Universality in the current decay and flux creep of Y-Ba-Cu-O high-temperature superconductors

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Recent data on flux creep in a variety of Y-Ba-Cu-O superconductors show a temperature-independent plateau in the magnetization decay  \( S = -d\ln M(t)/d\ln(t) \), with values clustered in the range  \( S \approx 0.020-0.035 \). This apparent universality in  \( S \), which appears at odds with conventional flux-creep theories, can be explained naturally if one assumes the existence of a truly superconducting ordered phase at low temperatures that has a strongly nonlinear (exponential)  \( I-V \) characteristic.

Early flux-creep data \(^1\text{-}^3\) on YBa\(_2\)Cu\(_3\)O\(_7\) (henceforth denoted Y-Ba-Cu-O) superconductors already showed an initially unexpected increase in the effective pinning barrier with temperature in an intermediate temperature range. We point out here that recent data, \(^4\text{-}^12\) taken in such a way as to avoid the additional complications of incomplete flux penetration \(^3\) and other experimental artifacts, \(^5\text{-}^7\text{,}^9\) show this increase with a remarkably universal slope. A first suggestion of this universality came from experiments on various kinds of crystals by Xue et al. \(^9\), but we show here that the result is considerably more general. Furthermore, we argue that this universality can be understood, provided one assumes that at low temperatures in the mixed state the voltage vanishes exponentially with current density \( J \), i.e., \( V \sim \exp[-(J/J_\mu)^\mu] \) (where \( \mu \) is an exponent and \( J_\mu \) a temperature-dependent parameter with units of current density). An exponential  \( I-V \) characteristic, which implies a truly superconducting state with strictly zero linear resistance, is the form predicted in the vortex-glass phase. \(^13\text{-}^{14}\)

Figure 1 summarizes Y-Ba-Cu-O data from many different laboratories on grain aligned powders, \(^6\text{-}^7\) flux-grown and melt-processed crystals \(^8\text{-}^{10}\) measured with field both parallel and perpendicular to the \( c \) axis, irradiated crystals \(^10\), and \( e \) beam and magnetron sputtered films, \(^4\text{-}^{11}\) which together span several orders of magnitude in critical current density, even at the same temperature and field. The data represent the normalized time-logarithmic slope of the magnetization \( S = -d\ln M(t)/d\ln(t) \). We emphasize that this is not the same as the extracted effective barrier height, where in certain cases authors \(^2\text{-}^4\text{,}^9\) have included a \( \ln(t/t_0) \) correction. In most cases \( S \) is reasonably approximated by \( -(1/M_f)dM/d\ln(t) \), where \( M_f \) is the initial magnetization measured at the beginning of the relaxation experiment.

The results generally show a plateau or at least a flat maximum over a large range of temperature, usually with a dropoff at lower temperature and either an enhancement or a dropoff at temperatures approaching \( T_c \). We concentrate our discussion here on the plateau region, which, with only a few exceptions, \(^9\) lies at approximately \( S \approx 0.020-0.035 \), a remarkable universality considering the range of \( J_\mu \)'s, applied field strengths, field orientations, and material properties represented by these samples.

The magnetization in the mixed state is proportional to the current density flowing in the sample \( J(t) \) which, due to Joule heating, decays in time as \( \partial J/\partial t \sim -V(J) \). A measurement of the magnetization decay is thus an indirect measurement of the system's  \( I-V \) characteristics. The standard Anderson-Kim flux-creep model \(^15\text{-}^{17}\) assumes an  \( I-V \) curve of the form \( V \sim \exp(-U_0/T) \times \sinh[(U_0/T)(J/J_0)] \), where \( U_0 \) is the energy barrier and \( J_0 \) is the critical current density without flux creep. This assumption leads to

\[
J(t) = J_0 \exp[-(U_0/T)\ln(t/t_0)], \quad t \ll t_{cr}, \tag{1a}
\]

\[
J(t) = J_0 \exp(-ct/t_{cr}), \quad t > t_{cr}, \tag{1b}
\]

with a crossover time \( t_{cr} = t_0 \exp(U_0/T) \), \( t_0 \) a microscopic hopping "attempt" time, and \( c \) a sample-dependent geometric constant, of the order of one for typical sample dimensions.

