Chiral Surface States in the Bulk Quantum Hall Effect

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In layered samples which exhibit a bulk quantum Hall effect (QHE), a two-dimensional (2D) surface "sheath" of gapless excitations is expected. These excitations comprise a novel 2D chiral quantum liquid which should dominate the low temperature transport along the field (z axis). For the integer QHE, we show that localization effects are completely absent in the "sheath," giving a metallic z-axis conductivity. For fractional filling $\nu = 1/3$, the sheath is a 2D non-Fermi-liquid, with incoherent z-axis transport and $\sigma_{zz} \sim T^3$. Experimental implications for the Bechgaard salts are discussed.

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Disorder has a profound effect on transport in two-dimensional (2D) electron systems [1]. In the absence of an applied magnetic field, all the electronic states are believed to be localized due to strong quantum interference effects. With weak disorder, when the conductivity σ is larger than e^2/h, there is a weak-localization regime described by diffusive behavior with logarithmic temperature corrections. In the low temperature limit, however, a crossover to strongly localized behavior with $\sigma \ll e^2/h$ is always expected. In this regime the conductivity drops rapidly with temperature and vanishes as $T \to 0$.

In the presence of a magnetic field, 2D localization can be circumvented by tuning to the center of Landau levels. At these isolated transitions between quantum Hall plateaus, extended states and a temperature independent conductivity of order e^2/h are predicted, consistent with experiment [2]. There are also other 2D systems which exhibit such "metallic" behavior at isolated transitions, for example, 2D films at the superconductor-insulator transition. But away from such transitions, when the conductivity is well below e^2/h, it is invariably strongly temperature dependent, and insulating at zero temperature.

In this paper, we describe a novel class of anisotropic 2D electronic phases, which surprisingly can exhibit metallic conductivities much smaller than e^2/h. These 2D phases arise at the *surface* of bulk *three-dimensional* quantum Hall samples. We consider layered samples which exhibit independent quantum Hall states in each layer when a large perpendicular magnetic field is applied. This requires an interlayer tunneling amplitude t small compared to the 2D quantum Hall gap E_g [$E_g = \hbar\omega_c$ in the conventional integral quantum Hall effect (QHE)]. In such materials, the surface of the sample is enveloped by a sheath of current-carrying states as depicted in Fig. 1. This chiral surface phase is the 2D analog of the 1D states at the edges of a single layer quantum Hall fluid. There are currently two candidate experimental realizations of such systems: 2D electron gas multilayers [3], and the Bechgaard salts, $(TMTSF)_2X$ (where TMTSF is tetramethyltetraselenafulvalene, and $X = PF_6$, ClO_4, or ReO_4), which exhibit simultaneous field-induced spin density waves (SDWs) and QHE [4,5].

Electronic transport parallel to the field direction is a powerful probe of these chiral surface phases. Our analysis reveals that for the integer quantum Hall effect (IQHE), the motion along the z direction is always diffusive, independent of the impurity scattering strength. Localization effects—weak or otherwise—are completely absent. The 2D resistivity ρ_{zz} is temperature independent as $T \to 0$, with a value which can be much *larger* than h/e^2. But perpendicular to the field, along the x axis, the transport is ballistic and the resistivity ρ_{xx} vanishes as $T \to 0$. Our predictions for various experimental quantities which should be accessible in the Bechgaard salts, are given in detail at the end of the paper.

The surface sheath in the fractional quantum Hall effect at filling $\nu = 1/3$ is even *more* anisotropic. In this case, we find that electron transport along the z axis is always incoherent. At low temperatures insulating behavior is predicted with a resistivity diverging as $\rho_{zz} \sim 1/T^3$, even in the absence of any impurity scattering. At $T = 0$ interlayer transport is completely absent, and the electrons are "confined" to 1D. This chiral surface sheath for $\nu = 1/3$ is a nice example of a 2D non-Fermi-liquid phase.

