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# Boson localization and superconductor–insulator transitions in films

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ABSTRACT: Summary of oral presentation

In the past decade substantial progress has been made in understanding Anderson localization in electron systems and the metal-insulator transition in dirty interacting Fermion systems (Lee and Ramakrishnan 1985). A central conclusion which has emerged, is that in two dimensions (2D) even weak disorder localizes all states: A true 2D metallic phase with non-zero conductivity is not possible at $T = 0$. In this extended abstract, I summarize some recent results (Fisher et al. 1990; Fisher 1990; Fisher et al. 1989a) on related phenomena in bosonic systems, namely, superconducting to insulating transitions in disordered systems. Such $T = 0$ transitions are direct bosonic analogs to the metal-insulator transition. Attention is focussed on the 2D case, since a number of recent experiments (Haviland et al. 1989; Haviland 1989; S J Lee et al. 1990; Tyc et al. 1990; Hebard et al. 1984) have probed this transition by systematically varying the thickness or disorder strength of amorphous or granular films. In this way the subtle interplay between localization and superconductivity can be examined.

Our central conclusion concerns the resistance right at the $T = 0$ superconductor-insulator transition in thin 2D films. A simple scaling argument shows (Fisher et al. 1990) that this resistance is finite and non-zero, ie. "metallic". Metallic conduction in 2D *at* $T = 0$ appears to be possible in this case, since the charge carrier is a boson (Cooper pair), and not a fermion as in normal electron systems. Moreover, the value of the resistance (per square) right at the transition is predicted (Fisher et al. 1990) to be universal, depending only on the universality class of the transition, and being insensitive to all microscopic details.

In addition to tuning through the 2D superconductor-insulator transition by varying film thickness or disorder strength, it has recently been argued (Fisher 1990) that a new and fundamentally different superconductor-insulator transition should be accessible by simply tuning the strength of a magnetic field applied to a thin amorphous film. Experimental results on this new field-tuned transition have recently been obtained in amorphous $\alpha InO_x$ films (Hebard et al. 1990).

The basis of a more detailed theoretical description of both the disorder-tuned and field-tuned superconductor-insulator transitions is the hypothesis that near the transition a description in terms of a model of charge-2e bosons moving in a random potential is adequate. Direct evidence for this comes from the 1D case (Giamarchi et al. 1988), where it is found that a model of electrons with a BCS attraction moving in a random potential, exhibits a superconductor-insulator transition in the same universality class as the superfluid- insulator transition in a model of repulsively interacting bosons, representing the Cooper pairs. More generally, in the insulating phase it is possible to define "localization lengths" for both single electrons and for Cooper pairs (Cha et al. in preparation): The pair length, $\xi$, which is essentially a superconducting coherence length, determines the spatial decay of $< \psi(x)\psi^{\dagger}(0) > \sim \exp(-x/\xi)$, where the Cooper pair operator $\psi$ is defined in terms of electron operators as $\psi(x) = c_{\uparrow}(x)c_{\downarrow}(x)$. A single electron localization length, $\xi_1$, follows from the decay of the ensemble average of the square of the imaginary part of the (retarded) single electron Greens function: $|G_R(x,0; E=0)|^2 \sim \exp(-x/\xi_1)$. On approach to a (continuous) superconducting transition from the insulating phase, the pair length diverges, but the single electron localization length is expected to remain finite (non-critical). Indeed, $\xi_1$ should also be finite in the superconducting phase, since electrons can only propogate coherently as pairs. (This scenario is quite closely analogous to nematic transitions in classical xy models, wherein $\exp(2i\phi)$ orders but the xy order parameter $\exp(i\phi)$ remains zero and the associated length non-critical (D H Lee et al. 1985)). Close to the transition where $\xi$ greatly exceeds $\xi_1$, the long lengthscale physics should be describable in terms of a composite Cooper-pair boson. On the insulating side, a description in terms of localized bosons is appropriate, since pairs are less localized than single electrons.

Theoretical support for the universality of the resistance right at the 2D transition comes from the Mott-Hubbard insulator to superconductor transition in a clean lattice model of interacting bosons at a commensurate density (Fisher et al. 1990; Fisher et al. 1989a; Wen et al. 1989; see also Granato et al. 1990). In 2D, this $T=0$ transition is in the same universality class as the 3D classical xy model. Although this is probably not the appropriate universality class for real disordered amorphous or granular films, or for 2d Josephson junction arrays, it's virtue is relative simplicity. Generalizing the xy (or O(2)) model to O(N), enables the resistance at the 2D transition to be calculated systematically in a large$_*$ N-expansion. At $N=\infty$ the value is (Fisher et al. 1990; Wen et al. 1989) $R^* = (8/\pi)R_Q$, where $R_Q = h/4e^2 \simeq 6.5$kOhm is the (pair) quantum unit of resistance, and the first 1/N correction is positive (Cha et al. in preparation). Recent Monte Carlo calculations (Cha et al. in preparation) give a value between 3 and 4 $R_Q$, consistent with the 1/N expansion and showing clearly that the resistance is *not* equal to $R_Q$, at least in this clean case.

At the magnetic field-tuned transition in disordered films, a simple argument can be given to estimate the value of the resistance (Fisher 1990). The physics near the transition can be either described in terms of Cooper pairs or in terms of vortices, which are induced in the pair wave function by the applied field. Normally vortices are treated as classical particles, but at a $T=0$ transition this is clearly inappropriate. Recent work shows (D H Lee et al. 1989; Fisher et al. 1989b; van Wees 1990), in fact, that vortices are themselves bosons. The magnetic field-tuned transition from superconductor to insulator arises from a competition between condensation of Cooper pairs and a condensation of vortices. In the superconducting phase, the pairs have condensed and the vortices, pinned by the impurities, are immobile (or localized), whereas the *vortices* have condensed and the pairs are localized in the insulating phase. Moreover, for a model of logarithmically interacting pairs (bosons) the field-tuned transition is self – dual[3]. This can be used to show that the sum of longitudinal and Hall resistivies at the transition satisfy[3]

$$(\rho^{*}_{xx})^2 + (\rho^{*}_{xy})^2 = R_Q^2.$$

This equation says in effect that for every Cooper pair crossing the system there is precisely *one* vortex crossing the system.

Since Cooper pairs do not interact logarithmically, but as 1/r, the above expression should only be taken as a rough estimate for the expected values for the resistivies in real amorphous films. Recent experiments on amorphous $\alpha InO_x$ films (Hebard et al. 1990), which probe down to 15mK and are apparently in the T=0 critical regime showing nice scaling behavior, find $\rho_{xx}$ in the range 4.5-5.5 kOhm and a negligibly small Hall resistance, values close to the above theoretical estimate.

It should be emphasized that in experiments, the resistance at the superconductor-insulator transition is only expected to be close to the universal value when temperatures are low enough that the system is in the T=0 critical regime. How cold this is in practice, will probably depend in detail on the properties of the films under study, and will be difficult to predict reliably beforehand. A necessary condition experimentally is that the resistance versus temperature curves near the transition scale appropriately, as for example the data at the field-tuned transition in Reference (9) appear to. Inspection of the data on the disorder-tuned (zero field) 2D superconductor-insulator transition (Haviland et al. 1989; S J Lee et al. 1990; Tyc et al. 1990; Hebard et al. 1984) suggests that none of the current experiments have gone cold enough to enter the critical region, and check for universal resistances. There is clearly much exciting work, both experimentally and theoretically, that remains to be done in the relatively virgin field of superconductor-insulator transitions.

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