Physica B 169 (1991) 85–90
North-Holland

Type-II superconductors in a magnetic field: fluctuations, pinning and transport

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The invited talk was given by M.P.A. Fisher.

The phase diagram and electrical transport properties of strongly type-II superconductors are examined in the presence of thermal fluctuations and random vortex pinning. Pinning destroys the vortex lattice phase, probably replacing it with a vortex glass phase that has spin-glass like off-diagonal order and strictly zero resistivity in contrast to the conventional picture. The properties of this novel phase and the phase transition into it are summarized and experiments on $YBa_2Cu_3O_7$ and $Bi_2Sr_2CaCu_2O_8$ are discussed in light of the results. The latter is argued to exhibit quasi-two-dimensional behavior over a wide range of temperature and magnetic field.

1. Introduction

In type-II superconductors above their lower critical field, H_{c1}, their defining property – vanishing resistivity – has never been adequately understood. Indeed, whether the linear resistivity actually vanishes at low temperatures or is merely exponentially small is not known. This question is largely academic for conventional bulk superconductors in which fluctuation effects are very small.

A series of recent transport measurements on the high-T_c cuprate superconductors show that in these materials, fluctuation effects are much larger: in $Bi_2Sr_2CaCu_2O_8$ (BSCCO) the linear resistivity in moderate magnetic fields is readily measurable down to less than a third of the zero field transition temperature T_c [1]. This has been attributed to melting, by thermal fluctuations, of the Abrikosov vortex lattice into a vortex fluid phase [2]. In the absence of vortex pinning, however, such melting has little effect on simple transport properties: under an applied current, both a vortex lattice and a vortex fluid will move leading to *linear* "flux flow" resistivity.

In this paper we explore the effects of thermal fluctuations and vortex pinning on the phase diagram and transport properties of strongly type-II anisotropic superconductors, focussing on measurements on BSCCO and YBa_2CuO_7 (YBCO) – which exhibit rather different behavior. A much more detailed presentation will appear in [3]. We see that YBCO behaves as a three-dimensional system and appears to have a truly superconducting vortex glass phase replacing the vortex lattice [3]. BSCCO, on the other hand, is much more anisotropic: the penetration length anisotropy $\gamma \equiv \lambda_\perp/\lambda_z$ (where $\perp$ and z denote in-plane and c-axis directions) is of order $\frac{1}{50}$ for BSCCO but only $\frac{1}{5}$ for YBCO [4]. This results in two-dimensional behavior over a wide range of the H–T plane in BSCCO, roughly for fields greater than ~ 10 kG, where the system consists of weakly coupled two-dimensional layers.

We first consider critical behavior at the zero-field transition since, for small magnetic fields, this determines where vortex correlations become strong and in the absence of disorder the vortex lattice melting line. The main effect of

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random pinning is to destroy the long-range order of the vortex lattice, which may lead to a glass-like phase with off-diagonal long-range order – the vortex glass [5]. Some properties of this phase are outlined, as are those of the transition from vortex glass to vortex fluid. These together yield a semi-quantitative understanding of many of the experiments on YBCO. To understand BSCCO we explore the role of quasi-two-dimensionality.

2. Critical fluctuations

In small magnetic fields, thermal fluctuations become important for $T < T_c$ when

$$\lambda_{\perp}^2/\gamma\xi_{\perp} \simeq C_c \Lambda_T\,, \tag{1}$$

where $\Lambda_T \equiv \phi_0^2/16\pi^2 T \simeq 2\times 10^8$ Å, K/T is the thermal length set by the flux quantum, $\phi_0 = hc/2e$, and $\xi_{\perp}$ is the in-plane coherence length. The number C_c depends on the precise criterion one uses, as we will discuss.

For strongly type-II layered systems with $\kappa \equiv \lambda_{\perp}/\xi_{\perp} \gg 1$ and $\gamma \ll 1$, the fluctuations can become important relatively far from T_c. The criterion (1) will mark the crossover to an XY critical regime. In this regime $\xi \sim |\epsilon|^{-\nu}$ and $\lambda \sim \rho_s^{1/2} \sim |\epsilon|^{-\nu/2}$ with $\epsilon \equiv (T - T_c)/T_c$ and $\nu \simeq \frac{2}{3}$. Only when $\xi_{\perp} \sim \lambda_{\perp}$ will fluctuations of the magnetic field become important; this only occurs *very* close to T_c in the cuprate superconductors, since $\kappa \gg 1$ outside the XY critical region. In the XY critical regime, eq. (1) is obeyed as an *equality* with a universal constant $C_c \simeq 0.8$ [6]. For YBCO, this yields a crossover between Ginzburg–Landau and XY critical regimes for $T < T_c$ at a reduced temperature $|\epsilon_X| \lesssim 10^{-1}$. Note this estimate yields a much larger critical regime than the conventional Ginzburg criterion for the specific heat. The reason is that the fluctuation specific heat has a rather small amplitude, grows only as $\sim\epsilon^{-1/2}$ in the Ginzburg–Landau regime, and only diverges as $|\ln \epsilon|$ in the critical regime. This results in a fluctuation specific heat that does not grow to be of order the mean-field specific heat discontinuity until *deep* in the critical regime [3]. The fluctuation conductivity behaves similarly.

