THEORY OF SUPERCONDUCTOR-INSULATOR TRANSITIONS IN DISORDERED FILMS

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A scaling theory for the zero temperature ($T=0$) superconductor to insulator transition in disordered films is described. Right at the transition, the system is predicted to be "metallic", with a resistance per square having a finite, non-zero value at $T=0$. This value, moreover, should be universal, independent of all microscopic details. In the presence of an applied magnetic field, an additional $T=0$ superconductor-insulator transition is accessible at which both resistivities ρ_{xx} and ρ_{xy} should be universal.

INTRODUCTION

In the past decade substantial progress has been made in understanding Anderson localization in electron systems and the metal-insulator transition in dirty interacting Fermion systems[1]. A central conclusion which has emerged, is that in two dimensions (2D) even weak disorder localizes all states: A true 2D metallic phase with non-zero conductivity is not possible at $T=0$. In this paper, I describe some recent results on related phenomena in bosonic systems[2] In particular, I consider possible superconducting to insulating transitions in disordered systems, the direct bosonic analog to the metal-insulator transition. Attention is focussed on the 2D case, since a number of recent experiments[3] have probed this transition by systematically varying the thickness of amorphous films. In this way the subtle interplay between localization and superconductivity can be examined.

The first part of the paper describes a scaling theory of the $T=0$ superconductor-insulator transition[2]. Quite remarkedly, as in Anderson localization, two dimensions again emerges as a special dimension. In (and only in) 2D, right at the transition separating superconducting from insulating behavior, the system should be "metallic", with a finite, non-zero resistivity. Thus, perhaps paradoxically, disordered (unpaired) electrons in 2D cannot diffuse at $T=0$, but Cooper pairs, perched on the brink of superconductivity, can. This 2D metallic state, moreover, is predicted to have a universal resistance per square[4], independent of all microscopic details, depending only on the universality class of the superconductor-insulator transition. These results are consistent with recent experiments on amorphous thin-film superconductors[3].

In the second section, the effects of an applied magnetic field on the low temperature properties of dirty superconducting films are considered. It is argued that an additional $T=0$ superconductor-to-insulator transition can be accessed, by varying the strength of the applied field. At low fields, vortices introduced by the field are localized and do not dissipate at $T=0$, so that the film can be superconducting, in a so-called vortex-glass phase[5]. With increasing field, the vortices should de-localize and the film undergo a $T=0$ transition into an insulator. Right at this transition, in addition to a universal resistivity ρ_{xx}, the Hall resistivity, ρ_{xy}, should be finite and universal.

SCALING THEORY OF TRANSITION

It has recently been argued that the $T=0$ superconductor-insulator phase transition in thin amorphous[2], films such as those of ref. 3, can be correctly described by a model of charge 2e bosons moving in a 2D random potential[6]. In the superconducting phase the electrons have bound to form Cooper pairs and a description of the low energy physics in terms of charge 2e composite bosons is presumably valid. In the insulating phase, where pairing is destroyed and the individual electrons are presumably localized by the disorder, such a description is inadequate. It seems nonetheless likely that the asymptotic critical properties of the transition are insensitive to the obvious difference between bosonic and fermionic insulating phases, i.e., between the bose glass[7] and fermi glass[1] phases. Indeed, Bose condensation and the superconducting transition in pure systems at finite T belong, e.g., in the same universality class, the difference between the normal bosonic and fermionic phases notwithstanding. Moreover, in 1D the $T=0$ superconductor-insulator transition can be studied directly in terms of a model of electrons with a BCS attractive interaction moving in a random potential. It is found[8] that the critical behavior of this $T=0$ phase transition is in the same universality class as the superfluid-insulator transition in a model of repulsively interacting bosons, representing the Cooper pairs, moving in a random potential. One therefore expects that the experimentally relevant superconductor-insulator transition in amorphous films can be properly described in terms of charge 2e bosons.

