Nonequilibrium Noise and Fractional Charge in the Quantum Hall Effect

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We study nonequilibrium edge state transport in the fractional quantum Hall regime at filling $g = 1/m$. Tunneling of Laughlin quasiparticles between edges is shown to generate both current and voltage shot noise with a universal ratio of noise amplitudes given by $(ge^2/h)^2$. In the weak backscattering limit we predict a current shot noise satisfying Poisson statistics with charge ge, which should enable a direct measurement of the fractional charge of the quasiparticle. On resonance the quasiparticles tunnel in pairs and the effective charge entering the current shot noise becomes $2ge$.

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The suggestion by Laughlin [1] that the quasiparticles in the fractional quantum Hall effect carry fractional charge was one of the most striking predictions in condensed matter physics. Nevertheless, despite the overall success of Laughlin's theory a direct and convincing experimental measurement of fractional charge has remained elusive [2]. The primary reason for this is that conventional dc transport measurements are not sensitive to the charge of the carrier. In contrast, nonequilibrium noise measurements, such as shot noise, are dependent on the absolute magnitude of the carrier's charge. In this Letter we develop a theory for nonequilibrium edge state transport in the fractional quantum Hall regime, focusing on fluctuations in the currents and voltages. In addition to a prediction that the current and voltage shot noise amplitudes should have a universal ratio, we find that in the weak backscattering limit the magnitude of the current shot noise depends directly on the fractional charge of the quasiparticles. This should enable a direct experimental measurement of its fractional charge.

In the past several years the Landauer approach to quantum transport [3] has been generalized to include a description of current fluctuations [4–6]. The central result, valid for a single channel noninteracting electron gas incident upon a barrier with transmission probability, T, is that the magnitude of the current shot noise is given by $(e^2/h)T(1-T)\mu$, where μ is the drop in chemical potential across the barrier. The classical theory of shot noise, which describes the uncorrelated transmission of discrete electrons, is thus valid only in the limit $T \ll 1$. In the opposite limit, $1-T \ll 1$, however, this result may be understood as classical "hole" shot noise, which is due to the uncorrelated *backscattering* of discrete electrons.

In an incompressible quantum Hall state transport is dominated by one-dimensional edge states, which suggests that this might be an ideal arena for experimental shot noise measurements [6]. Indeed, a suppression of shot noise below the classical prediction has been observed in the integer quantum Hall effect [7]. In the fractional quantum Hall effect, however, the edge excitations are not Fermi liquids, and Landauer transport theory may not be straightforwardly applied. As emphasized by Wen [8], the edge excitations of a $\nu = 1/m$ fractional quantum Hall state form a single channel chiral Luttinger liquid, characterized by a dimensionless conductance $g = 1/m$. We develop below a nonequilibrium transport theory for interacting Luttinger liquids, and apply it to the fractional quantum Hall effect to calculate both current and voltage fluctuations.

The specific geometry we have in mind, depicted schematically in Fig. 1, consists of a quantum Hall bar with a gated constriction separating the source from the drain. In the presence of a finite source drain voltage, V_{sd}, we assume that the top and bottom edges which are incident from the drain and source are in thermal equilibrium at chemical potentials separated by eV_{sd}. This can be checked in an unconstricted channel by verifying that the excess current noise vanishes at low frequencies. When the gate is constricted, though, there will be nonequilibrium current and voltage fluctuations. Specifically, backscattering at the constriction will give rise both to a "glitch" in the source drain current, I, and a glitch in the dissipative voltage, V, across the weak link mea-

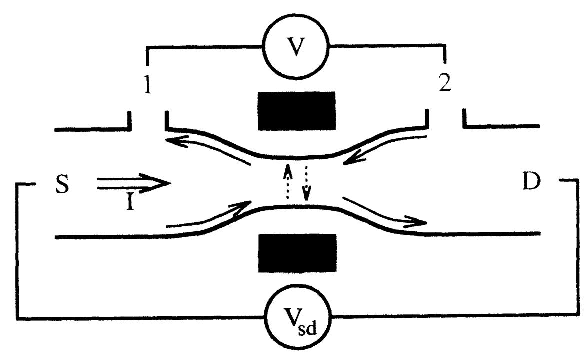

FIG. 1. Schematic of quantum Hall bar with gated constriction (G) separating source from drain. Of interest are nonequilibrium fluctuations in the current, I, and voltage, V, in the presence of an applied source-to-drain voltage, V_{sd}.

sured between leads 1 and 2 in the figure. The central point is that since the backscattered particles are Laughlin quasiparticles consisting of a fraction g of an electron bound to one vortex [9], the ratio of the current to voltage glitches is precisely ge^2/h. This physics manifests itself in a simple relationship between the low frequency nonequilibrium noise characteristics of the current and voltage, which we derive below. Moreover, in the weak backscattering limit, the absolute magnitude of the current fluctuations is predicted to take a classical shot noise form with charge ge, enabling a direct measurement of the quasiparticle's fractional charge.