The non-linear current–voltage (I–V) characteristics in the Meissner phase should also show the effects of fluctuations. Thermally activated nucleation of vortex loops [7] leads to non-linear dissipation and an electric field

$$E \sim \exp[-(J_T/J)^{\mu}] \tag{2}$$

for small current density J, with $\mu = 1$ and $J_T^{\mu} \sim 1/T$ at low temperatures. Above a characteristic current density J_F, the dissipation rapidly increases due to non-activated phase slip processes. In the mean-field regime $J_T \gg J_F$, so that J_F appears as a sharp critical current. In the critical regime, however, $J_T \sim J_F$ so the I–V curves are smoother.

3. Flux lattice melting

Above the lower critical field H_{c1} (which $\sim(T_c - T)^{\nu}$ in the XY critical region) vortices penetrate the sample. In the absence of random pinning, a vortex lattice can then form, but thermal fluctuations of the vortices may melt this lattice [2]. At fields just above H_{c1} the inter-vortex interactions are weak, resulting in a narrow fluid phase near H_{c1}. Here we restrict consideration entirely to fields in the z-direction well in excess of H_{c1} so near-neighbor vortices interact logarithmically. The lattice melts at a field $B_M(T)$ which may be well below the mean field H_{c2} [2]. The latter becomes merely a crossover from a normal metal to a vortex fluid.

The melting field may be estimated using the Lindemann criterion [2]. The wave vector of the dominant vortex line fluctuation is of order the zone boundary π/a_v in the plane, (with vortex lattice constant $a_v = \sqrt{\phi_0/B}$) but a factor of γ smaller in the z-direction [3]. Thus we must include the discreteness of the layers if the layer spacing $d \gtrsim \gamma a_v$. This quasi-two-dimensional regime applies in YBCO only for enormous fields $\gtrsim 300$ kG, however BSCCO should become quasi-two-dimensional for $B \gtrsim 3$ kG.

3.1. *Three-dimensional regime*

At low temperatures the melting boundary will approach $H_{c2}(T)$, but as T increases it can drop to considerably lower fields. In the XY critical region, the melting field scales as [3] $B_M \approx C_M \phi_0^2 / 2\pi \xi_\perp^2 \sim (T_c - T)^{2\nu}$, with $C_M \lesssim 0.1$. The melting temperature, T_M, is likely to show a maximum very close to T_c – near the magnetic critical fluctuation regime – before bending back down to join the reentrant low-field melting line near H_{c1}.

3.2. *Two-dimensional regime*

At high fields in strongly anisotropic materials, both the direct and magnetic interlayer coupling between the vortices are ineffective. The system then acts as a stack of weakly coupled two-dimensional layers with fluctuating point vortices in each layer. In this regime, the melting temperature in a clean system will be roughly given by the two-dimensional Kosterlitz–Thouless melting temperature [8] which is *only weakly field dependent* for $H_{c2} \gg B \gg H_{c1}$: $T_M^{2D} \approx 1 - 2 \times 10^{-2} \phi_0^2 d / 16\pi^2 \lambda_\perp^2 \approx 15\text{–}30\,\mathrm{K}$ with the $\lambda_\perp \approx 1400\,\mathrm{\AA}$ as measured at low temperatures in YBCO [9].

A similar $\lambda_\perp$ for BSCCO (the experimental value is controversial), implies a low melting temperature over a wide range of fields ($B \gtrsim 3\,\mathrm{kG}$) which then increases for lower fields, eventually approaching T_c for B small. In this quasi-two-dimensional system, however, the stronger fluctuation effects may be enough to keep the maximum T_M considerably below T_c.

4. Effects of pinning

The motion of vortex lines and the concomitant electrical resistance can be impeded by both *macroscopic* defects such as widely spaced grain boundaries, or *microscopic* impurities (or possibly microtwins). If only the former were present, the linear resistivity would drop sharply when the vortex lattice forms; instead a more gradual drop is seen in experiments [1]. In addition, flux line imaging at low fields in BSCCO [10] and in twin free crystals of YBCO [11] shows evidence for pinning by small scale disorder. Although macroscopic defects may be important for obtaining high critical currents, we will concentrate on the effects of microscopic pinning which should be more universal.

