Cooper-pair tunneling into a quantum Hall fluid

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Transport through a tunnel junction connecting a superconductor to a spin-aligned quantum Hall fluid at filling ν is studied theoretically. The dominant transport channel at low temperatures is the tunneling of Cooper pairs into edge states of the quantum Hall fluid. This process, which is greatly suppressed at low energies due to both Coulomb and Pauli exclusion effects, leads to a tunneling conductance which vanishes with temperature as $T^{4/\nu-2}$, for ν^{-1} an odd integer. For integer fillings with $\nu > 1$ the "Pauli blockade" is circumvented and a nonvanishing conductance is predicted.

There are some striking parallels between the phenomena of the fractional quantum Hall effect[1] and superconductivity. In both cases, one has a system that exhibits a dissipationless flow of electrical current. In each case, the physics was elucidated initially by a many-body wave function, involving a new charge carrier. More recently a Ginzburg-Landau approach to the quantum Hall effect has identified an underlying condensed boson,[2] responsible for the dissipationless flow, highlighting the analogy between the two phenomena. Very recently, attention has shifted to the study of weak links or point contacts in the quantum Hall effect,[3] the loose analog of a Josephson junction.

Since 1992, several experimental groups[4-6] have succeeded in making low resistance contacts between superconducting leads and a two-dimensional electron gas (2DEG) in a semiconductor heterostructure. Evidence has been found for Andreev reflection,[7] in which Cooper pairs are converted into unpaired electrons and transported into the 2DEG. These experiments raise the exciting possibility of making and studying a tunnel junction between a superconductor and a dissipationless quantum Hall fluid. Motivated by this, I consider in this paper the simplest possible tunneling model for a such a tunnel junction. I focus exclusively on low temperature dc electrical transport through the junction, which is dominated by pair tunneling.

An immediate obstacle arises when considering a tunnel junction between a superconductor and a Hall fluid. The large magnetic field needed to put the 2DEG into the quantum Hall fluid will tend to suppress the superconductivity. One is thus restricted to type-II superconductors, with upper critical fields H_{c2} in the 5–10 T range or above. In this case, magnetic vortices will penetrate the superconductor, but provided they are strongly pinned (i.e., in the vortex-glass phase[8]) they will not contribute to the low temperature transport of interest.

In addition to penetrating the superconductor, the large magnetic field will, of course, tend to spin polarize the 2DEG. Here I consider only the case of a fully spin-polarized electron gas. Then a spin singlet Cooper pair passing out of the superconductor into the 2DEG must not only break apart into two unpaired electrons, as in a conventional Andreev process,[7] but must undergo a spin-flip process to align the spin of both electrons with the magnetic field. The suppression of the tunnel current due to this spin-flip process can be studied independently of the quantum Hall effect by placing the external magnetic field in the plane of the 2DEG. Of interest here, though, is when the field is perpendicular, and the 2DEG is in a dissipationless quantum Hall fluid. Then the transport current in the 2DEG will be confined to edge states.[9,10] Consequently, at low temperatures the dominant transport process through the junction should be tunneling of a Cooper pair into the edge state.

In this paper, I introduce and analyze a simple model for Cooper pairs tunneling into a quantum Hall edge state. I focus primarily on odd integer ν^{-1}, where the edge state is believed to be a single-channel chiral Luttinger liquid.[10] As recently emphasized,[11] the tunneling of electrons into a Luttinger liquid is greatly suppressed at low energies, and leads to a true Coulomb blockade with vanishing tunneling conductance at $T = 0$. Here, though, *two* electrons (a Cooper pair) are trying to tunnel *simultaneously* into the Luttinger liquid edge state. The central point of this paper, established below, is that the Pauli exclusion principle operating between the *pair* of electrons leads to an additional suppression of the tunneling conductance, over and above the Coulomb blockade. Together, the Coulomb blockade and this "Pauli blockade" effect lead to a conductance that is predicted to vanish with a large power of temperature: $G \sim T^{4/\nu-2}$, for ν^{-1} an odd integer. Strikingly, the predicted conductance vanishes even for $\nu = 1$, when the edge state is a Fermi liquid.[9,10] In this case, the vanishing conductance is due entirely to a Pauli exclusion between the pair of electrons. For integer filling with $\nu > 1$, where the edge consisits of more than one Fermi liquid channel, the Pauli blockade is circumvented, and a nonvanishing low temperature tunneling conductance is predicted.

