal activation or macroscopic quantum tunneling models with the dissipation as a fitting parameter. These fitting procedures are indirect because the contribution of dissipation is masked by the exponential dependence of the transition rate on the barrier potential. Small errors in the barrier potential energy can therefore produce large variations in the values for dissipation inferred in this way. Recently, several theoretical workers<sup>10,11</sup> have pointed out a more direct way to measure subgap dissipation in Josephson junctions. This method involves measuring  $I_r$ , the current at which the junction returns to the zero-voltage state from the finite-voltage state. In the present work, which is the first quantitative experimental use of this approach, we have measured the

subgap dissipation in very high-quality Nb-PbAuIn tunnel junctions. We find that the observed return currents follow closely the theoretically predicted behavior for an ideal-tunnel-junction model.<sup>11</sup> This demonstrates for the first time that, for sufficiently high-quality junctions, the subgap dissipation is dominated by the tunneling of thermally excited quasiparticles.

In the resistively shunted junction (RSJ) model<sup>12</sup> the behavior of a Josephson junction is approximated by an ideal Josephson junction in parallel with a shunt resistance and capacitance. The Josephson current  $I_j = I_c \sin \phi$ , where  $I_c$  is the Josephson critical current and  $\phi$  is the relative phase difference between the superconducting order parameters of the two electrodes making up the junction. In the RSJ model the equation of motion for  $\phi$  is like that of a driven damped pendulum:

$$\frac{d^2 \phi}{d\tau^2} + \frac{1}{\omega_j R_j C_j} \frac{d\phi}{d\tau} = \alpha - \sin \phi. \quad (1)$$

Here  $\tau = \omega_j t$  is the time in units of the plasma frequency  $\omega_j = (2\pi I_c / C_j \Phi_0)^{1/2}$ ,  $C_j$  is the junction capacitance,  $\Phi_0$  is the flux quantum  $h/2e$ ,  $R_j$  is the effective shunt resistance, and  $\alpha = I/I_c$  is the reduced bias current through the junction. If the damping coefficient  $(\omega_j R_j C_j)^{-1}$  is sufficiently small, there are two solutions to Eq. (1). The zero-voltage state corresponds to the phase oscillating around a particular potential minimum. The voltage state corresponds to the phase increasing monotonically with time, leading to a nonzero time-averaged voltage through the relation  $V = (\Phi_0/2\pi) d\phi/dt$ . As the bias current is reduced from a large value with the junction in the voltage state, a point is reached at which the energy lost to dissipation is just greater than the energy fed into the system by the external bias current, and the junction switches to the zero-voltage state. This condition is satisfied in the RSJ model in the weak-damping limit ( $I_r \ll I_c$ ) at a return current

$$I_r^{\text{RSJ}} = \frac{4I_c}{\pi \omega_j R_j C_j}. \quad (2)$$

In the RSJ model the shunt resistance is taken to be

independent of voltage. Recently Chen, Fisher, and Leggett<sup>11</sup> (CFL) have given an expression for the return current appropriate for a low-dissipation, nonlinear, ideal tunnel junction. This theory accounts for the frequency and phase dependences of contributions to the dissipation from both the quasiparticle current and the quasiparticle-pair interference current [due to the “ $\cos(\phi)$  term”]. For an ideal junction specified by a Bardeen tunneling Hamiltonian the quasiparticle current is given by<sup>13</sup>

$$I_{qp}(eV) = \frac{1}{eR_n} \int_{-\infty}^{\infty} n_L(E) n_R(E - eV) [f(E - eV) - f(E)] dE, \quad (3)$$

where  $R_n$  is the normal-state resistance, the BCS densities of states for the two electrodes are given by

$$n_i(E) = \Theta(|E| - \Delta_i) |E| / (E^2 - \Delta_i^2)^{1/2},$$

where  $\Theta(x) = 1$  for  $x > 0$  and 0 otherwise, the  $\Delta_i$ 's are the (positive real) superconducting energy gaps of the two electrodes, and  $f(E)$  is the Fermi function  $f(E) = (1 + e^{E/k_B T})^{-1}$ . The quasiparticle-pair interference-term current  $I_{J2}$  is given by a similar expression with the quasiparticle density of states  $n_i(E)$  replaced by the Cooper-pair density of states

$$p_i(E) = \Theta(|E| - \Delta_i) \Delta_i \operatorname{sgn}(E) / (E^2 - \Delta_i^2)^{1/2},$$

where  $\operatorname{sgn}(E) = 1$  for  $x > 0$  and  $-1$  otherwise. In what follows the subscript  $L$  corresponds to the Nb electrode, and  $R$  to the PbAuIn electrode.