Larkin and Ovchinikov [12] showed that vortex-lattice long-range order is destroyed by random pinning for distances larger than a length scale L_p which is long for weak pinning. What happens on longer length scales has never been understood: the usual discussions of "flux creep" [13] implicitly assume that the system is a strongly pinned *fluid* with independent creep of flux bundles up to a maximum characteristic size – yielding a *finite* linear conductivity. Alternatively, the system may be *ordered* on long length scales but with a specific non-crystalline frozen-in configuration of the vortex lines and spinglass-like off-diagonal long-range order [5]. Before studying the properties of this vortex glass phase, we discuss the effective strength of the pinning.

In the interesting regime of temperature, the thermal motion of the vortex lines averages the random vortex core pinning over some volume [14]. This is particularly effective at low fields. At fixed temperature the relative size, L_p/a_v, of the crystalline regions *decreases* with increasing field for $B \gtrsim 2H_{c1}$. (Very near H_{c1}, the effects of pinning can again become more important.) Thus the direct observation of fairly well ordered flux lattices [10, 11] at low fields $B \lesssim 200\ g$ in both YBCO and BSCCO does *not* imply that these correlations persist at the much higher fields where the transport measurments have generally been carried out [1, 9, 15]. Differences between samples probably also play an important role.

In an experiment on a YBCO film [15] (three-dimensional regime), a gradual drop in the resistivity was observed roughly coinciding with the expected position of the melting line in a clean system [2]. This gradual drop implies that the actual transition is *dominated* by pinning for that sample. In a single crystal YBCO [16] on the other hand, the resistivity was seen to drop much more sharply with T, consistent with weaker pinning, and a larger L_p/a_v near the clean system's melting temperature.

In the zero-field XY critical region, the pinning is marginally irrelevant, so that its effective strength should be almost independent of $T_c - T$, implying that *all characteristic fields* ($\geqslant H_{c1}$) will scale as $(T_c - T)^{2\nu}$ with $\nu \simeq \frac{2}{3}$, consistent with various recent experiments [17]. In this regime, the effective pinning strength can be quite large due to the small amplitude, $\tilde{v}_{CO}$ of the correlation volume [3]: $v_{CO} \sim \tilde{v}_{CO}\epsilon^{-3\nu}$.

5. Vortex glass phase

In the three-dimensional regime, the vortex lattice is likely to be replaced by a truly superconducting vortex glass phase with spin-glass-like-off-diagonal long-range order [3, 5, 18] below a temperature T_G. We briefly summarize some of the properties of this new phase.

5.1. Nonlinear response

With an applied current density, the pinning causes barriers $B(J) \sim J^{-\mu}$ for nucleation of vortex loop or vortex bundle excitations, resulting in an I–V curve of the form eq. (2) with $\mu \leqslant 1$ and a vanishing linear resistivity [3, 5]. At higher currents above some J_F, the vortex lines will become depinned and flow. For low temperatures, $J_F \ll J_T$ and J_F appears as a sharp critical current. However, as for the Meissner phase, $J_T \sim J_F$ near T_G, and there will not be a sharp critical current. In both the Meissner and vortex glass phases, a non-equilibrium current will decay as $J(t) \approx J_F[1 + (T/U)\ln(t/t_0)]^{-1/\mu}$, with $T/U \equiv (J_F/J_T)^{\mu}$ and $t_0 \sim 10^{-9}$ s a microscopic "attempt" time [19]. This form yields a $\partial J/\partial \ln t$ which is *non*-monotonic in temperature, in contrast to the Anderson–Kim theory and consistent with experiments on YBCO crystals [20]. Due to the strong non-linearities and the broad distribution of energy barriers, hysteresis and other non-equilibrium effects will be predominant in the vortex glass phase, consistent with experimental findings. (We have ignored finite penetration length effects and the resulting current inhomogeneities which can be important in experiments on the vortex glass. For $\lambda \gg a_v$, however, our results apply directly to small samples.)

5.2. Linear response

The response of the vortex glass phase to a linear AC applied current or magnetic field is rather subtle. Since any additional DC field will penetrate the sample, one might expect that the $\omega \to 0$ limit of the complex conductivity $\sigma(\omega)$ would qualitatively differ from that of the Meissner phase. Nevertheless, we find that [3]

$$\sigma(\omega) \sim \frac{\rho_s}{-i\omega + \epsilon} + O\left(\frac{1}{\omega|\ln \omega|^{\lambda}}\right), \tag{3}$$

so that, in fact, there exists a linear-response superfluid density ρ_s! Penetration of an applied AC field should be rather inhomogeneous due to rare "soft spots" where the field penetrates arbitrarily far, so that the penetrating field typically falls off exponentially even as $\omega \to 0$, although its *mean* decays as a power law [3]. Thus, depending on the measurement, one could conclude that either the AC penetration length $\lambda_G(\omega)$ goes to a constant as $\omega \to 0$ as implied by eq. (3), or that it diverges as a power of $|\ln \omega|$. A non-zero superfluid density nevertheless still exists because the large thermally active excitations ("soft spots") occupy only a small fraction of the volume. Thus even a low-frequency current can circumvent those regions resulting in only a finite reduction of ρ_s [3], in contrast to earlier expectations [18].

