Hall Effect near the Vortex-Glass Transition in High-Temperature Superconductors

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We discuss the Hall effect in the mixed state of type-II superconductors with an emphasis on the regime near the vortex-glass transition. This transition is argued to have a "hidden" particle-hole symmetry, and a new exponent is introduced to characterize the irrelevance of the particle-hole asymmetry. This leads to a natural explanation of the scaling of the Hall and longitudinal resistivities observed in $YBa_2Cu_3O_7$. At the transition, the nonlinear Hall voltage is predicted to vary with a universal power of the current, and the linear ac Hall conductivity with a universal power of the frequency.

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Since the discovery of the high-temperature superconductors there has been tremendous interest in the properties of the mixed state of these fascinating materials. It was already clear from early experiments [1] that, in contrast to conventional superconductors [2], magnetic field-induced vortices are mobile over a substantial range of temperatures and fields. In this vortex-liquid regime the mobile vortices cause resistive losses and destroy superconductivity. A central question which has emerged is whether or not this vortex liquid freezes into a frozen superconducting vortex-glass phase upon cooling [3]. There is now mounting experimental evidence in films [4,5], crystals [6], and ceramics [7], from both nonlinear and frequency-dependent [5] transport measurements, of a phase transition into a superconducting phase. Detailed scaling predictions [3,8] for transport properties close to a putative vortex-glass transition have been verified experimentally. Numerical simulations [9] give supporting theoretical evidence for the existence of the vortex-glass phase.

Recent attention has focused on the Hall effect in the vortex-liquid regime of the oxide superconductors which has been found to exhibit a number of surprising features. Specifically, the Hall effect appears to undergo a sign change upon cooling into the vortex-liquid regime from the normal state [10]. Moreover, upon further cooling, Luo *et al.* [11] have observed a striking power-law dependence between the Hall and the longitudinal resistivities in $YBa_2Cu_3O_7$ (YBCO) films. They find $\rho_{xy} \propto \rho_{xx}^a$ with $a \simeq 1.7$.

In this paper we study theoretically the Hall effect near the putative vortex-glass transition, where the Hall voltage should exhibit universal scaling behavior. Insight is gained by analysis of the zero-field superconducting transition. Explicit renormalization-group calculations show that terms in the equations of motion which break particle-hole symmetry are irrelevant at this transition. More specifically, the effective particle-hole asymmetry approaches zero with a universal power of the correlation length. We hypothesize that the same scenario holds at the vortex-glass transition, except with a different value for the exponent. From this hypothesis follows a number of explicit and testable predictions for the Hall effect near the vortex-glass transition: (1) The Hall resistivity is expected to vanish as a universal power of the longitudinal resistivity, consistent with Luo *et al.* [11]. (2) The nonlinear Hall electric field E_y should exhibit universal scaling, and right at the transition should vanish with a universal power of the current J_x. (3) At the transition the linear ac Hall conductivity should vary with a universal power of frequency. In each of the three cases above, the universal exponent depends on a known way on the dynamical exponent z (which can be measured independently from the I-V curves) and the new exponent which describes the irrelevance of particle-hole asymmetry. Thus, even in the absence of a calculation for this new exponent, measuring one of the above three exponents leads to a quantitative prediction for the other two. Unfortunately our scaling analysis cannot predict the *sign* of the Hall effect in the vortex flow regime. Indeed, the sign of the irrelevant operator which breaks particle-hole symmetry and leads to a Hall effect is *nonuniversal* and can in general be material specific. For this reason, we believe an understanding of the sign change in the Hall effect in the oxide superconductors might only be possible once a microscopic pairing theory is in hand.

To describe the static properties of the mixed state we adopt the usual Ginzburg-Landau description with Hamiltonian

$$\mathcal{H} = \int d^d x \left[\frac{\hbar^2}{2m} \left| \left(\nabla - \frac{ie}{\hbar} \mathbf{A} \right) \psi \right|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{8\pi}|\nabla \times \mathbf{A} - \mathbf{H}|^2 \right], \quad (1)$$

where ψ is the pair wave function (or order parameter), $\mathbf{B} = \nabla \times \mathbf{A}$ is the induction field, and $\mathbf{H}$ is an applied magnetic field. Random pinning can be included in the coefficient of the quadratic term, via $a \to a + a_1(\mathbf{x})$, with