Consider first the IQHE. The electronic states at the edge can be modeled simply as chiral fermions [6]. In the

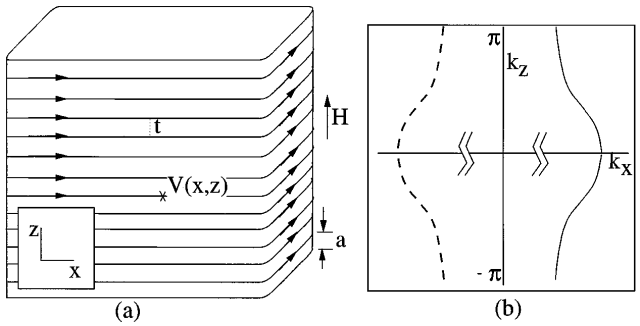

FIG. 1. (a) Geometry of a 3D quantum Hall sample. z-axis transport is included via the tunneling amplitude t, and impurity scattering by the random potential V. (b) Associated Fermi surface for the sheath. It differs from that of a conventional open Fermi-surface metal by the absence of the left-moving (dashed) half.

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3D case of interest here, there is a single edge state per layer (x-y plane) per filled Landau level (see Fig. 1). We focus on the case $\nu = 1$; higher integer filling fractions behave similarly. Including a small interlayer matrix element t leads to the noninteracting Hamiltonian

$$H = \sum_i \int dx\, \psi_i^\dagger i v \partial_x \psi_i - t(\psi_i^\dagger \psi_{i+1} + \psi_{i+1}^\dagger \psi_i) \quad (1)$$

$$= \int_p (v p_x - 2t \cos p_z a) \psi^\dagger(p_x, p_z) \psi(p_x, p_z), \quad (2)$$

where $\int_p \equiv \int_{-\infty}^{\infty} (dp_x/2\pi) \int_{-\pi}^{\pi} dp_z/2\pi$ and $\psi_i^\dagger$ is an electron creation operator for the edge state in the ith layer. Here we have set the layer spacing a equal to 1. Calculations for the field-induced SDW model show that Eq. (2) also describes the edge modes in that case, with a proper choice of v and t [7]. Equation (2) shows that interlayer hopping induces a small dispersion along the layering direction, resulting in "half" of an open Fermi surface [see Fig. 1(b)].

Before including impurity scattering, we reexamine the validity of Fermi-liquid theory (FL) with electron-electron interactions present. Despite the chiral nature of the sheath, the phase-space restrictions which stabilize the FL for a conventional Fermi surface continue to apply here, in particular, the quasiparticle decay rate $\mathrm{Im}\Sigma(\omega) \sim \omega^2 \ln\omega$, the dc conductivity $\sigma_{zz} \sim 1/T^2$, and the specific heat $C \sim T/av$. Long-range Coulomb interactions are screened in the static ($\omega \to 0$) limit.

To study dc transport at low temperatures, we must include impurity scattering at the surface, which we include by adding to the Hamiltonian a term of the form

$$H_{\mathrm{imp}} = \sum_i \int dx\, V(x, ia) \psi_i^\dagger \psi_i . \quad (3)$$

For simplicity, we assume the random potential to be Gaussian and uncorrelated, $\overline{V(x,z)V_j(x',z')} = \Delta \delta(x - x') \delta_{z,z'}$. In contrast to the results for the pure system, the chiral nature of the dirty sheath leads to dramatic differences from ordinary dirty 2D metallic behavior. In particular, localization along the x direction is precluded *a priori* since the wave functions Φ are necessarily delocalized along the x direction. This follows from the continuity equation $\partial_x |\Phi|^2 = \nabla_z \cdot J$, where $J(x,z) = (2t/v)\,\mathrm{Im}[\Phi^*(x,z)\Phi(x,z+a)]$, valid for arbitrary random potential V. To study possible localization effects along the z axis, we consider the usual averaged product of advanced and retarded Green's functions for noninteracting electrons,

$$\mathcal{D}(\mathbf{r}; \Omega) \equiv \overline{G_+(\mathbf{0}, \mathbf{r}; E) G_-(\mathbf{r}, \mathbf{0}; E + \Omega)} . \quad (4)$$