The frequency components of the dissipation important to the retrapping process are  $0 < \omega \lesssim \omega_j$ , since these are the dominant Fourier components of  $d\phi/dt$  as  $\phi$  moves from one barrier of the washboard potential to the next before retrapping. For the present experiments  $\hbar\omega_j \ll \Delta_i$ ,  $\Delta_L - \Delta_R$ , and  $k_B T$ , so that to a good approximation  $I_{qp}(\omega) = \hbar\omega/eR_{qp}$ , and  $I_{J2}(\omega) = \hbar\omega/eR_{J2}$ , for  $\omega < \omega_j$ , where  $1/R_{qp}$  and  $1/R_{J2}$  are frequency-independent low-voltage quasiparticle and quasiparticle-pair interference conductances, respectively. Under these conditions and in the weak-damping limit the CFL expression for the return current becomes simply

$$I_r^{\text{CFL}} = \frac{4I_c}{\pi\omega_j C_j} \left[ \frac{1}{R_{qp}} + \frac{1}{3R_{J2}} \right]. \quad (4)$$

In the limits  $\hbar\omega_j \ll k_B T \ll \Delta_i$ ,  $\Delta_L - \Delta_R \ll \Delta_L$ , and with  $\Delta_L > \Delta_R$ , the quasiparticle component of the conductance is given by

$$\frac{1}{R_{qp}} = \frac{1}{R_n} \frac{\Delta_L}{k_B T} g \left( \frac{\Delta_L - \Delta_R}{k_B T} \right) e^{-\Delta_L/k_B T}, \quad (5)$$

where  $g(z) = e^{z/2} K_0(z/2)$ , and  $K_0$  is the  $\nu=0$  modified Bessel function.<sup>14</sup> In these same limits the quasiparticle-pair interference-term conductance is given by  $1/R_{J2} = (\Delta_R/\Delta_L)1/R_{qp}$ . Inserting these expressions into Eq. (4) gives the following result for the return current:

$$I_r^{\text{CFL}} = \frac{4I_c}{\pi\omega_j C_j R_n} \frac{\Delta_L + \Delta_R/3}{k_B T} g \left( \frac{\Delta_L - \Delta_R}{k_B T} \right) e^{-\Delta_L/k_B T}. \quad (6)$$

Notice that  $I_r$  is dominated by the exponential tempera-

ture dependence which arises from tunneling of thermally activated quasiparticles.

It should be emphasized that Eq. (6) is for a deterministic equation of motion which ignores thermal (or quantum) fluctuations. Ben-Jacob *et al.*<sup>10</sup> have given a simple expression for the lifetime of a junction in the voltage state before it is thermally activated into the zero-voltage state:

$$\tau_s = RC(\pi k_B T / \Delta w)^{1/2} e^{\Delta w/k_B T}, \quad (7)$$

where  $\Delta w = (I - I_r)^2 R^2 C$ . In the presence of thermal fluctuations a number of measurements of the switching current for a finite current sweep rate  $dI/dt$  has a distribution of values<sup>1</sup>  $P(I)$  centered around a current higher than the deterministic result  $I_r$ .  $P(I)$  is related to the lifetime  $\tau_s$  by

$$P(I) = \tau_s^{-1} (dI/dt)^{-1} \left[ 1 - \int_I^{\infty} P(i) di \right].$$

The predicted return currents of Eq. (6) therefore have to be corrected for this shift before a direct comparison with experiment can be made.

The samples used in this study were three 25- $\mu\text{m}$ -long by 0.3- $\mu\text{m}$ -wide Nb-PbAuIn edge junctions, with current densities of about 10 A/cm<sup>2</sup>, fabricated with use of techniques described previously.<sup>15</sup> The junctions were measured in a single-shot, closed-cycle <sup>3</sup>He refrigerator capable of reaching temperatures as low as 0.45 K. The Dewar was shielded from external magnetic fields by a double layer of Mumetal. High-capacitance cabling, twisted pair leads, and 1-k $\Omega$  resistors near the samples at low temperatures were used to isolate the samples from room-temperature noise. The measurements were made in a screened room with batteries powering the current sweep and the preamplifiers, thereby isolating the samples from the electronics outside of the screened room.

