Temporal Order in Dirty Driven Periodic Media

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We consider the nonequilibrium steady states of a driven charge density wave in the presence of impurities and noise. In three dimensions at strong drive, a dynamical phase transition into a temporally periodic state with quasi–long–range translational order is predicted. In two dimensions, impurity induced phase slips are argued to destroy the "moving solid" phase. Implications for narrow band noise measurements and relevance to other driven periodic media, e.g., vortex lattices, are discussed.

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The influence of quenched impurities on a periodic medium can lead to very rich physics. Examples include charge density wave (CDW) systems [1] and the mixed state of type II superconductors [2] in which the vortices form a periodic lattice. In both these cases it has been argued that the impurities ultimately destroy the long-ranged periodicity, and pin the periodic medium. However, with an applied force, provided by an electric field or current, the periodic structure depins and becomes mobile. Once in motion, the impurities are less effective at destroying the periodicity [3]. Indeed, recent experiments have shown evidence for a first order melting transition of the *moving* vortex lattice [4]. Vinokur and Koshelev have interpreted the data on NbSe2, in terms of a true nonequilibrium phase transition, from a moving liquid phase to a "moving solid phase" [5].

This experiment raises a number of questions about the nonequilibrium steady states of such noisy driven systems with impurities. The most basic concerns the very existence of a moving solid phase. A solid in equilibrium is usually characterized by the presence of long-ranged crystalline correlations (Bragg peaks). But other criteria also suffice, such as the presence of a nonzero shear modulus or the absence of unbound dislocation loops. Under what circumstances, if any, is it possible to have a true moving solid, separate from a driven liquid with plastic flow? If the moving solid phase is possible, what are its characteristics and experimental signatures?

In this Letter we attempt to answer these questions, focusing for simplicity on the CDW. Many of our conclusions, however, apply also to the driven vortex lattice. In 2D, we find that a moving solid phase driven through impurities is always unstable to a proliferation of dislocations. The system becomes equivalent (in symmetry) to a driven liquid. In 3D, a moving solid phase appears to be stable at large velocities, as illustrated in the schematic phase diagram (Fig. 1). However, the solid phase does not have true long-ranged positional correlations (LRO) as in an equilibrium (3D) crystal. Rather, algebraic power law positional correlations are predicted as in a 2D equilibrium crystal. Likewise, unbound dislocation loops are absent in the moving solid. Despite the absence of spatial LRO, the moving solid phase is periodic in time—and hence has long-ranged temporal correlations. The experimental signature of such a periodic state is narrow band noise (NBN) at the "washboard" frequency [16].

Upon inclusion of thermal effects or phase slips, the CDW depinning transition [7] is predicted to be rounded [8], becoming a crossover (see Fig. 1). For electric fields E above this crossover, the CDW is in a plastic flow regime. With increasing E, we predict a true phase transition into a temporally periodic moving solid phase. For the CDW, this dynamical transition is likely to be continuous. As shown below, scaling arguments then predict NBN characteristics near the transition. The phase diagram for the driven vortex lattice should be similar, with current replacing electric field. However, in this case the transition into the moving solid phase is likely to be first order, at least in the large current limit.

Charge density waves tend to form in very anisotropic metals, consisting of weakly coupled metallic chains [1]. The electronic density in a CDW has a periodic modulation along the chain (x) direction:

$$\rho(\mathbf{x}) = \rho_0 + \mathrm{Re}\,\psi e^{i2k_F x}, \qquad (1)$$

with k_F the in-chain Fermi wave vector. Long-ranged order of the CDW is manifest in the complex order parameter field, $\psi(\mathbf{x}) = \rho_1 e^{i\phi}$.