Here $G_\pm = [i v \partial_x - t \nabla_z^2 - E \mp i\eta + V(\mathbf{r})]^{-1}$, with ∇_z^2 the discrete Laplacian, and $\eta = 0^+$. A standard summation of ladder diagrams, which gives a diffusive form for nonchiral fermions, yields the approximation

$$\mathcal{D}_{\mathrm{ladder}}(p_x, p_z; \Omega) = \frac{2\pi\rho}{-i(\Omega + v p_x) + D p_z^2 + 2\eta} , \quad (5)$$

where the density of states $\rho = 1/2\pi v a$ and $D = t^2 \tau a^2$, with the relaxation time $\tau = 2v/\Delta$. The anisotropic form of the denominator in Eq. (5) reflects the difference in propagation along x (ballistic) and z (diffusive). The Einstein relation $\sigma_{zz} = e^2 \rho D$ can be verified by explicit computation. The derivation of the diffusive transverse motion in Eq. (5) has neglected possibly important quantum interference corrections. Indeed, for isotropic 2D systems, such corrections invalidate diffusive behavior at low temperatures, due to localization. To study possible interference effects here, we use the Q-matrix approach, which allows a systematic treatment of corrections to Eq. (5). Following McKane and Stone [8], the averaged correlation functions are obtained from the replicated partition function $Z = \int [d\bar{\phi}][d\phi][dQ] \exp(-S)$, where

$$S = \sum_i \int dx \Big\{ i \bar{\phi} \Lambda [i v \partial_x - t \nabla_\perp^2 - E - \Delta Q] \phi + \frac{\Delta}{2} \mathrm{Tr} Q^2 + \eta \bar{\phi} \phi \Big\} . \quad (6)$$

Here ϕ is a $2n$-component complex vector, Q is a $2n$ by $2n$ Hermitian matrix, and $\Lambda = 1 \otimes \sigma_z$. Equation (6) follows from introducing $\phi_\pm$ fields to generate retarded and advanced Green's functions, averaging the nth moment of the original generating function Z_0, and decoupling the resulting quartic interaction using the Q matrix. The η term infinitesimally breaks the $U(n,n)$ symmetry of the remaining action. Physical correlators, which are derived from the logarithm of Z_0 [$\ln Z_0 = \lim_{n\to 0}(Z_0^n - 1)/n$], must be computed in the limit $n \to 0$. Writing $Q = \begin{pmatrix} Q_{++} & Q_{+-} \\ Q_{-+} & Q_{--} \end{pmatrix}$, the density of states is, e.g., $\rho = (1/\pi)\,\mathrm{Im}\langle Q_{++;\alpha\alpha}\rangle$, where no sum is implied over the (arbitrary) replica index $1 \le \alpha \le n$. Up to nonsingular contact terms, the correlator $\mathcal{D}(\mathbf{r};0) = \langle Q_{+-;\alpha\alpha}(\mathbf{r}) Q_{-+;\alpha\alpha}(\mathbf{0})\rangle$.

The ladder approximation is recovered as the leading term in a saddle point expansion of the effective action for Q, obtained by integrating out the bosonic fields. At the mean-field level, $\bar{Q}_{++} = -\bar{Q}_{--} = i/2va$, reflecting the finite density of states.