Our junctions showed high-quality current-voltage behavior (Fig. 1). However, the junction characteristics were nonideal for  $eV > \Delta_{\text{PbAuIn}}$ , as has been noted earlier.<sup>16</sup> There was an overshoot at the voltage  $V_s = (\Delta_{\text{PbAuIn}} + \Delta_{\text{Nb}})/e$ , and some excess current for  $\Delta_{\text{PbAuIn}} < eV < \Delta_{\text{PbAuIn}} + \Delta_{\text{Nb}}$ . The normal resistance  $R_n$  of the junction of Fig. 1 was 1820  $\Omega$ , obtained by fitting the  $I$ - $V$  characteristic with Eq. (3) for voltages above  $V_s$ . At low temperatures the product  $I_c R_n$  is expected to have a value of 2.25 mV,<sup>17</sup> indicating a critical current of 1.24  $\mu\text{A}$ . The measured low-temperature critical

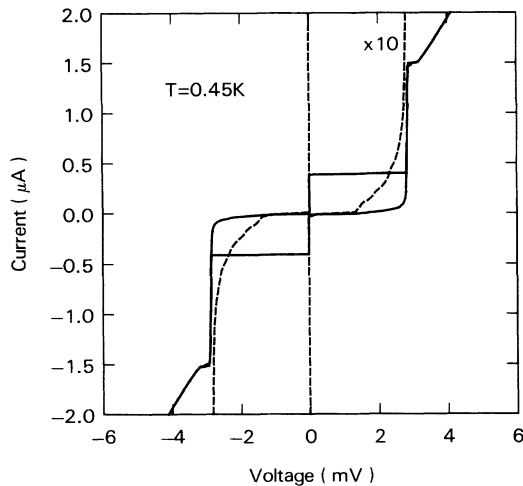


FIG. 1. Current-voltage characteristic at  $T=0.45$  K for an  $R_n=1820\text{-}\Omega$  junction. The dashed curve is expanded in current by a factor of 10.

current, from fitting distributions of transitions into the voltage state,<sup>18</sup> is  $0.6 \mu\text{A}$ . This 50% reduction of the critical current from the ideal is larger than the roughly 30% reduction that had been observed previously for the Nb-PbAuIn technology, and attributed to strong-coupling effects.<sup>16</sup>

Figure 2 shows the low-voltage current-voltage characteristics for an  $R_n=1820\text{-}\Omega$  junction at selected temperatures. These data were taken with the current bias decreasing from large positive currents, and shows switching from the voltage state into the zero-voltage state. The dashed curves in Fig. 2 were generated numerically from Eq. (3). The zero-temperature gaps of the two electrodes were taken to be  $\Delta_R(T=0)=1.35$  meV and  $\Delta_L(T=0)=1.52$  meV. These are established values for PbAuIn and Nb, respectively, and correctly predict the voltage positions of the current step and logarithmic cusp in the current-voltage characteristic. The temperature dependences of the gaps were assumed to follow the BCS theory, with critical temperatures of 7.0 and 9.3 K for the PbAuIn and Nb, respectively. The approximate analytical expression for the quasiparticle conductance [Eq. (5)] is about 15% different at low voltages from the numerical results shown in Fig. 2. Since our data were taken with a constant current bias, the negative resistance regions above  $V_d=[\Delta_{\text{Nb}}(T) - \Delta_{\text{PbAuIn}}(T)]/e$  are unstable to fluctuations,<sup>11</sup> implying a "voltage gap"—as seen in the experimental curves. Below  $V_d$  the data agree quite well with the predicted curves. It should be emphasized that the dashed curves in Fig. 2 were derived with no adjustable parameters, since  $R_n$  and the  $\Delta_i$ 's were read from the experimental data above  $V_s$ .

The solid dots in Fig. 3 show the measured return

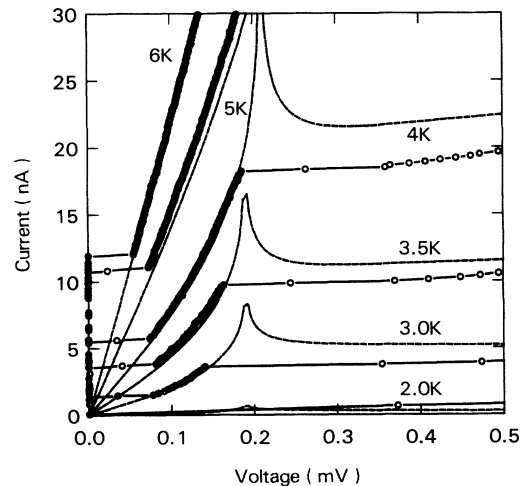


FIG. 2. Current-voltage characteristics in the region of return from the voltage state to the zero-voltage state of an  $R_n=1820\text{-}\Omega$  junction. The open circles are the experimental points. The dashed curves are theoretical predictions in the absence of switching, as described in the text.