$a_1(\mathbf{x})$ an appropriate quenched random potential. In a neutral superfluid, such as ^{4}He, the critical dynamics is appropriately described by model-F dynamics, in which the order parameter ψ is coupled to a conserved density [12]. Such a coupling is necessary in order to capture the hydrodynamic (second or fourth) sound mode. In a superconductor this sound mode is pushed to the plasma frequency due to the Coulomb interaction and need not be included in the low-frequency dynamics. We thus assume purely relaxational dynamics (model A) for ψ [13]:

$$\Gamma^{-1}(1+iv)\partial_t\psi(x,t)=-\delta\mathcal{H}/\delta\psi^*(x,t)+\zeta(x,t)\,, \qquad (2)$$

where the noise has Gaussian white-noise correlations

$$\langle\zeta^*(x,t)\zeta(x',t')\rangle=2k_BT\Gamma^{-1}\delta^d(x,x')\delta(t-t')\,. \qquad (3)$$

The appropriate equation of motion for the vector potential is $\sigma_n\partial_t\mathbf{A}=-\delta\mathcal{H}/\delta\mathbf{A}+\boldsymbol{\eta}$, where σ_n is a "normal-state" conductivity and $\boldsymbol{\eta}$ is a current noise satisfying

$$\langle\eta_\alpha(x,t)\eta_\beta(x',t')\rangle=2k_BT\sigma_n\delta_{\alpha\beta}\delta^d(x-x')\delta(t-t')\,.$$

An essential feature of (2) is the imaginary part of the damping coefficient v, on the left-hand side. In the absence of this term the equations exhibit a "particle-hole" symmetry which can be used to show that the Hall conductivity is identically zero. Specifically, with $v=0$, the equations of motion are invariant under the transformation $\psi\to\psi^*$, $H\to -H$, and $B\to -B$. Under this transformation the current $\mathbf{J}\propto\mathrm{Im}\psi^*\nabla\psi-(e\mathbf{A}/\hbar)|\psi|^2$ changes sign; when combined with a Kubo formula for the conductivity, in which the current enters quadratically, it follows that $\sigma_{xy}(H)=\sigma_{xy}(-H)$. On the other hand, microscopic time-reversal invariance requires the conductivity to satisfy an Onsager relation $\sigma_{xy}(H)=\sigma_{yx}(-H)$. If we further assume that the sample is invariant under rotations (by 90°) around the z axis with $\mathbf{H}=H\hat{\mathbf{z}}$, or for the random case is at least statistically so, then we have $\sigma_{xy}(H)=-\sigma_{yx}(H)$. Upon combining these three conditions we deduce, $\sigma_{xy}(H)=-\sigma_{xy}(H)$, i.e., a zero Hall conductivity.

When the damping in (2) is complex ($v\neq 0$) the Hall effect will in general be nonzero. Indeed, for small v we expect $\sigma_{xy}\propto v$, so that the sign of v determines the sign of the Hall effect. This can be seen in the low-field limit ($H\ll H_{c2}$), where a description in terms of single vortices is appropriate. It is possible to derive from (2) a single-vortex equation of motion which takes the form [14]

$$\mathbf{J}=-\frac{m}{\hbar^2\Gamma}\frac{H_{c2}}{4\pi\kappa^2}[\alpha_1(\hat{\mathbf{z}}\times\mathbf{V}_L)+\alpha_2 v\mathbf{V}_L]\,, \qquad (4)$$

with κ the Ginzburg-Landau parameter, $\mathbf{J}$ the transport current, $\mathbf{V}_L$ the vortex line velocity, and α_1 and α_2 constants which depend on the structure of the vortex. This, combined with $\mathbf{V}_L=\mathbf{E}\times\hat{\mathbf{z}}/B$, gives the longitudinal and Hall conductivity with $\sigma_{xy}\propto v$ from the second term on the right-hand side of (4).

What determines the sign of v? Some insight comes from BCS theory. In the dirty limit it is possible to derive a Ginzburg-Landau-type equation of motion, as in (2), starting from a microscopic BCS theory. A small imaginary part to the damping is found which is proportional to an energy derivative of the electron density of states at the Fermi surface [15], $\partial_E N(E_F)$. Thus v is sensitive to the microscopic particle-hole asymmetry of the electrons in the normal state. In this particular model the Hall effect in the mixed state will have the same sign as in the normal state. In general, though, this can be complicated by more subtle band-structure effects. An understanding of the sign change in the Hall effect of the oxide superconductors will probably require a detailed microscopic pairing theory which incorporates particle-hole asymmetries in both the normal and mixed states. Although the *sign* of the Hall effect will depend on band-structure effects, we argue below that the temperature dependence should nevertheless exhibit some striking universal features near the vortex-glass transition.