Consider then the Hamiltonian for a system of charged bosons: $H = H_0 + H_1$ with,

$$H_0 = \int d^d x[(\hbar^2/2m)\,|\nabla\psi|^2 + U(x)\,|\psi(x)|^2] \tag{1a}$$

$$H_1 = \int d^d x d^d x' V(x-x')[\,|\psi(x)|^2 - n_0][\,|\psi(x')|^2 - n_0], \tag{1b}$$

and $\psi(x)$ the usual boson field operator. Here $V(x) = (2e)^2/|x|$ is a Coulomb interaction between the bosons, with n_0 a compensating positive charge background (charge neutrality fixing the boson density at n_0), and $U(x)$ a random potential. Since the 2D screening length is typically macroscopic for thin films (λ^2/d, with d=film thickness), coupling to a fluctuating gauge field in (1) can be ignored.

As the boson density n_0 is increased through some critical density n_c, a $T=0$ phase transition from a localized bose glass phase[7] to a superconducting phase with $<\psi> \neq 0$ is expected. It is convenient to introduce a parameter $\delta = n_0 - n_c$, which measures the distance to this $T=0$ transition. In experiments on real amorphous films one could take $\delta = d - d_c$, with d the film thickness, or $\delta = R_N - R_{N,c}$, where R_N is the film's "normal state" resistance per square taken at some convenient reference temperature above the bulk transition temperature. Provided this $T=0$ transition is continuous, it is characterized by a superconducting correlation length which diverges as $\xi \sim \delta^{-\nu}$, with an exponent satisfying the bound[7] $\nu \geq 2/d$. There is also a characteristic frequency Ω which vanishes at criticality as $\Omega \sim \xi^{-z}$ where z is the dynamical exponent.[7] Near the T=0 critical point all frequencies and the temperature scale[7] with Ω. Thus the Kosterlitz-Thouless[13] transition temperature T_c , at which the (2D) system becomes normal for $\delta > 0$, will scale as

$$T_c \sim \delta^{z\nu}, \tag{2}$$

for $\delta \to 0^+$.

In the superconducting phase the second sound (phonon) mode has a plasmon- like dispersion relation[9,10] $\omega \sim k^{(3-d)/2}$ due to the long-range Coulomb interaction. This mode can be described by an effective imaginary time action[7] which depends only on the phase ϕ of the order parameter $\psi = |\psi| \exp(i\phi)$,

$$S_\phi = (1/2)\int d^d k d\omega [(\rho_s \hbar/2m)k^2 + \hbar\omega^2 |k|^{d-1}/e_R^2] |\phi(k, \omega)|^2 . \qquad (3)$$

Here ρ_s is the fully renormalized superfluid density and e_R a "fully renormalized" charge, $e_R^2 \equiv \lim_{k \to 0} |k|^{d-1}/C_{nn}(k, \omega = 0)$, with $C_{nn}(k, \omega) \equiv \delta < n(k, \omega) > /\delta\mu(k, \omega)$ the density-density response function. Near the T=0 superconductor-insulator transition[11] ρ_s vanishes as $\rho_s \sim \xi^{-(d+z-2)}$. This essentially follows from power counting in the first term in (3), noting that both S_ϕ and $\phi(x)$ are dimensionless[7]. Likewise, the second term in (3) implies that the charge e_R should scale near the transition as $e_R^2 \sim \xi^{1-z}$. It can be argued[2] that at the superconductor to localized Bose-glass transition, e_R will have a finite value. This implies

$$z = 1, \qquad (4)$$

which should hold in all dimensions. Eqn. (4) is the generalization to charged systems of the relation $z = d$, which has been argued to hold[7] at the T=0 superfluid-insulator transition in disordered charge neutral boson systems, such as ^{4}He in porous media.