We characterize the nonequilibrium current and voltage fluctuations by correlation and response functions,

$$C_A(\omega) = \frac{1}{2}\int dt\, e^{i\omega t}\langle\{A(t), A(0)\}\rangle, \quad (1)$$

$$R_A(\omega) = \frac{1}{2}\int dt\, e^{i\omega t}\langle[A(t), A(0)]\rangle, \quad (2)$$

where the operator A can be either I or V. In equilibrium at temperature Θ these are related by the fluctuation dissipation theorem, $C(\omega) = \coth(\hbar\omega/2k_B\Theta)R(\omega)$, but no such general relation holds in the presence of a nonequilibrium source-to-drain voltage.

Nevertheless, specializing to incompressible quantum Hall states with $g = 1/m$, so that each edge state is a single branch chiral Luttinger liquid, we derive below a general relation between the nonequilibrium current and voltage fluctuations:

$$C_I(\omega) - \coth\left(\frac{\hbar\omega}{2k_B\Theta}\right)R_I(\omega) = \left(\frac{ge^2}{h}\right)^2\left[C_V(\omega) - \coth\left(\frac{\hbar\omega}{2k_B\Theta}\right)R_V(\omega)\right]. \quad (3)$$

In equilibrium both sides of this equation vanish but in the presence of a nonzero $V_{\rm sd}$ it has nontrivial content. To see this consider the zero frequency limit, at small but nonzero temperatures. The response functions can then be directly related to the differential current-voltage characteristics: $\lim_{\omega\to 0}\coth(\hbar\omega/2k_B\Theta)R_I(\omega) = (2T/\hbar)dI/dV_{\rm sd}$ and $\lim_{\omega\to 0}(ge^2/h) \times \coth(\hbar\omega/2k_B\Theta)R_V(\omega) = (2T/\hbar)dV/dV_{\rm sd}$. Since both $dI/dV_{\rm sd}$ and $dV/dV_{\rm sd}$ are finite for nonzero $V_{\rm sd}$, in the zero temperature limit Eq. (3) reduces to

$$C_I(\omega\to 0) = \left(\frac{ge^2}{h}\right)^2 C_V(\omega\to 0), \quad (4)$$

which relates directly the amplitude of the current and voltage shot noise.

The striking result (4) is a direct consequence of the binding of charge to vortices [9]. A Laughlin quasiparticle consists of one vortex bound to a gth of an electron. Tunneling of quasiparticles between edges should thus lead to a current shot noise which is a factor g^2 smaller than the voltage shot noise, when expressed in units of e and h/e. The binding of charge to vortices can be demonstrated even more convincingly by considering the fluctuations in the quantity $I - (ge^2/h)V$. We find that

$$C_{I-gV}(\omega) = \frac{ge^2}{h}\hbar\omega\coth\left(\frac{\hbar\omega}{2k_B\Theta}\right). \quad (5)$$

Thus in the dc limit as the temperature is taken to zero all fluctuations in $I - gV$ vanish completely. The nonequilibrium current and voltage fluctuations are locked together. This is in contrast to what is expected in a normal resistor in which thermal dephasing effects can give rise to nontrivial fluctuations in $I - gV$, for example, of a $1/f$ form, which may be interpreted as temporal fluctuations in the value of the conductance itself. In the quantum Hall effect, however, the conductance is a topological invariant and the flucutations are given explicitly by (5), reflecting the binding of charge and flux.

Before describing how we derive the general expressions (3) and (5), we consider the limit of very weak and very strong backscattering where one expects to recover classical shot noise. We find that in the limit of strong backscattering where the constriction is almost completely pinched off, $V_{\rm sd} - \langle V\rangle \ll V_{\rm sd}$, the current fluctuations satisfy a classical (Poissonian) shot noise form,

$$C_I(\omega\to 0) = e\coth\left(\frac{eV_{\rm sd}}{2k_B\Theta}\right)\langle I\rangle + \cdots, \quad (6)$$

which is due to uncorrelated tunneling of electrons through the constriction from source to drain. This form is identical to that for noninteracting electrons (i.e., a Fermi liquid) [6], and shows that in this limit successive electrons tunnel so infrequently that the electron-electron interactions are unimportant. The coth factor in (6) comes from the uncorrelated "upstream" tunneling of electrons, which costs an energy $eV_{\rm sd}$.