In the absence of impurities and an applied electric field, the CDW exhibits long-ranged order in the pair

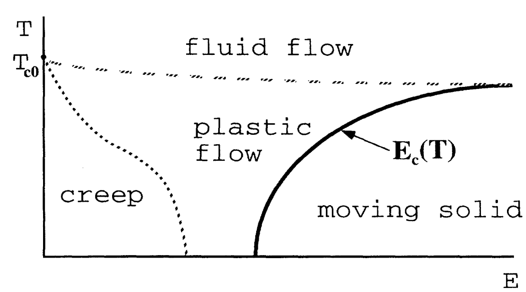

FIG. 1. Schematic phase diagram for the three-dimensional CDW. Hashed lines indicate (possible sharp) crossovers.

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correlation function

$$G(\mathbf{x}) = \langle \psi^*(\mathbf{x},t)\psi(\mathbf{0},t)\rangle_t \,, \tag{2}$$

with $G(x \to \infty) \neq 0$. Here the subscript t denotes a time average. Lee and Rice have argued that quenched impurities destroy the LRO of G for physical dimensions $d < 4$ [9]. However, when the CDW is driven and moving, the Lee-Rice argument is not valid, and LRO of G is not precluded (but see below).

In the moving nonequilibrium steady state, temporal correlations in ψ also serve to characterize the CDW order. Consider the pair correlation function

$$C(t) = \langle \psi^*(\mathbf{x},t)\psi(\mathbf{x},0)\rangle_x \,, \tag{3}$$

where the subscript x denotes a spatial average. Temporal LRO is signaled by a periodic, and nondecaying, behavior, $C(t + t_0) = C(t)$, with t_0 the washboard period. Middleton has shown that for a large class of dynamical models, which exclude phase slip and thermal noise, the steady state is (microscopically) periodic [10]. With noise, microscopic periodicity is destroyed, but the statistical correlation function $C(t)$ is still periodic. However, when phase slip is allowed, the robustness of the periodic state is much less clear, as we discuss below.

To proceed, we assume local CDW order and construct a description in terms of ψ. In the absence of phase slips, which we consider first, amplitude fluctuations can be ignored, and the dynamics involves the phase field ϕ. A common starting point is the Fukuyama-Lee-Rice (FLR) model [11] with equation of motion

$$\partial_t\phi = D\nabla^2\phi + V(\mathbf{x})\sin(2k_F x + \phi) + v_0 2k_F \,. \tag{4}$$

The spatial coordinates transverse to the chains have been rescaled to give an isotropic diffusion constant $D = \tau v_F^2$, with scattering time τ and Fermi velocity v_F. The second term on the right side represents the effect of quenched random impurities. The last term is present in an applied electric field E, with "bare" velocity $v_0 = (e\tau/m)E$. This term can be shifted away, $\phi \to \phi + v_0 2k_F t$, reducing the FLR equation to an equilibrium form $\partial_t\phi = -\delta H/\delta\phi$ with (time–dependent) Hamiltonian H. However, there are additional terms that can, and should, be added to FLR which are manifestly nonequilibrium.

The most important such term is $\sigma\partial_x\phi$, allowed by symmetry once the CDW is in motion along the x direction. Assuming dissipation occurs independently for each electron, it arises simply from replacing ∂_t by the convective derivative $D_t = \partial_t - v\partial_x$, where v is the actual CDW velocity, so $\sigma = v$.

In general, there are other missing terms, for example, of the Kardar-Parisi-Zhang form $(\partial_x\phi)^2$ [12]. This term, involving more gradients and powers of ϕ, is less relevant than $\partial_x\phi$. In the following we drop this term, although it can play an important role [13].

To study the properties of the moving state, it is appropriate to use a coarse-grained (in space and time) equation of motion. It is tempting to argue that the impurity term $V(x)$, oscillatory after shifting $\phi \to \phi + v2k_F t$, will average to zero at long times. However, as pointed out in Refs. [8,14], it is not legitimate to ignore completely the effect of impurities which modify the local mobility μ of the phase field. (This can be seen explicitly via a high velocity expansion.) The random V term can then be replaced by $\delta\mu(x)E2k_F = F(x)$, where $\delta\mu$ denotes the fluctuating part of the mobility. We take F Gaussian with $\overline{F} = 0$ and $\overline{F(\mathbf{x})F(\mathbf{0})} = g\delta(\mathbf{x})$.