The magnetization decay is measured in a window of times, from some initial time \( t_i \) to a final time \( t_f \). Provided that \( t_0 \) is much larger than both of these times, logarithmic decay is predicted \(^2\) with \( S \approx -d\ln J(t)/d\ln(t) \equiv T/U_0 \), an inverse measure of the barrier height. In low-\( T_c \) superconductors this is often the case. In the high-\( T_c \) superconductors the barrier height \( U_0 \), extracted at low temperatures from \((1a), \) is often sufficiently small (e.g., \( 20 \text{ meV} \)) that \( t_{cr} \) already drops below the window of observation times at moderate temperatures (assuming a constant or decreasing barrier \( U_0 \) with increasing temperature). In this case \((1b)\) would predict that since \( t \gg t_{cr}, \) the magnetization should have completely decayed to its equilibrium value by the time the observation begins. The observed persistence of logarithmic time decay at higher temperatures thus indicates that the simple  \( I-V \) characteristic with a single barrier height assumed by Anderson-Kim is not appropriate in these materials, as pointed out earlier. \(^18\)

One approach to explaining the logarithmic time depen-
Universality in the Current Decay and Flux Creep

Figure 1. Normalized time-logarithmic magnetic relaxation rate $S = -d \ln M(t)/d \ln (t)$ vs temperature $T$ for a variety of YBa$_2$Cu$_3$O$_7$ samples. Solid lines emphasize plateaus and show temperature range averaged to give values shown at right. (a) Data of Xu et al. (Ref. 6) on grain-aligned powders with $H$ parallel to 1 T (squares) and 2 T (circles). (b) Data of Campbell et al. (Ref. 7) on grain-aligned powders with $H$ parallel to 0 T (crosses) and 2 T (squares). (c) Data of Keller et al. (Ref. 8) on melt-processed crystals at remanence after application of saturating fields along the $c$ (squares) and $ab$ (circles) axis. (d) Data of Civale et al. (Ref. 10) on unirradiated (circles) and 3 MeV-proton-irradiated (squares) flux-grown crystals for $H$ parallel to 1 T. (e) Data of Sun et al. (Ref. 11) (crosses) on magnetron-sputtered thin films on MgO substrates with $H$ parallel to 0.9 T, and of Stollman et al. (Ref. 4) (pluses) on $e$-beam evaporated and annealed thin films on SrTiO$_3$ substrates at 0 T.

Dependence at higher $T$ and the plateau in $S(T)$ has been to invoke distributions of various kinds, either in the energy barriers or in the critical current. Except for one case with a rather special inverse power-law distribution of energy barriers [which also gives the interpolation formula of Eq. (3) below], the predicted relaxation will depend on details of the distribution and so is difficult to reconcile with the apparent universality of the data.

Another approach to account for the temperature dependence of $S$ has involved modeling the physics in terms of a single physical barrier, but with a more complicated shape than in the Anderson-Kim theory. If one ignores the motion of flux against the Lorentz force, replacing the sinh by an exponential, the Anderson-Kim $I$-$V$ characteristics can be written in the suggestive form $V(J) = \exp[-U(J)/T]$, with an effective current-dependent barrier $U(J) = U_0(1 - J/J_c)$. A number of papers have proposed other forms for $U(J)$, all in the context of a single well with a more complicated barrier shape. These fall into two classes: Those with $U(J)$ which approach a finite limit as $J \to 0$, as in Anderson and Kim, and those in which $U(J)$ diverges in the zero-current limit. In the former class, at times long compared to the crossover time $t_{ct} = t_0 \exp[U(J = 0)/T]$, the magnetization will decay exponentially with time as in (1b). At shorter times a $\ln(t)$ time decay is expected, with a form which will depend in detail on the shape of the barrier leading to $U(J)$. It is hard for us to see how this approach can then account for the apparent universality of the experimental results for $S$, especially since the pinning barrier is likely to be collective and should depend on both the strength and density of the microscopic pinning centers. Models in the latter class, based on a single well for a finite flux bundle hopping a finite distance, but with a barrier $U(J)$ which diverges as $J \to 0$ seem rather unphysical to us. Nevertheles, the formal results which follow from the assumption of a single well with a $U(J)$ varying as an inverse power of $J$, will closely parallel the approach we advocate below.

Several recent theoretical papers go beyond the simple model of a single well, and consider directly the statistical mechanics of many interacting vortex lines in an environment of dense pinning centers. In one approach, a truly superconducting phase is predicted, named a vortex glass, in which highly nonlinear $I$-$V$ characteristics are expected, with an exponential dependence at low currents, $V(J) \propto \exp[-(J/J_c)^\mu]$. Here $\mu$ is an exponent characteristic of the low-temperature phase, which must satisfy $\mu \geq 1$ for $J \to 0$. Feigel'man et al. and Natterman using an elastic medium approach to collective flux creep, also predict an exponential $I$-$V$ characteristic, but under some conditions there can be several regimes in current density, each with an exponential form but with different exponents. We point out below that such an exponential $I$-$V$ characteristic leads directly to a plateau in $S(T)$ with a value which should be essentially sample independent.