The origin of this universality becomes clear upon analyzing the zero-field superconducting transition as described by the Ginzburg-Landau theory above. For simplicity consider the extreme type-II limit, ignoring all fluctuations in the gauge field (model-A dynamics). The dynamics with $v=0$ has been studied extensively [12] using an $\varepsilon=4-d$ expansion, with d the spatial dimension. How does an imaginary part of the damping modify this critical behavior? In a mean-field description, valid for $d>4$, v will be unrenormalized and will be nonzero (and nonuniversal) at the transition. Below $d=4$ it is possible to study the role of v in an ε expansion. The renormalization-group (RG) flows for v can in fact be taken directly from de Dominicis and Peliti [16], who studied complex damping in model F. One deduces [16] that v is an irrelevant perturbation at the transition, flowing to zero as

$$\partial v/\partial l=-\lambda_v v+O(v^2)\,, \qquad (5)$$

with an eigenvalue $\lambda_v=\frac{2}{25}\ln(\frac{4}{3})\varepsilon^2+O(\varepsilon^2)$.

This flow equation implies that right at the zero-field transition the long length- and time-scale physics is described by a particle-hole symmetric theory ($v=0$). Right at the transition the dynamics of the system develops a symmetry which is not present in the underlying equations of motion. The phase transition exhibits a higher (dynamical) symmetry than either of the two phases. A similar result was found at the ($T=0$) superconductor-insulator transition, where a 1D calculation demonstrated that the transition is particle-hole symmetric [17]. Based on these results, *we hypothesize that the vortex-glass transition in the mixed state likewise possesses a particle-hole symmetry.* Moreover, we assume that the asymmetry v has an associated positive eigenvalue λ_v, just as in (5), with a value which is determined by the RG flows near the vortex-glass transition. As we show below, these assumptions are entirely consistent with the recent data by Luo *et al.* [11]. In principle, the irrelevancy of particle-hole asymmetry at the vortex-glass transition could be checked via a systematic $\varepsilon = 6-d$ expansion [18] about the mean-field theory [19]. However, to find a nonvanishing contribution might require an $O(\varepsilon^2)$ calculation, as in the zero-field case (5), which would be extremely formidable. Below we explore the consequences of the above assumptions.

Scaling expressions for the conductivities near the vortex-glass transition can be obtained from the fact that only a single divergent length scale is expected, a vortex-glass correlation length, which diverges with an exponent ν, $\xi \sim |T-T_g|^{-\nu}$. This length is a measure of the distance over which the phase of the superconducting wave function ψ is correlated. There is also a diverging time scale, varying as ξ^z, with z the dynamical exponent. Near the vortex-glass transition, the *I-V* curves, both nonlinear and frequency dependent, should scale with the appropriate powers of the critical length and time scales [3,8]. Here we generalize this scaling approach to include the Hall effect.

Consider first the dc linear conductivity. Near T_g the longitudinal conductivity diverges [3,19] as $\sigma_{xx} \sim \xi^{z+2-d}$. For the Hall conductivity we expect the same dependence except that the asymmetry parameter v must enter:

$$\sigma_{xy} \approx \xi^{z+2-d} \Sigma_{xy}(v\xi^{-\lambda_v}) . \tag{6}$$

Here $\Sigma_{xy}(X)$ is a scaling function whose argument $X = v\xi^{-\lambda_v}$, which is a measure of the effective particle-hole asymmetry, scales to zero with an exponent λ_v as the transition is approached. Now, we have argued that when $v=0$ the Hall conductivity vanishes, and that the sign is determined by the sign of v. Thus it is natural to expect that $\Sigma_{xy}(X)$ is linear for small argument, which implies $\sigma_{xy} \sim v\xi^{-\lambda_v}\sigma_{xx}$. (Recall that a linearity in v followed from the single-vortex equation motion in the low-field limit.) The Hall angle thus scales to zero upon approaching the vortex-glass transition. The Hall resistivity in turn drops to zero more rapidly than the longitudinal resistance:

$$\rho_{xy} \sim v\xi^{-\lambda_v}\rho_{xx} \sim \rho_{xx}^a , \tag{7}$$

with $a = 1+\lambda_v/(z+2-d)$. The latter expression predicts a Hall resistivity which vanishes as a universal power of the resistance upon approaching the vortex-glass transition. This form is particularly convenient experimentally since the glass transition temperature, which is normally taken as a fitting parameter, is not needed. In measurements on YBaCuO films, Luo *et al.* [11] found such a power-law behavior with $a \simeq 1.7$. With a value of $z \simeq 5$ [4-7], this gives $\lambda_v \simeq 3$ for the asymmetry exponent. In an alternate scenario, in which v scales to a constant at a new fixed point, one expects a constant and (probably) universal Hall angle with $a=1$ in (7). This appears to be inconsistent with Luo *et al.*'s data.