Scaling of the frequency-dependent conductivity near the superconductor -- insulator transition can be obtained from the relation[12]

$$\sigma(\omega) = (2e)^2 \rho_s(-i\omega)/(-im\omega) \ , \qquad (5)$$

where $\rho_s(\omega)$ is a generalized frequency dependent superfluid density defined in terms of a current-current correlation function. Since all frequencies should be scaled by the characteristic frequency Ω near the transition one can write the scaling relation

$$\rho_s(\omega, \xi) = \xi^{-d}(\xi/a)^{2-z}\tilde{\rho}_s(\omega/\Omega), \qquad (6)$$

where $\tilde{\rho}_s$ is an appropriate dimensionless scaling function and $\Omega = (\hbar/ma^2)(a/\xi)^z$, with a a short distance cutoff. For $x \equiv \omega/\Omega \to 0$ we must recover the result $\rho_s \sim \xi^{-(d+z-2)}$, so $\tilde{\rho}_s(x)$ must approach a constant. The form as $x \to \infty$ is set by the requirement that at criticality, where both ξ and $\Omega^{-1} \sim \xi^z$ are infinite, $\rho_s(\omega, \xi = \infty)$ is finite:

$\tilde{\rho}_S(x) = c_d x^{(d+z-2)/z}$, with c_d a dimensionless constant. Combining this with (5) implies that at criticality

$$\sigma(\omega, \xi = \infty) = c_d (e^2/\hbar) a^{2-d} (-i\hbar\omega/ma^2)^{(d-2)/z} . \tag{7}$$

Similarly, at the transition, the finite temperature d.c. conductivity should scale as $\sigma(T, \xi = \infty) \sim T^{(d-2)/z}$.

Eqn. (7) indicates that in 2D the $T = 0$ conductivity at criticality is a finite constant, $c_d e^2/\hbar$, in the d.c. limit. (Logarithmic corrections in ω are not expected in 2D, since $d = 1$ is the lower critical dimension for the transition[7].) Thus, at the superconductor-insulator transition the system exhibits true metallic conduction at T=0, something not possible in 2D normal fermion systems. The Cooper pairs, poised on the brink of becoming superconducting, are capable of ordinary diffusion.

Likewise, the resistance per square at the transition $R^* \equiv 1/\sigma(\omega = 0, \xi = \infty)$, when expressed in units of h/e^2, is a pure number. Since this number is given by the $k = \omega = 0$ limit of a (current-current) response function evaluated at the critical point[12], standard renormalization group (RG) arguments imply that, like critical exponents, it is universal: Its value will depend only on the universality class of the transition, and not on microscopic details.

EFFECTS OF MAGNETIC FIELD

In this section I consider the effects of an applied magnetic field on the low temperature properties of thin amorphous films. Consider a film which in zero field is superconducting below some Kosterlitz-Thouless transition temperature, T_c. In the superconducting state, an applied field will induce vortices in the Cooper pair wave function, all of the same sign. These vortices will interact with one another logarithmically[13] out to the two-dimensional screening length, $\lambda_{2D} = \lambda^2/d$, which in practice is usually macroscopic[14]. In the presence of disorder, which will tend to pin the vortices, the Abrikosov vortex-lattice phase will be destroyed[15]. At finite temperatures the film will then not be a true zero-resistance superconductor: Thermally activated vortex creep will lead to a dissipative (linear) resistance[16]. What happens when the system is cooled to $T = 0$? A classical description of vortex dynamics would predict that all vortex motion ceases in this limit, and the resistance should vanish. This $T = 0$ superconducting phase will exhibit Edwards-Anderson spin-glass type order[17] in the boson field ψ , and as a result is referred to as a "vortex-glass"[18]. But what about quantum fluctuations of the vortices?

Recently a theoretical framework has been developed for treating vortex quantum fluctuations in 2D bosonic systems[19] It is found that the vortices are in fact themselves bosonic objects, with a dynamics not altogether different from that of the Cooper pairs. At zero temperature these vortices can be localized by inhomogeneties, just as regular bosons or fermions can. Thus, the 2D superconducitng vortex-glass phase at $T=0$ should survive quantum fluctuations.

As the applied magnetic field is increased, though, a tantalizing possibility arises. The quantum gas of point vortices, increasing in density with applied field, should eventually "bose" condense, at some critical field H_c. (It turns out that this condensation is only possible at $T=0$.) What properties does a "superfluid" of vortices possess? Since vortex motion causes voltage fluctuations, this phase is, not surprisingly, an insulator with infinite resistance. This phase can alternatively be described in more conventional terms, as a localized fermionic (or Cooper pair) insulator. In any case, it should be possible, by simply varying the strength of the external field, to tune through this (T=0) superconductor-insulator transition.