5.3. Vortex glass transition

Near the vortex-glass to vortex-fluid transition, at T_G, the critical behavior of the non-linear I–V curve has the scaling form [15]

$$E \sim \xi_G^{-z-1}\mathscr{E}_{\pm}(J\xi_G^2\phi_0/cT_G), \tag{4}$$

in three-dimensions where $\xi_G \sim |\epsilon_G|^{-\nu_G}$ is the vortex glass correlation length which can be indirectly inferred from the scale of the current density in eq. (4). Here ϵ_G is the reduced temperature $(T-T_G)/T_G$. For $T<T_G$ and $J \to 0$, the scaling function $\mathscr{E}_-$ reduces to eq. (2); for $T>T_G$ and small J, $\mathscr{E}_+ \sim J$ yielding $\sigma \sim \epsilon_G^{-(z-1)\nu}$; while for $T=T_G$, $E \sim J^{(z+1)/2}$, this power law behavior thus providing a good criterion for T_G. The expression eq. (4) has been successfully used [15] to analyze recent data from YBCO films yielding $z \simeq 5$ and $\nu_G \simeq 1.8$. Behavior consistent with eq. (4) was found [15] over the field range 20–40 kG. We note that the small length scales inferred from eq. (4) (~500 Å at 40 kG) suggest the pinning dominates at length scales $\gtrsim a_v$, in the sample of [15].

To strengthen the case for a true phase transition into a vortex glass phase, it would be very helpful to also measure the AC linear conductivity on the same samples. This should scale as

$$\sigma(\omega, t) \sim \xi_G^{z-1} S_\pm(\omega \tau_G) , \tag{5}$$

where the characteristic time scale $\tau_G \sim \xi_G^z$. At low frequencies, for $T<T_G$, this yields $\sigma \sim \mathrm{i}\rho_s/\omega$ with [17] $\rho_s \sim \xi_G^{-1}$; for $T>T_G$, $\sigma(\omega) \to$ real constant; while for $T=T_G$, $\sigma(\omega) \sim \mathrm{e}^{\mathrm{i}\phi_\sigma} \omega^{-(z-1)/z}$ with [21] $\phi_\sigma \approx \pi(z-1)/2z \simeq 70°$, assuming $z \simeq 5$.

5.4. Two-dimensional regime

We now briefly turn to the strongly anisotropic case of BSCCO. At very low fields the magnetic interlayer coupling causes a three-dimensional vortex lattice similar to YBCO, as found in imaging experiments [10]. For fields of a few kG and above, the interlayer coupling becomes ineffective, resulting in quasi-two-dimensional behavior. In this two-dimensional regime, there will be a finite in-plane vortex glass correlation length $\xi_G \sim 1/T^{\nu_2}$, and only weak interlayer correlations. Concommitantly, free energy barriers, B, for the motion of point vortices over distances of order ξ_G will gradually grow as T decreases, but these barriers cannot grow faster than $B \sim \ln \xi_G \sim \ln(1/T)$. This gives rise to a characteristic relaxational time $\tau_G \sim \exp(B(T)/T)$, which grows at most slightly faster than Arrhenius. The conductivity will diverge in the same manner. Both of these are consistent with linear AC magnetic screening measurements [10] in the range $15\,\mathrm{K} < T < 80\,\mathrm{K}$ and $10\,\mathrm{kG} < H < 120\,\mathrm{kG}$. At low temperatures, probably below T_M^{2D}, the in-plane correlation length will become long enough that the interlayer coupling become important, presumably driving the system to a three-dimensional vortex glass phase. Careful non-linear I–V data taken in conjunction with the linear AC transport on BSCCO is needed.

Finally, we note that the best systems to study some of these issues are probably artificially layered materials in which many more parameters can be controlled. However, it does appear that the high-T_c materials have already given substantial clues to the answer of the question: Are superconductors in a magnetic field *really* superconducting?

Acknowledgement

We would like to thank numerous experimental colleagues for useful discussions. D.S. Fisher is partially supported by the NSF, DMR 8719523 and the A.P. Sloan Foundation.

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