currents for an  $R_n=7300\text{-}\Omega$  junction as a function of inverse temperature. The open circles show the predictions for the deterministic  $I_r$  of Eq. (6), with no adjustable parameters. In the evaluation of Eq. (6)  $I_c(T)$  was taken from Ambegaokar and Baratoff,<sup>17</sup> the  $\Delta_i$ 's were assumed to have a BCS temperature dependence, and  $R_n$  and  $\Delta_i(T=0)$  were deduced directly from the  $I$ - $V$  curves. The only remaining parameter was  $C_j$ , which we estimate to be 1.8 pF by scaling capacitance measurements<sup>19</sup> of previous samples made with the Nb-PbAuIn edge junction technology to the dimensions of our sam-

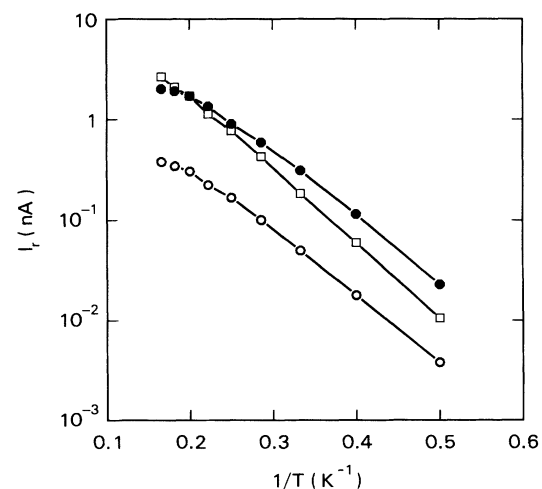


FIG. 3. Return currents vs inverse temperature for an  $R_n=7300\text{-}\Omega$  junction. The solid dots are experimental data. The open circles are the predictions of the theory of Chen, Fisher, and Leggett. The open squares are the theoretical predictions corrected for the effects of thermal fluctuations.

ples. Although the absolute magnitude predicted for  $I_r$  from Eq. (6) is low, the temperature dependence agrees remarkably well with the experimental points. In particular, theory and experiment show the same exponential temperature dependence at low temperatures. The discrepancy in absolute magnitude is reduced when thermal fluctuations are accounted for. The open squares in Fig. 3 are the peak currents for the predicted thermally activated switching distribution  $P(I)$ , with use of an effective resistance  $R_{\text{eff}}^{-1} = R_{\text{qp}}^{-1} + (3R_{J2})^{-1}$  in Eq. (7),  $I_r$  from Eq. (6), and the experimental current sweep rate  $10^{-11}$  A/sec, with the other parameters the same as listed above. The widths of the predicted (and measured) return-current switching distributions  $P(I)$  are smaller than the symbols used in Fig. 3, and their positions are insensitive to the sweep rate chosen. For example, increasing the sweep rate by a factor of  $10^4$  lowers the peak currents of the predicted return-current distributions by about 30%.

Given the uncertainty of our estimate for the junction capacitance ( $\sim 30\%$ ) and the approximations involved in correcting for the effects of thermal fluctuations, the CFL predictions for the return currents are in excellent agreement with experiment. These results indicate that the mechanism for subgap dissipation in sufficiently high-quality junctions is the tunneling of thermally activated quasiparticles. Moreover, although our results do not directly demonstrate which dissipative mechanism is important for macroscopic quantum tunneling (MQT), the important frequencies for MQT are comparable to or smaller than those involved in retrapping. Therefore our results suggest that previous attempts to measure the dissipation using MQT have been dominated by nonideal dissipation in low-quality junctions in those cases when loading effects from external circuitry were not identified as dominating.<sup>6</sup>

The effective resistances that we infer from these measurements [using Eqs. (4) and (7)] agree to within a factor of 2 with those inferred from the  $I$ - $V$  characteristics, and show no evidence of saturation to the lowest measurable return currents. Our measurements place a lower

limit of  $R_{\text{eff}}(T=0) > 1 \text{ M}\Omega$  for the  $R_n = 7300\text{-}\Omega$  junction. This is comparable to the effective subgap dissipative resistance desirable for the observation of macroscopic quantum coherence effects.<sup>9</sup>

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