We thereby arrive at a generalization of FLR,

$$\partial_t\phi = D\nabla^2\phi + v2k_F + \sigma\partial_x\phi + F(\mathbf{x}) + \eta(\mathbf{x},t) \,, \tag{5}$$

where the true velocity v is reduced from v_0 at finite E by impurity drag. The stochastic noise term is assumed to be Gaussian with $\langle\eta\rangle = 0$ and $\langle\eta(\mathbf{x},t)\eta(\mathbf{x}',t')\rangle = k_B T\delta(\mathbf{x} - \mathbf{x}')\delta(t - t')$. Although $\sigma \approx v$, we will consider it as an independent parameter to emphasize the role of convective effects.

Finally, we modify the model to allow for phase slip. One way to incorporate phase slip is to put the model on a lattice and replace $\partial_x\phi \to a^{-1}\sin[\phi(x + a) - \phi(x)]$, etc. Alternatively, amplitude fluctuations can be included using field ψ. The appropriate soft-spin model, which reduces to Eq. (5) in the spin-wave limit, is

$$\partial_t\psi = [D\nabla^2 + \sigma\partial_x + M + r(\mathbf{x}) + i\omega_0 + iF(\mathbf{x})]\psi - u\psi|\psi|^2 + \xi(\mathbf{x},t) \,, \tag{6}$$

with the definition $\omega_0 = v2k_F$. Here M is a "mass" term which controls the magnitude of the order parameter, and $\xi(\mathbf{x},t)$ is a complex stochastic noise term. We have also included a spatially random component to the mass, denoted $r(\mathbf{x})$, which we take to be a zero-mean Gaussian random variable with $\overline{r(\mathbf{x})r(\mathbf{0})} = \Delta\delta(\mathbf{x})$.

First, consider the system with phase slips suppressed. In this spin-wave limit, Eq. (5) is linear in ϕ and can be solved via Fourier transforms

$$\tilde{\phi}(\mathbf{p},\omega) = \frac{F(\mathbf{p})}{Dp^2 + i\sigma p_x} 2\pi\delta(\omega) + \frac{\eta(\mathbf{p},\omega)}{i\omega + Dp^2 + i\sigma p_x} \,, \tag{7}$$

where $\tilde{\phi}(t) = \phi(t) - \omega_0 t$. The first term in Eq. (7) represents a *static* distortion in $\phi(x)$, induced by the random mobility, while the second gives "noisy" dynamical fluctuations around this mean. The static phase variations diverge algebraically with system size L for $d < 3$, leading to the (stretched) exponential decay of $G(x)$. Thus even without phase slips, a 2D driven CDW lacks translational LRO. For the 3D case, Eq. (7) gives $G_{\rm sw}(x) \sim x^{-\eta}$, corresponding to power law peaks in the static structure function and translational quasi-LRO (QLRO). Notice that the presence of the nonzero $\sigma\partial_x\phi$ term in Eq. (5) is critical here. Indeed, for a driven periodic system with reflection symmetry $\sigma = 0$, Eq. (7) implies exponential decay of $G(x)$ for all $d < 4$. Because the disorder term in Eq. (7) is static, the dynamical properties are determined by the thermal noise term, with $C_{\rm sw}(\omega) \sim \delta(\omega - \omega_0)$ for all $d > 2$, indicating temporal LRO of $C(t)$. In 2D, spin waves imply temporal QLRO for $C(t)$.

We now address the stability of these spin-wave results to phase slips. In equilibrium $\sigma = F = 0$, Eq. (6) describes an *XY* model with relaxational dynamics. In this case the unbinding of *topological* defects (i.e., vortices) coincides with the loss of translational LRO due to spin-wave fluctuations. For $d > 2$, the vortices form $d - 2$ dimensional subspaces (lines in 3D). With a core energy growing with size L as L^{d-2}, they are bound at low temperatures. In equilibrium, 2D is marginal for both spin waves, which give QLRO, and vortex unbinding. But in the nonequilibrium case of interest, the unbinding of phase slips and vortices needs to be readdressed.