The arguments above have assumed the existence of a vortex-glass transition. However, a vortex-glass transition is not expected in a strictly two-dimensional (2D) system [20] (except at $T=0$). Therefore Bi-Sr-Ca-Cu-O (BSCCO), which approximates a 2D system due to its large anisotropy [3], should behave in a manner markedly different from YBCO. At high temperatures in the 2D regime [3], one expects that BSCCO will have an activated Hall resistivity, as seen by Artemenko, Gorlova, and Latyshev [21]. Upon cooling vortex-glass correlations will grow, eventually coupling the planes and leading to a fully 3D vortex-glass transition [3]. Measurements of the longitudinal resistivity in BSCCO using a SQUID picovoltmeter [22] appear to support this scenario; it would be interesting to have companion results on the Hall effect. One expects the same (universal) value for the exponent λ_v in BSCCO as in YBCO.

Consider next the nonlinear Hall voltage. Imagine applying a current along the x direction, with current density J_x and measuring the Hall voltage, or electric field E_y, with $J_y = 0$. From (7) and the fact that the current density should be scaled by [3] ξ^{d-1}, we expect that near T_g, E_y should satisfy a scaling form:

$$E_y \approx v\xi^{d-2-z-\lambda_v} J_x \tilde{E}_\pm (\phi_0 J_x \xi^{d-1}/T) . \tag{8}$$

Here $\tilde{E}_\pm(X)$ is a scaling function above (+) or below (−) the transition and $\phi_0 = h/2e$. For small arguments $\tilde{E}_+(X)$ should approach a constant, corresponding to an Ohmic Hall effect in the vortex-liquid phase, but should vanish, probably exponentially, in the vortex-glass phase: $\tilde{E}_-(X) \sim \exp(-X^{-1/\mu})$. The behavior of $\tilde{E}_\pm(X)$ at large argument follows from the requirement of a finite electric field E_y even as ξ diverges: $\tilde{E}_\pm(X) \sim X^{(z+2-d+\lambda_v)/(d-1)}$ for $X \to \infty$. This implies a power-law Hall *I-V* curve right at the vortex-glass transition:

$$E_y(T=T_g) \sim J_x^{(z+1+\lambda_v)/(d-1)} . \tag{9}$$

Note that the power in (9) is larger than the corresponding power for E_x, which is [3] $(z+1)/(d-1)$. A measurement of z from the longitudinal *I-V* curves and the particle-hole asymmetry exponent λ_v from the linear resistivities, Eq. (7), then leads to a direct prediction for the Hall voltage power law in (9).

Finally, we discuss the frequency-dependent Hall resistivity. Since the frequency should be scaled by the critical time scale [3] ξ^z, the following scaling form is expected to be valid for low frequencies near T_g:

$$\sigma_{xy}(T,\omega) \approx \xi^{z+2-d-\lambda_v} \tilde{\Sigma}_\pm(\omega\xi^z) . \tag{10}$$

The scaling function above the transition, $\tilde{\Sigma}_+(X)$, should approach a constant at small arguments, being Ohmic in the dc limit, and in the critical regime must vary for large argument as $X^{(z+2-d+\lambda_v)/z}$, in order to give a finite limit as ξ diverges for $\omega \neq 0$. This implies a universal power law at the transition,

$$\sigma_{xy}(T_g,\omega) \sim (-i\omega)^{(d+\lambda_v-z-2)/z} . \tag{11}$$

Note that causality requires the real and imaginary parts to be related as shown [8]. A critical power law at T_g has recently been observed in the frequency-dependent longitudinal conductivity [5]. Measuring a frequency-dependent Hall resistance should prove even more challenging.

To summarize, we have developed a scaling theory for the Hall conductivity near the vortex-glass transition in type-II superconductors. One consequence is that the Hall and longitudinal resistivities should scale with a universal power, in agreement with the recent results of Luo *et al.* [11]. We also predict that the nonlinear Hall field should scale with a universal power of the current at the vortex-glass transition, and that the linear ac Hall conductivity should scale with a universal power of the frequency. The observed sign change of the Hall effect [10] was argued to be a nonuniversal feature which will likely require detailed microscopic calculations for its resolution.

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