Films with low (H=0) Kosterlitz-Thouless transition temperatures, will presumably have correspondingly low "critical fields", H_c , above which they are insulating (at $T=0$). The most natural scenario is that as the superconductivity is weakened, by making the film thinner, say, T_c and H_c will vanish together at the (multi-critical) $T=H=0$ superconductor-insulator transition. In this limit, the critical field will vanish as $H_c \sim \xi^{-2}$, where ξ is the superconducting correlation length of the $T=H=0$ transition, introduced before Eqn. 2. Combining this with (2) implies that near this transition

$$H_c \sim T_c^{2/z}. \tag{8}$$

This provides a direct way to measure the dynamical exponent, and should allow for a check on the theoretical prediction, $z=1$, for charged systems.

I now address the expected properties of the system near, and at, the $H \neq 0$ vortex-glass superconductor-to-insulator transition. Most of the scaling results discussed in Sec. 2 apply equally well to this transition. Once again, near the transition one expects a diverging superconducting correlation length, $\tilde{\xi} \sim (H-H_c)^{-\tilde{\nu}}$, and vanishing characteristic frequency, $\tilde{\Omega} \sim \tilde{\xi}^{-\tilde{z}}$, where the tilde's are used to differentiate these quantities from their $H=0$-transition analogs. One expects $\tilde{z}=1$, $\tilde{\nu} \geq 2/d$ (but presumably different from ν), and a universal resistivity at the transition (in 2D). Since the critical fixed point describing this transition is different than the $H=0$- transition fixed-point, different values for the universal resistivities would be expected.

Due to the applied field, a Hall resistivity ρ_{xy} is also expected at the $H \neq 0$ vortex-glass to insulator transition. Like ρ_{xx}, ρ_{xy} should have a universal value at the transition. In the scaling regime close to this transition (ie. $H \to H_c$ and $T \to 0$) both resistivities should satisfy scaling forms,

$$\rho_{x\alpha} = (h/4e^2)\tilde{R}_{x\alpha}[c(H-H_c)/T^{1/\tilde{z}\tilde{\nu}}], \tag{9}$$

with $\alpha = x,y$ and, c a non-universal constant. Here $\tilde{R}_{x\alpha}[X]$ are dimensionless scaling functions, which take constant values at $X \equiv c(H-H_c)/T^{1/\tilde{z}\tilde{\nu}} = 0$, and diverge or vanish exponentially in X (to some power) for large positive and negative X, respectively. Plotting $\tilde{R}_{xx}$ <u>versus</u> $\tilde{R}_{xy}$ would eliminate the (shared) non-universal constant c, giving a unique universal function, $R_{xx}(R_{xy})$. (A similar trick is used in plotting resistivities near the phase transition between plateaus in the integer quantum Hall effect[20].)

Can one estimate the universal resistivities, $\rho^*_{x\alpha} \equiv (h/4e^2)\tilde{R}_{x\alpha}[X=0]$, at this transition? As alluded to above, Cooper pairs and vortices play a dual role near the vortex-glass -to-insulator transition: In the superconducting phase the Cooper pairs have Bose condensed, whereas the vortices are condensed in the insulating phase. It turns out that the $T=0$ vortex-glass to localized Bose-glass transition, in a model system of <u>logarithmically</u> interacting bosons moving in a random potential, is, in fact, <u>self-dual</u>[21]. This can be used to show that the universal resistivities at the transition satisfy,

$$(\rho^*_{xx})^2 + (\rho^*_{xy})^2 = (h/4e^2)^2. \tag{10}$$

Since Cooper pairs <u>do not</u> interact logarithmically, but as 1/r , (10) will presumably not be identically satisfied in a real physical system. I suspect, though, that a more appropriate model, with 1/r interactions, might well give a value not substantially different than in (10) (although presumably not a simple rational times $h/4e^2$). In any event, (10) should serve as a useful guide for experimental investigations of this transition.

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