Expanding $Q = \bar{Q} + \tilde{Q}$, the $\tilde{Q}_{++}$ and $\tilde{Q}_{--}$ fields have only massive fluctuations around the saddle, but the Q_{+-} and Q_{-+} fields are massless Goldstone bosons for $\eta = 0$. Following the conventional nonlinear sigma model approach, we ignore the massive fluctuations and focus on the Goldstone modes, staying within the saddle point manifold by eliminating Q_{++} and Q_{--} via the zero momentum, tree equations of motion. The resulting theory may be expressed solely in terms of the off-diagonal submatrices of $\tilde{Q}$, and is governed by an action

of the form

$$S_{\text{eff}} = \frac{1}{2\pi\rho}\int_p (-i\upsilon p_x + Dp_z^2)\,\mathrm{Tr}[Q_{+-}(\mathbf{p})Q_{-+}(-\mathbf{p})] + \prod_{i=1}^{4}\int_{\mathbf{p}_i}\Gamma(\mathbf{p}_i)\,\mathrm{Tr}[Q_{+-}Q_{-+}Q_{+-}Q_{-+}], \quad (7)$$

to $O(Q^4_{\pm\mp})$. The interaction vertex $\Gamma(\mathbf{p}_i) = \delta(\sum \mathbf{p}_i) f(\mathbf{p}_i)$, with $f(\mathbf{p}) \sim c_1 p_x + c_2 p_z^2$ for small momenta, and all higher vertices must have similar momentum structure owing to the continuous $U(n,n)$ symmetry. Truncating at quadratic order leads to the ladder result for $\mathcal{D}$, while retaining the anharmonic corrections is analogous to the usual expansion of the appropriate nonlinear sigma model.

With this formulation, the stability of the "diffusive" metal can be evaluated from simple power counting. To make the quadratic action dimensionless, we rescale $z \to bz$, $x \to b^2 x$, and $Q \to b^{-1/2}Q$ (with $b > 1$ to focus on long-wavelength behavior). Under this transformation the quartic coupling amplitude $\Gamma_0 \to \Gamma_0/b$, implying the *irrelevance* of anharmonic terms in Q_{+-} and the stability of the metallic phase. This is in contrast to the usual case of a nonchiral 2D dirty conductor, in which the metallic fixed point is marginally unstable. Physically, the absence of localization effects may be attributed to the ballistic motion along the x axis, which suppresses multiple scattering. Although it is possible that strong enough disorder could induce some kind of transverse localization, we have been unable to find evidence of such a phase, and suspect that metallic behavior persists for all impurity concentrations.

In other 2D electron models which exhibit diffusion at $T = 0$, the delocalized wave functions typically exhibit multifractal scaling [2]. A classic example is the IQHE plateau transition, which manifests multifractal behavior via anomalous scaling of $\mathcal{D}(p,\Omega)$, in the limit $Dp^2 \ll \Omega$. But in the present case $\mathcal{D}(p,\Omega)$ scales trivially with Ω and p_z, implying that the wave functions are *not* multifractal.

Having established the stability of the metallic phase in the noninteracting case, we turn to the combined influence of interactions and disorder. As shown by Altschuler and Aronov (see Ref. [1] and references therein), interaction effects are much enhanced by diffusive motion in conventional metals. For example, diffusive relaxation of density fluctuations leads to singularities in the tunneling density of states. For our semiballistic metal, one expects this effect to be absent, since any charge buildup is swept away at the chiral Fermi velocity. Indeed, one can show that the diagrams responsible for the singularity yield only smooth contributions in the chiral case.

The z-axis transport for the surface sheath in the IQHE behaves much as a conventional metal. Dramatically different results are obtained for fractional filling factors, as we now describe. We focus on the odd-denominator fractional states, particularly $\nu = 1/3$. The edge states are then chiral Luttinger liquids of charge ν Laughlin quasiparticles [9]. Because these quasiparticles have integrity only within a single quantum Hall plane, interlayer transport must involve the tunneling of physical *electrons*, or equivalently $1/\nu$ quasiparticles. The surface "sheath" can be described using the bosonized Euclidean action