It is perhaps surprising that for the fractional Hall state at filling $\nu = 1/3$ the tunneling conductance is predicted to vanish with such an enormous power of temperature, $G \sim T^{10}$. Despite the fact that both the superconductor and quantum Hall fluid have zero resisitance, the junction between the two is predicted to be an extremely good *insulator*. Ultimately, this is because very different bosons are condensing in the two systems, a Cooper pair

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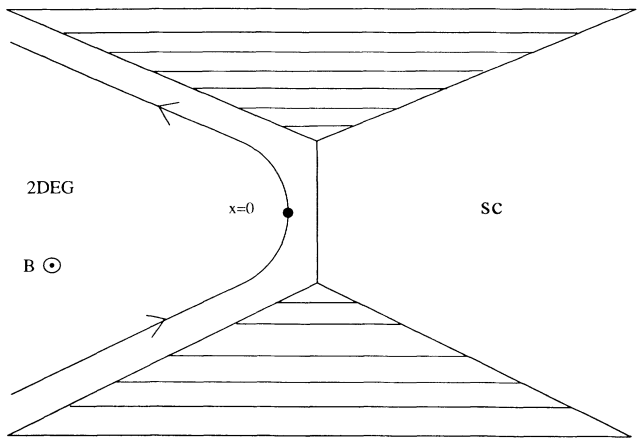

FIG. 1. Schematic of a tunnel junction from superconductors (SC) to a quantum Hall fluid. Cooper pairs tunnel into the edge at $x = 0$.

in one and a vortex-electron composite[2] in the other.

Consider then a tunnel junction, or point contact, between a singlet superconductor and an incompressible quantum Hall fluid at filling ν, as depicted schematically in Fig. 1. The total Hamiltonian is expressed as a sum of three pieces: $H = H_{\rm QHE} + H_{\rm SC} + H_{\rm PERT}$, where $H_{\rm QHE}$ is the Hamiltonian for the spin-polarized quantum Hall fluid, $H_{\rm SC}$ is the Hamiltonian for the superconductor and $H_{\rm PERT}$ a perturbation that couples them together. Being interested only in temperatures well below the superconducting gap, it is adequate to model the superconductor in terms of a bosonic pair field $\hat{c}$, which exhibits long-ranged (vortex-glass[8]) order and has a nonzero condensate, $\langle \hat{c} \rangle = \Delta$. Moreover, for temperature scales well below the gap in the quantum Hall fluid, $H_{\rm QHE}$ can be taken as an edge Hamiltonian (see below). For the perturbation term at the tunnel junction we take initially

$$H_{\rm PERT} = \int_{x,x'} t_1(x,x')[\psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x')\hat{c}(x=0) + \text{H.c.}] + \int_x t_2(x)[\psi_\uparrow^\dagger(x)\psi_\downarrow(x) + \text{H.c.}], \quad (1)$$

where x is a $1d$ spatial coordinate that runs along the edge of the quantum Hall fluid. The first term hops a Cooper pair from the superconductor through the point contact (at $x = 0$) into the edge of the Hall fluid. Since the Cooper pair is a singlet, the two electrons deposited are of opposite spin. The pair "wave function," $t_1(x,x')$, is symmetric under the interchange of x with x'. It is assumed to fall off exponentially for x and x' large compared to the pair size—essentially the superconducting coherence length ξ. Since the 2DEG is completely spin polarized by the magnetic field (spin up), the edge state only transports spin-up electrons. It is thus necessary to consider spin-flip processes. We model these phenomenologically by the second term in (1), which flips the electron spin with amplitude t_2 at position x along the edge. Physically, t_2 will probably be dominated by spin-orbit-mediated scattering in the 2DEG. (It is also possible that nearby magnetic impurities mediate the spin-flip process.)

It is apparent from $H_{\rm PERT}$ in Eq. (1) that at second order in t_j a term will be generated that destroys a Cooper pair in the superconductor and creates two up-spin electrons in the 2DEG. Retaining only this composite process, we let $H_{\rm PERT} \to H_{\rm TUNN}$ with

$$H_{\rm TUNN} = \int_{x,x'} t(x,x')[\psi_\uparrow^\dagger(x)\psi_\uparrow^\dagger(x')\hat{c}(x=0) + \text{H.c.}], \quad (2)$$

where $t(x,x') = t_1(x,x')[t_2(x) - t_2(x')]$. Since $t_1(x,x')$ vanishes rapidly for x, x' larger than the coherence length ξ, the integrals above will be dominated by small x and x'. For simplicity, we replace these integrals by a single term,

$$H_{\rm TUNN} = t[\psi_\uparrow^\dagger(x=\xi)\psi_\uparrow^\dagger(x=0)\hat{c}(x=0) + \text{H.c.}], \quad (3)$$

where the spatial arguments are separated by the coherence length, ξ. The qualitative results obtained below do not depend on this simplification. The remaining parameter t characterizes the strength of the pair tunneling. In the following, we will drop the spin subscript on the up-spin electrons.