For simplicity, consider first the case $\sigma = 0$. Then Eq. (5) can actually be cast into an equilibrium form, $\partial_t \phi = -\delta E_{\rm eff}/\delta\phi + \eta$, with the proviso that the "energy," $E_{\rm eff} = \int\{D|\nabla\phi|^2/2 - F\phi\}$, is a multivalued (i.e., nonperiodic) function of the phase.

It is clear that spin-wave conformations of the phase are highly constrained. Imagine subdividing the system into regions of linear size L. Each such region experiences a net random torque $\int d^d\mathbf{r} F(\mathbf{r})$ of order $\pm\sqrt{gL^d}$. The torque in neighboring regions is generally different so that the local phases are pushed at different rates. In the absence of phase slips, however, all regions must rotate synchronously or build up enormous strains. Eq. (7) describes the resulting steady state in which the strains increase to counteract the nonuniform applied torques.

Once phase slips are allowed, however, such a situation is clearly metastable. If the net torque in a particular region is positive, then the energy of the spin-wave state is lowered simply by increasing all the phases in the region L by 2π. This decreases the random energy but does not alter the strain energy (which is now periodic in ϕ). For finite L and nonzero T, this process will, therefore, occur with an activated rate $1/\tau \sim \exp(-U/k_B T)$, where U is the energy barrier for the phase slip process.

The energy U is estimated from the elastic energy midway through the process, i.e., when there exist phase shifts of order π on scale L. Adding the elastic and random contributions to the energy gives $U(L) \sim DL^{d-2} - \sqrt{gL^d}$. A more microscopic picture is that of vortex nucleation. The phase slip is achieved by nucleating a small neutral topological defect (vortex-antivortex pair or vortex loop in $d = 2, 3$) which expands and slips over the region. The elastic contribution to the barrier energy is just the binding potential of the defect, $V_{\rm defect}(L) \sim DL^{d-2}$, up to possible $\ln L$ dependence. For $d > 4$ and small g, $U(L)$ is positive. Moreover, since $U(L)$ grows with L, large phase slips are exponentially suppressed, indicating stability of the spin-wave phase for $d > 4$. For $d < 4$, however, $U(L)$ becomes negative for $L > L_c \sim (D^2/g)^{1/(4-d)}$, and arbitrarily large unbound phase slip processes will presumably be nucleated. On scales much bigger than L_c, it is then inconsistent to assume a well defined average phase. The relaxation time for the phase slips on scale L_c is $\tau_c \sim \exp[DL_c^{d-2}/k_B T]$. Beyond this time scale, phases separated by distance that is large compared to L_c will become dephased, destroying the temporal LRO.

For $\sigma \neq 0$, the argument is trickier. First, transform to the moving frame via $x \to x - \sigma t$, which removes the $\sigma\partial_x\phi$ term in Eq. (5). The elastic force is invariant under such a transformation, but $F(\mathbf{r}) \to F(x - \sigma t, \mathbf{r}_\perp)$, so that the random torque field appears to move with velocity σ. Again, dividing the system into regions of size L, we see that a statistically uncorrelated realization of F moves into a particular region in a time $t_0 = L/\sigma$. For large L this decorrelation time t_0 is much smaller than the typical diffusive time for phase changes $t_\phi \sim L^2/D$. Only on time scales longer than t_ϕ can a phase change take advantage of the random torques spread out over the entire region. The random energy gained is thus averaged over at least t_ϕ/t_0 realizations of the torques, leading to a net torque on the entire region of $F_{\rm net} \lesssim \sqrt{g/\sigma}\, L^{(d-1)/2}$. Assuming an equality here, the energy balance is $U(L) \sim DL^{d-2} - \sqrt{g/\sigma}\, L^{(d-1)/2}$, and phase slips proliferate for $d < 3$. Directly in $d = 3$, these naive arguments suggest a transition between an ordered state for $v > v_c$ (large σ) and a disordered state for $v < v_c$, with $v_c \sim \sigma_c \sim g/D^2$.