$$S = \sum_i \int_{-\infty}^{\infty} dx \int_0^{\beta} d\tau \Big\{ \frac{1}{4\pi\nu}\,\partial_x\phi_i(i\partial_\tau\phi_i + \upsilon\partial_x\phi_i) - t\cos[(\phi_i - \phi_{i+1})/\nu]\Big\}, \quad (8)$$

where ϕ is a boson field and $\beta = 1/T$. The charge density is $n_i(x) = \partial_x\phi_i/2\pi$, and the operator $e^{i\phi(x)}$ creates a quasiparticle (2π soliton in ϕ) at position x. The cosine term describes interlayer tunneling with amplitude t. In the ideal $\mathbf{H} \parallel \hat{z}$ geometry, no flux penetrates between successive edge modes; oscillatory phase factors are therefore absent from this term. To study the effects of tunneling, we employ standard renormalization group methods perturbative in t. After introducing a cutoff Λ in q_x, one integrates out modes with $\Lambda e^{-d\ell} < |q_x| < \Lambda$. Upon rescaling $x \to e^{d\ell}x$ and imaginary time $\tau \to e^{d\ell}\tau$, the quadratic action is brought back to its original form. The tunneling amplitude is, however, renormalized [10]

$$\frac{dt}{d\ell} = (2 - 1/\nu)t, \quad (9)$$

and is technically an irrelevant perturbation. Equation (9) isunchanged even in the presence of long-range Coulomb interactions (not explicitly included) [7]. To extract the z-axis conductivity imagine renormalizing until the rescaled temperature is of order of the quantum Hall gap E_g, which implies a rescaling factor $e^{\ell} = E_g/T$. From scaling one then obtains an interlayer tunneling time $\tau_{\text{in}} \sim \tau_\phi[E_g/t(\ell)]^2$, with $t(\ell) \sim t(T/E_g)^{1/\nu-2}$. Here $\tau_\phi \sim 1/T$ is a thermal dephasing time. Since $t(\ell)$ scales to zero at low temperatures, $\tau_{\text{in}} \gg \tau_\phi$ and the interlayer transport is incoherent. The z-axis conductivity can then be obtained from the diffusion constant $D \sim a^2/\tau_{\text{in}}$ and the Einstein relation $\sigma_{zz} = e^2\rho D$. This gives $\sigma_{zz} \sim (e^2/h)a/\upsilon\tau_{\text{in}} = \text{const} \times T^{2/\nu-3}$, which *vanishes* as $T \to 0$.

We now summarize our predictions for transport and thermodynamic quantities. For temperatures well below the bulk QHE gap, the surface sheath dominates the z-axis transport. In this regime, we expect a surface sheath from each full Landau level, each contributing a temperature independent (sheet) conductivity to give a total $\sigma_{zz}(\nu = N) \approx N(e^2t^2\tau a)/2\pi\upsilon$. The $\nu = 1/3$ state, if present, has a vanishing z-axis conductivity, $\sigma_{zz} \sim T^3$. In contrast to the quantized Hall plateaus, the "plateaus" in σ_{zz} are unquantized, and will probably exhibit small nonvanishing slopes even at the lowest temperatures.

The full behavior for σ_{zz} as a function of field will depend on the behavior in the regions connecting adjacent Hall states. There are several possible scenarios. In clean

$(TMTSF)_2X$, direct first order transitions between adjacent Hall states are possible, with the SDW period changing discontinuously. In this case, discontinuous jumps in σ_{zz} are expected. A second possibility is a direct but continuous transition between plateaus, as in 2D quantum Hall systems. Indeed, model calculations for clean $(TMTSF)_2X$ predict continuous $T = 0$ transitions for some parameter values. A continuous plateau transition would reveal universal scaling features for the steplike features in σ_{zz} (see Fig. 2 and below). Recent data on $(TMTSF)_2ClO_4$ [11] is roughly consistent with this scenario. Finally, a bulk 3D metallic state may intervene between adjacent quantized Hall states. A noninteracting description leads naturally to this latter behavior, as discussed in Ref. [12]. With disorder, the transition from Hall state to bulk metal should be continuous, with a mobility edge. In this scenario, the metallic state would contribute a very large (extensive) contribution to σ_{zz}, giving a large peak between Hall states, as depicted in the Fig. 2. Data on 2D gas multilayers is roughly consistent with this, although lower temperature data are necessary.