In the opposite limit of weak backscattering where the deviations from perfectly quantized source-to-drain conductance are small, $\langle V\rangle \ll V_{\rm sd}$, the dominant backscattering channel is the tunneling of single Laughlin quasiparticles. This is valid at low temperatures for $g = 1/3$ because the single quasiparticle backscattering operator is the only one whose amplitude does not vanish in the low temperature limit [10,11]. Provided the temperature is high enough that the single quasiparticle backscattering has not grown too large, the tunneling of Laughlin quasiparticles between edges will still be infrequent and should be described by a similar Poissonian distribution. Indeed, in this limit we find *voltage* fluctuations which satisfy a classical shot noise form:

$$C_V(\omega\to 0) = \frac{h}{e}\coth\left(\frac{geV_{\rm sd}}{2k_B\Theta}\right)\langle V\rangle + \cdots. \quad (7)$$

The coefficient h/e can be understood by noting that when a Laughlin quasiparticle, which consists of a vortex (bound to gth of an electron), tunnels between edges it causes a voltage glitch with integrated strength $\int V\,dt = h/e$.

Since each Laughlin quasiparticle carries fractional charge, in the weak backscattering limit we expect the current noise should satisfy a classical shot noise expression with charge ge. This can be obtained upon combining (4) and (7) with an expression which relates the average current to average voltage,

$$\langle I\rangle = \frac{ge^2}{h}(V_{\rm sd} - \langle V\rangle), \tag{8}$$

yielding for the current shot noise at zero temperature

$$C_I(\omega\to 0) = (ge)(I_{\max} - \langle I\rangle) + \cdots , \tag{9}$$

where the maximum current is $I_{\max} = (ge^2/h)V_{\rm sd}$. Equation (9) is valid in the weak backscattering limit, $I_{\max} - \langle I\rangle \ll I_{\max}$, and reduces to the known result for noninteracting electrons [6] when $g = 1$. Physically it arises from discrete uncorrelated *backscattering* of charge ge quasiparticles. Observation of current shot noise in this regime would thus yield a direct measurement of fractional charge.

How is the shot noise modified near a resonance? In the above analysis we have assumed that the dominant tunneling channel is single Laughlin quasiparticles. However, in a configuration where the incident current passes through two constrictions in series, it is possible (e.g., by varying a gate voltage) to achieve a destructive interference for backscattering, so that the amplitude for tunneling single quasiparticles vanishes [10,11]. At the resulting resonance the dominant tunneling process will be pairs of quasiparticles. We then find that on resonance Eq. (7) becomes

$$C_V^{\rm res}(\omega\to 0) = 2\frac{h}{e}\coth\left(\frac{2geV_{\rm sd}}{2k_B\Theta}\right)\langle V\rangle + \cdots . \tag{10}$$

Thus in this weak backscattering regime, upon tuning through a resonance, we expect the voltage shot noise amplitude, when scaled by the voltage, to double in magnitude. Of course the voltage drop itself will decrease on resonance. Likewise, on resonance Eq. (9) becomes

$$C_I^{\rm res}(\omega\to 0) = (2ge)(I_{\max} - \langle I\rangle) + \cdots , \tag{11}$$

a classical shot noise form with charge $2ge$.

We now briefly describe the formalism which enables us to establish the above results. We first note that for the $g = 1/m$ quantum Hall state the right and left moving edge states (at the bottom and top, respectively, of the Hall bar in Fig. 1) form a Luttinger liquid with dimensionless conductance g [8]. The constriction, which causes interedge backscattering, plays the role of a defect in the 1D Luttinger liquid [10,11]. "Bosonizing" the Luttinger liquid yields an effective Lagrangian, $L = (1/2g)(\partial_\mu\theta)^2$, which can be used to describe the nonequilibrium transport through the constriction. Upon integrating out those fluctuations away from the constriction, at $x = 0$, we arrive at an effective Euclidean Lagrangian in terms of $\theta = \theta(x = 0)$ [12]:

$$S_E = \frac{1}{g}\sum_{\omega_n} |\,\omega_n\,||\,\theta(\omega_n)\,|^2 - v\int d\tau\,\cos[2\sqrt{\pi}\theta(\tau)]\,. \tag{12}$$

Here v is the amplitude for tunneling of a Laughlin quasiparticle between edges, and the electron current through the constriction is $I = \dot\theta/\sqrt{\pi}$. Since we are interested in nonequilibrium fluctuations, however, we need a real time formulation. Fortunately, Eq. (12) is identical to a Caldeira-Leggett model of a damped Josephson junction [13], so we can adopt directly the Feynman-Vernon or Keldysh approach used so successfully in this context [14]. We introduce a generating functional expressed as an integral over forward and backwards paths, $\theta_\pm(t)$, with t running between $\pm\infty$:

$$Z = \int D\theta_+\,D\theta_-\,e^{S(\theta_\pm)}\,. \tag{13}$$

The real-time effective action can then be expressed in terms of new fields $\theta_\pm = \theta \pm (1/2)\tilde\theta$, as $S = S_0 + S_1 + S_2$ with