For the integer quantum Hall state, with $\nu = n$, the edge states, which carry away the spin-polarized electrons, are noninteracting Fermi liquids.[9] The appropriate effective (Euclidian) action in this case is simply

$$S_{\rm QHE} = \sum_{j=1}^{n} \int dx\, d\tau\, \psi_j^\dagger(\partial_\tau - i v_j \partial_x)\psi_j, \quad (4)$$

where τ is imaginary time. Here ψ_j denotes the (spin-up) electron in edge branch j and v_j is the corresponding edge velocity. For fractional states at odd integer ν^{-1}, the edge state is expected to be a single-channel chiral Luttinger liquid. The appropriate Euclidian action in terms of a chiral boson field ϕ is[10]

$$S_{\rm QHE} = \frac{1}{4\pi\nu} \int dx\, d\tau\, \partial_x\phi[i\partial_\tau\phi + v\partial_x\phi], \quad (5)$$

with ν^{-1} an odd integer. The electron operator is given by $\psi \sim e^{i\phi/\nu}$.

Consider now the effect of the tunneling term Eq. (3) in transferring charge across the junction. Our approach is perturbative in the tunneling amplitude t. Before calculating the tunneling conductance, it is instructive to consider a simple renormalization group (RG) transformation[11] which tells us how the tunneling amplitude t varies with the energy (or temperature) scale. Since $H_{\rm TUNN}$ in Eq. (3) involves fields near $x = 0$, it is useful to integrate out the degrees of freedom for $x \neq 0$ in both the edge action above, and in the superconductor. Since the superconductor has a nonzero condensate, it is legitimate to simply replace the operator $\hat{c}$ in Eq. (3) by the c number Δ. The remaining field is the spin-up electron near $x = 0$, which depends on imaginary time τ. Consider a RG transformation which integrates out a shell of Matsubara frequencies between Λ/b and Λ, where Λ is a high frequency cutoff. The resulting RG flow equation for the tunneling amplitude t is given to leading order by ($l = e^b$):

$$\partial t/\partial l = (1-\delta)t, \tag{6}$$

where δ is the scaling dimension of the tunneling operator, $O = \psi^\dagger(x=\xi)\psi^\dagger(x=0)$. This dimension can be evaluated from the large (imaginary) time decay of the correlation function

$$\langle O^\dagger(\tau)O(\tau=0)\rangle \sim \tau^{-2\delta}, \tag{7}$$

using the edge action in Eq. (4) or (5).

Specializing to odd integer ν^{-1}, the tunneling operator can be expressed in terms of the boson field ϕ, as $O \sim e^{2i\phi(x=0)/\nu}$, where the "2" is because two electrons are tunneling. Performing the average in (7) using the quadratic edge action (5) gives $\delta = 2/\nu$. Thus, for all odd integer ν^{-1}, the tunneling amplitude t flows to zero at low energies. For $\nu = 1$ this result is perhaps surprising, since one might have expected a constant (energy-independent) tunneling amplitude for a Fermi liquid edge state.

At nonzero temperatures the RG flows are cut off by T, and one obtains an effective temperature-dependent tunneling amplitude, $t_{\rm EFF}(T) \sim tT^{2/\nu-1}$. One expects the tunneling conductance through the junction to vary as $t_{\rm EFF}^2$, which gives the result

$$G(T) \sim t^2 T^{4/\nu-2}, \qquad \nu^{-1} = \text{odd integer}. \tag{8}$$

This result can be verified directly by calculating the conductance via a Kubo-type formula[11] (see below). One can also calculate the nonlinear current-voltage ($I-V$) curve throught the point contact,[11] and at $T=0$ one obtains $I \sim V^{4/\nu-1}$ for small V.

Equation (8) indicates that the conductance of a junction separating a dissipationless superconductor from a dissipationless quantum Hall fluid vanishes as $T \to 0$. The point contact is insulating. For the $\nu = 1/3$ state, the conductance vanishes with an enormous power, $G \sim T^{10}$. For the integer state $\nu = 1$ the power is smaller, with $G(T) \sim T^2$, but even a vanishing conductance is surprising since the edge state is a Fermi liquid in this case.

In order to understand the above result for $\nu = 1$, it is helpful to calculate explicitly the junction conductance using the noninteracting electron action in Eq. (4). The junction conductance is defined as

$$G = \lim_{\omega\to 0}\left[\frac{1}{\hbar\omega_n}\int_0^\beta d\tau\, e^{i\omega_n\tau} \times\langle T_\tau I(\tau)I(0)\rangle\right]\Big|_{i\omega_n\to\omega+i\epsilon}, \tag{9}$$

where the junction current operator is

$$I = 2eit[\psi^\dagger(x=\xi)\psi^\dagger(x=0)\hat{c}(x=0) - \text{H.c.}]. \tag{10}$$