For the latter two scenarios, one expects universal scaling properties near the continous transitions, albeit with different exponents. Here we focus on contributions from the surface sheath. Upon approaching either transition from within a quantized Hall state, one expects an (in-plane) localization length which diverges as $\xi \sim |\delta B|^{-\nu}$. The z-axis localization length is assumed to vary as $\xi_z \sim \xi^{\zeta}$ (with $\zeta \leq 1$ on physical grounds), whereas characteristic energy scales vanish as $\omega \sim \xi^{-z}$ with z a dynamical exponent. Following standard methods [13], the bulk conductivity scales as $\sigma_{zz}^{\text{bulk}} \sim \xi^{\zeta-2}\Xi(T\xi^z)$. Near the transition, the thickness ξ of the surface sheath becomes large, and we may estimate the z-axis sheet conductivity as $\sigma_{zz} \sim \sigma_{zz}^{\text{bulk}}\xi$. Using the bulk scaling results, this implies singular behavior at $T = 0$ for the "steps" in σ_{zz}: $\sigma_{zz} \sim \xi^{\zeta-1} \sim |\delta B|^{(1-\zeta)\nu}$.

The SDW compounds provide a unique opportunity to study the specific heat of quantum Hall edge modes, in this case those of the whole sheath. In 2D electron gases, the edge contribution to C is masked by that of the localized states in the interior. While this should also be the case in multilayers, the SDW gap implies that the bulk contribution to C is activated at low temperatures. The gapless surface modes then dominate, contributing a linear temperature dependence to the specific heat per area: $C_A = N(\pi k_B^2 T/6\hbar a v)$. Since the ratio C/T is linear in the number N of full Landau levels, one expects steps in this quantity as well. Recent specific heat data in the ClO_4 Bechgaard salt appear roughly consistent with this variation [14].

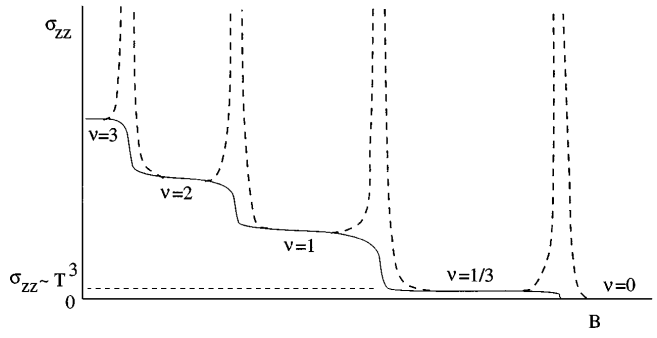

FIG. 2. Predicted behavior of σ_{zz} for an isolated transiton (solid line) and with an intervening metallic phase (dashed curve). See text for details.

Finally, we consider the effects of a transverse magnetic field ($B_y \neq 0$) on the integer surface sheath, induced (albeit nonuniformly) by tilting the sample. In the gauge $A_x = -B_y z$, this field shifts the relative Fermi momenta between edge modes in adjacent layers by $\delta k_F = eaB_y$. In the absence of impurities, momentum conserving interlayer tunneling is not possible below an energy scale, $\Delta_B \sim veaB_y$. This leads to activated z-axis transport, $\sigma_{zz} \sim \sigma_{zz,0}\exp(-\Delta_B/T)$. With impurity scattering, a finite conductivity will be restored at $T = 0$. But for a relatively clean surface, a *positive magneto resistance* for z-axis transport is expected.

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Note added.—After this work was completed we became aware of recent related results by Chalker and Dohmen [12].

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