$$S_0 = -\frac{1}{g}\int d\omega\,\omega\coth\left(\frac{\omega}{2T}\right)|\,\tilde\theta(\omega)\,|^2 + \frac{2i}{g}\int dt\,\tilde\theta(t)\dot\theta(t), \tag{14}$$

$$S_1 = -iv\int dt(\cos 2\sqrt{\pi}\theta_+ - \cos 2\sqrt{\pi}\theta_-), \tag{15}$$

$$S_2 = \frac{i}{\sqrt{\pi}}\int dt[a(t)\dot{\tilde\theta}(t) + \eta(t)\dot\theta(t)], \tag{16}$$

where $\eta(t)$ is a source field and the source drain voltage is given by $V_{\rm sd} = \dot a$.

Functional differentiation with respect to the source field $\eta(t)$ generates an expectation value of the current operator, $\langle I\rangle = -i\delta\,\ln(Z)/\delta\eta$, whereas correlation and response functions are given as

$$C_I(\omega) = -\frac{1}{Z}\frac{\delta^2 Z}{\delta\eta(\omega)\delta\eta(-\omega)}, \tag{17}$$

$$R_I(\omega) = -\frac{1}{Z}\left(\frac{\delta^2 Z}{\delta a(\omega)\delta\eta(-\omega)} - \frac{\delta^2 Z}{\delta\eta(\omega)\delta a(-\omega)}\right). \tag{18}$$

In order to identify the voltage operator it is useful to perform a shift in θ and $\tilde\theta$ to eliminate terms linear in the source fields $a(t)$ and $\eta(t)$ which appear in (16). Then S_1 and S_2 become

$$S_1 = -iv\sum_\pm(\pm)\int dt\,\cos[2\sqrt{\pi}\theta_\pm + ga(t) - gP^\pm\eta], \tag{19}$$

$$S_2 = -\frac{g}{4\pi}\int d\omega\,\omega\coth\left(\frac{\hbar\omega}{2k_B\Theta}\right)|\eta(\omega)|^2 + \frac{ig}{2\pi}\int dt\,\eta\dot{a}, \tag{20}$$

where we have defined

$$P^{\pm}\eta = \int d\omega\, e^{-i\omega t}\left[\pm\frac{1}{2} + \coth\left(\frac{\hbar\omega}{2k_B\Theta}\right)\right]\eta(\omega). \tag{21}$$

Then differentiating Z with respect to η to obtain $\langle I\rangle$ and comparing with (8) implies that as expected the voltage operator is proportional to the quasiparticle tunneling term:

$$\hat{V} = (h/e)v\sin(2\sqrt{\pi}\theta + ga). \tag{22}$$

It is now straightforward to establish the central results described above. The general expression (3) relating current and voltage noise can be obtained by formally evaluating the current correlation and response functions using (17) and (18) and the action in (19) and (20), and then noting the form of the voltage operator given in (22). Equation (7) can be established by evaluating both the voltage noise and $\langle V\rangle$ perturbatively in v to second order, and comparing. On resonance the amplitude for tunneling a single quasiparticle vanishes [11], $v = 0$, and we must include a pair tunneling term $v_2\cos(4\sqrt{\pi}\theta)$ in the action (12). Then following the steps just outlined leads directly to the expression for the voltage shot noise on resonance, given in Eq. (10). Finally, in order to obtain the current shot noise expression in the strong backscattering limit, Eq. (6), it is convenient to employ a representation "dual" to that in Eqs. (12)–(16) [12,14]. Following the steps which lead to Eq. (7) in this dual representation immediately gives Eq. (6).

In summary, we have described a general framework for treating nonequilibrium transport phenomena in Luttinger liquids and applied it to extract current and voltage fluctuations in the fractional quantum Hall effect. This approach, which can be readily extended to incorporate electron spin, might also be useful in modeling nonequilibrium transport in narrow quantum wires.

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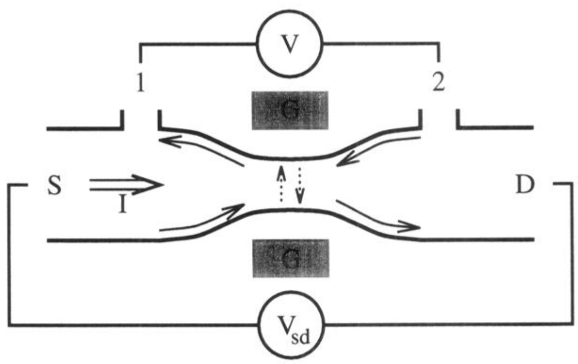

FIG. 1. Schematic of quantum Hall bar with gated constriction (G) separating source from drain. Of interest are nonequilibrium fluctuations in the current, I, and voltage, V, in the presence of an applied source-to-drain voltage, V_{sd}.