The above arguments are consistent with the spin-wave calculations. As in equilibrium, they suggest that vortex unbinding coincides with the loss of translational LRO due to spin-wave variations. Further support for this conclusion follows from an analysis of the soft spin model, Eq. (6), which we now describe.

In the absence of randomness, the soft spin model contains two phases for $d \geq 2$. Fluctuation effects near the transition, negligible above $d = 4$, can be studied for small $\epsilon = 4 - d$ via the renormalization group (RG). The RG, including $r(\mathbf{x})$, has been studied in the context of random–bond *XY* magnets [15]; we generalize this calculation to include F in the dynamics. After transforming $\psi \to e^{i\omega_0 t}\psi$, we employ standard dynamical RG methods. The resulting differential RG flow equations to quadratic order are (for $\sigma = 0$)

$$\begin{aligned} \dot{u} &= u(\epsilon - 5u + 6\Delta), \\ \dot{\Delta} &= \Delta(\epsilon - 4u + 4\Delta - 2g) + 2g^2, \\ \dot{g} &= g(\epsilon - 2g + 6\Delta), \end{aligned} \tag{8}$$

with $\dot{u} = du/d\ln b$, etc., where $b = e^\ell$ is the rescaling factor. A simple analysis shows that the Gaussian ($u = \Delta = g = 0$), pure ($\Delta = g = 0$), and dirty equilibrium ($g = 0$) fixed points are all unstable, and, further, that no other fixed points exist. Instead, the couplings diverge as $\ell \to \infty$. In particular, $u \to +\infty$ (*not* $-\infty$), so the instability does not appear to indicate a fluctuation induced first order transition. Instead, the strong divergence of the disorder strengths g and Δ are consistent with the scenario that the ordered phase is absent.

For $\sigma \neq 0$, changing to comoving coordinates $x \to x - \sigma t$ reduces Eq. (6) to the previous case but with $r(\mathbf{x}) \to r(x - \sigma t, \mathbf{x}_\perp)$ and $F(\mathbf{x}) \to F(x - \sigma t, \mathbf{x}_\perp)$. Because $z = 2 + O(\epsilon)$ at the pure XY fixed point, the weaker x dependence of r and F may be ignored. Perturbations of the form $r(-\sigma t, \mathbf{x}_\perp)\psi$ and $iF(-\sigma t, \mathbf{x}_\perp)\psi$ are strongly irrelevant near $d = 4$, consistent with the spin-wave analysis and our earlier scaling arguments which gave $d = 3$ as the lower critical dimension for the ordered phase. To study $d = 3$ in the soft spin representation, we employ nonperturbative techniques. We therefore consider a generalized model containing N complex fields ψ_i, obeying Eq. (6) but with $|\psi|^2 \to \sum_i |\psi_i|^2$. We analyze the stability of the pure $N = \infty$ fixed point to the random perturbations. From scaling, the singular part of the mean energy density varies as $\overline{\langle \psi^2 \rangle} = \xi^{1/\nu - d} f(\Delta \xi^{y_\Delta}, g\xi^{y_g})$, where $\xi \sim M^{-1/\nu}$ is the (pure) correlation length, and y_Δ and y_g are the RG eigenvalues of Δ and g, respectively. At $N = \infty$, $1/\nu - d = -2$. Differentiation implies $A_\Delta \equiv \partial_\Delta \overline{\langle \psi^2 \rangle}|_{\Delta, g=0} \sim \xi^{y_\Delta - 2}$, $A_g \equiv \partial_g \overline{\langle \psi^2 \rangle}|_{\Delta, g=0} \sim \xi^{y_g - 2}$. These quantities are computed at $N = \infty$, using saddle point techniques and the Martin-Siggia-Rose dynamical formalism [16]. We find $y_\Delta = d - 5$ and $y_g = 3 - d$. [For $\sigma = 0$, $y_\Delta = d - 4$ and $y_g = 4 - d$, in agreement with the usual Harris criterion and the ϵ expansion, Eq. (8).] Thus for $d < 3$, the equilibrium critical point is unstable to random F, consistent with the absence of an ordered phase.