First note that with an $I$-$V$ curve of the form $V(J) \propto \exp[-(J/J_c)^\mu]$, the current decay, which follows by integrating the equation $\partial J(t)/\partial t = -V(J)$, varies at long times as $J(t) \approx J_c/(\ln(t/t_0))^{\frac{1}{\mu}}$. Provided that one is in this “long-time” limit (see below), this immediately implies in turn that

$$S = -d \ln J/d \ln (t) = [\mu \ln(t/t_0)]^{-1}.$$

This key result, though implicit in earlier work, now displays explicitly the universality of the magnetic relaxation. This is because it depends only on $\mu$, which is a universal exponent in this interpretation, and only logarithmically on the observation time and the attempt time, which presumably do not vary enormously from system to system or from experiment to experiment. With $\mu = 1$, an attempt time of order $10^{-10}$ s, and an observation time of $t = 1000$ s, one obtains $S = 0.033$, in remarkable agreement with experiment. If this explanation of the plateau in $S(T)$ is correct, the observed values of $S$ in Fig. 1 would suggest an exponent $\mu$ close to the theoretical
upper bound of one.

The physical picture underlying this result involves vortex lines at low temperature in the mixed state creeping in an environment of dense randomly located pinning centers. In the vortex-glass phase and in the absence of an applied current, "vortex-loop" excitations of size $L$ (which can be thought of roughly as a way to represent the displacement of a length-$L$ segment of given vortex line) cost an energy which increases with some positive power of $L$. When a current is applied, loop excitations larger than some minimum size, $L_{\text{min}}$, varying as an inverse power of $J$, are no longer metastable due to the energy gained from the Lorentz force. Formation of these minimum size loops requires passing over barriers $U(L_{\text{min}})$ which typically scale as a power in $L_{\text{min}}$ and hence with an inverse power of $J$. A voltage results from nucleation (and then growth) of these minimum size loops, with magnitude $V(J) \sim \exp(-U(J)/T) \sim \exp(-U_f/J)^{1/\mu}$, i.e., exponentially in current.

In a decay measurement, the current will relax via nonactivated processes to the fluctuationless critical current density $J_c$ on a microscopic time scale, $t_0$. An interpolation formula has been proposed which connects the short-time regime ($t \gtrsim t_0$), where the decay is dominated by vortex-loop excitations with a fixed (current independent) size, comparable to the vortex-glass coherence length, and a fixed barrier height $U_0$, to the asymptotically long-time limit where the size of the dominant vortex-loop excitation ($L_{\text{min}}$) varies with current and the current decays with an inverse power of $\ln(t)$:

$$J(t) = J_c/[1 + (\mu kT/U_0)\ln(t/t_0)]^{1/\mu}.$$  

At short times, compared to the crossover time $t_{cr} = t_0 \times \exp(U_0/kT)$, the denominator in Eq. (3) can be expanded and reduces to the Anderson-Kim form (1a). In the long-time limit, though, (3) crosses over to an inverse power-law decay in $\ln(t)$. In practice, since the window of observation times is naturally restricted, this "long-time" limit can only be accessed by increasing the temperature until $t_{cr}$ is shorter than $t_{obs}$, i.e., $T > T^* = U_0/\ln(t_{obs}/t_0)$. Below this temperature, Eq. (3) predicts a linear temperature dependence of $S$, crossing over into a temperature-independent plateau for $T > T^*$. This accounts naturally for the decrease in $S(T)$ observed at temperatures below the plateau in the data (see Fig. 1).

It should be emphasized that although Eq. (3) predicts a nonlinear function of $\ln(t)$ (power law at long times), the dependence can in practice appear linear in a restricted window of observation time, since $(kT/U_0)\ln(t/t_f)$ [with $t_f$ (final) initial observation times] can be small compared to one, even though $(kT/U_0)\ln(t_f/t_0)$ is larger than one. Some experiments, though, in fact detect appreciable curvature when $M(t)$ is plotted versus $\ln(t)$.

Indeed, Xu et al. (and also Svedlindh et al., working on PbMoO$_4$Sb$_2$) find that when their data shows curvature, $M(t)$ is approximately linear in $\ln(t)$, consistent with Eq. (3), and does not fit the dependence predicted in (1b).

Finally we comment that factor-of-two variations in the relaxation rate $S$ in the plateau region are observed in certain cases, for instance with applied field in the work of Campbell et al. and Xue et al. It remains to be established to what extent these are due to differences in experiment, to progressive violation of the limits discussed above or to possible limits on the range of validity of the vortex glass model.

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