To leading (second) order in the tunneling amplitude it is sufficient to evaluate the correlation function in Eq. (9) using the free fermion edge action Eq. (4). Once again the superconducting pair field operator $\hat{c}$ can be replaced by Δ. We thereby obtain a perturbative expression for the junction conductance when $\nu = 1$

$$G = (4e^2/\pi v)(t\Delta)^2\beta\int_{-\infty}^{\infty}[1+\cosh(\beta E)]^{-1}\rho(E). \tag{11}$$

Here we have defined a "pair-tunneling" density of states, $\rho(E) = [1-\cos(2\xi E/\hbar v)]/(\hbar v)$. Evaluating this expression in the low-temperature limit, $k_B T \ll \hbar v/\xi$, gives

$$G(T) = (4e^2/\hbar)(4\pi/3)(t\Delta\xi/\hbar v^2)^2(k_B T)^2. \tag{12}$$

Notice that the T^2 dependence can be traced directly to the supression of the pair-tunneling density of states $\rho(E)$, which vanishes as E^2 for $E \ll \hbar v/\xi$. This can also be seen in the expression for the tunnel current at finite voltage, which at low temperatures is found to take the form

$$I \sim \int_{-\infty}^{\infty} dE\,\rho(E)[f(E-2eV) - f(E+2eV)] \sim V^3, \tag{13}$$

with $f(E)$ a Fermi function. Physically, the suppression of the pair-tunneling density of states can be attributed to the Pauli exclusion between the *pair* of electrons. After tunneling one spin-up electron into the edge state, tunneling of the second electron is suppressed by the Pauli exclusion principle up to a time ξ/v, at which point the first electron has been carried away a distance ξ by the edge current. This leads in turn to a suppression in the pair-tunneling density of states, $\rho(E)$, below the energy scale $\hbar v/\xi$.

It is amusing that this "Pauli blockade" of pair-tunneling, effective when $\nu = 1$, can be circumvented when tunneling into a $\nu = 2$ state, which has two edge channels. Specifically, the pair of electrons can simultaneously tunnel into the two different edge channels. This can be quantified as follows. Let p denote the probability that an electron in the pair will tunnel into the first ($j=1$) edge mode, and $1-p$ the probability to tunnel into the second ($j=2$). The electron operators entering into the tunneling Hamiltonian Eq. (3), can then be expressed as $\psi = \sqrt{p}\psi_1 + \sqrt{(1-p)}\psi_2$, where ψ_j is the electron operator in the jth edge state. To evaluate the junction conductance one needs the edge Green's functions, which follow from the action in (4) and are given by

$$\langle T_\tau \psi_j(x,\tau)\psi_j^\dagger(0,0)\rangle = \frac{e^{ik_j x}}{v_j\tau + ix}. \tag{14}$$

Notice that we have included a phase factor into the above Green's functions, with k_j playing the role of an edge (Fermi) momentum. In general, this edge momentum is gauge dependent, but the difference, $k_{12} = k_1 - k_2$, is gauge invariant and determined by the magnetic flux which penetrates between the two edge branches.[9] With l denoting the spatial separation between the two branches one has $k_{12} = eBl/\hbar$, with B the magnetic field.

Using the above Green's functions, it is straightforward to evaluate the junction conductance for the case $\nu = 2$, to leading order in the tunneling amplitude, t. A straightforward calculation using Eq. (9) gives the conductance at zero temperature

$$G(T=0) = (16e^2/\pi\hbar)(t\Delta/v)^2 p(1-p)[1-\cos(k_{12}\xi)], \quad \nu = 2. \tag{15}$$

Note that, in contrast to the $\nu = 1$ case, the tunneling conductance for $\nu = 2$ does *not* vanish at $T = 0$. Pauli exclusion is less effective here, with the two electrons tunneling into different edge states. The conductance is proportional to $p(1-p)$, and so vanishes when the tunneling is completely into one or the other of the two edge modes. Note, moreover, that the edge momentum difference k_{12} plays a crucial role here. Indeed, in the limit $k_{12} \to 0$, in which the edge states sit atop one another, the tunnel conductance vanishes. In this limt one recovers the "Pauli blockade."

In summary, I have introduced and analyzed a simple model for a tunnel junction between a superconductor and a quantum Hall fluid. The low temperature transport, dominated by pair tunneling, is suppressed due to both Coulomb blockade effects and the Pauli exclusion principle. The tunneling conductance has been found to vanish as a power law in temperature, which should be verifiable in experiments on superconductor-2DEG samples. Numerous interesting issues have not been addressed here, such as ac transport, resonant tunneling, and nonequilibrium noise at the superconductor–quantum-Hall junction. It would also be very interesting to study pair tunneling into a spinful 1D Luttinger liquid, appropriate for a superconducting contact to a 1D quantum wire in zero magnetic field.

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