Our predictions for the 3D phase diagram are summarized in Fig. 1. Upon lowering the temperature at weak drive E, substantial CDW amplitude develops at the mean-field transition temperature T_{c0}. At long distances and times, however, both $G(x)$ and $C(t)$ decay exponentially to zero. With increasing drive, the CDW undergoes a sharp nonequilibrium phase transition into an ordered "periodic state" with spatial QLRO and temporal LRO. Our arguments strongly suggest that for 2D CDW systems, the ordered phase is absent.

Experimentally, temporal LRO in the solid phase manifests itself in NBN. Consider current fluctuations in the presence of a fixed bias voltage; other setups are qualitatively similar. The current density $j(\mathbf{x}) = (en_\perp/\pi)\partial_t \phi[1 + (\rho_1/2k_F)\cos(2k_F x + \phi)]$, where $n_\perp$ is the areal chain density. In a sample of cross-sectional area A, the instantaneous CDW current through the plane $x = 0$ is $I_\times(t) = \int_A d^2\mathbf{x}_\perp j(x = 0, \mathbf{x}_\perp, t)$. The oscillatory part of the NBN correlator $S(t) \equiv \langle I_\times(t) I_\times(0) \rangle$ is

$$S_o(t) \approx I_0^2 \int_{\mathbf{x}_\perp, \mathbf{x}'_\perp} \mathrm{Re}\{e^{i\omega_0 t} \langle \psi(\mathbf{x}_\perp, t)\psi * (\mathbf{x}'_\perp, 0)\rangle\}, \quad (9)$$

where $I_0 = en_\perp \omega_0 / 2\sqrt{2}\, k_F \pi$. We consider this quantity in the bulk, and expect that measured current fluctuations (in the external leads) exhibit proportional behavior. Temporal LRO in the solid phase, therefore, implies a sharp (resolution limited) delta function peak in $S(\omega) \sim A^{2-\eta/2}\delta(\omega - \omega_0)$. Deep in the liquid phase, ψ correlations are short range in space and time, which gives the mean-field result $S(\omega) \sim A\Omega/[\Omega^2 + (\omega - \omega_0)^2]$. Near the transition field $E_c(T)$, provided the transition is continuous, we expect a scaling form $S_\pm(\omega, \delta E) \sim |\delta E|^a f_\pm[(\omega - \omega_0)/|\delta E|^{z\nu}, A|\delta E|^{2\nu}]$, where z and ν are the dynamical and correlation length exponents, a is an additional scaling exponent, and $\delta E = E - E_c$. Matching to the infinite area limit implies that the amplitude of the delta function frequency peak for $E > E_c$ vanishes as $|\delta E|^{a+(4+z-\eta)\nu}$. For $E < E_c$, the (generally non-Lorentzian) line shape $S(\omega) \sim A|\delta E|^{a+2\nu} s[(\omega - \omega_0)/|\delta E|^{z\nu}]$. In 2D, $S(\omega)$ has an intrinsic width for all fields and temperatures.

Although the discussion has focused on CDWs, most of the ideas employed here apply to more general periodic media. Of particular current experimental interest are vortex lattices and 2D Wigner crystals [17]. In all cases, translational and temporal LRO may be destabilized by both phonons (phase fluctuations) and topological defects (phase slips). Provided reflection invariance is broken by an external drive field, we expect linear gradient terms (e.g., $\sigma\partial_x\phi$) in the equations of motion. Preliminary investigation of driven lattices suggests that such terms play a similar role in that case [13].

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