Dynamics of a heavy particle in a Luttinger liquid

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We study the dynamics of a heavy particle of mass M moving in a one-dimensional repulsively interacting Fermi gas. The Fermi gas is described using the Luttinger model and bosonization. By transforming to a frame comoving with the heavy particle, we map the model onto a generalized "quantum impurity problem." A renormalization-group calculation reveals a crossover from strong to weak coupling upon scaling down in temperature. Above the crossover temperature scale $T^*=(m/M)E_F$, the particle's mobility μ is found to be (roughly) temperature independent and proportional to the dimensionless conductance g, characterizing the one-dimensional Luttinger liquid. Here $m(\ll M)$ is the fermion mass, and E_F the Fermi energy. Below T^*, in the weak-coupling regime, the mobility grows and diverges as $\mu(T)\sim T^{-4}$ in the $T\rightarrow 0$ limit.

I. INTRODUCTION

The quantum dynamics of a heavy particle moving through a fluid has been of longstanding interest. Most of the effort has focused on three-dimensional quantum fluids, either Fermi liquids such as ^{3}He or superfluids such as ^{4}He.[1] Recently, there has been a resurgence of interest in nonconventional quantum liquids. A paradigm is the Luttinger model,[2] which describes a one-dimensional interacting Fermi gas.

In this paper we study in detail the dynamics of a single heavy particle moving through a one-dimensional (1D) Luttinger liquid. Of interest is the temperature dependence of the heavy particle's mobility. Our motivation is twofold. Firstly, since the excitations in a 1D Luttinger liquid are profoundly different from in a Fermi liquid, one might anticipate that the dynamics of an immersed heavy particle would likewise be qualitatively modified. Secondly, powerful non-perturbative methods in 1D, such as bosonization, might be fruitfully employed to analyze the dynamics of a strongly coupled heavy particle.

Our main results are as follows. After introducing the model in Sec. II, we transform to a frame of reference comoving with the heavy particle in Sec. III. In this frame, the heavy particle sits at the origin. In the limit that $M\rightarrow\infty$ the model then becomes equivalent to a Luttinger liquid scattering off a static localized impurity. This problem has been analyzed in great detail recently, and is now well understood.[3] In the zero-temperature limit, the impurity effectively "breaks" the Luttinger liquid into two semi-infinite decoupled pieces. Fermions incident on the impurity are completely reflected. To analyze the case with finite mass M, a natural starting point is thus a limit in which the amplitude t for incident fermions to tunnel through the heavy particle is set to zero. Provided $t=0$, the mobility can be computed for arbitrary M, and one finds a temperature-independent value, $\mu=\pi g/(2\hbar k_F^2)$. At low temperatures, though, this limit is unstable to nonzero tunneling, t. A renormalization-group calculation reveals a crossover to a regime where the fermions are transmitted readily through the heavy particle. This leads to a decoupling between the dynamics of the heavy particle and the Fermi sea. At zero temperature, the only effect of the Fermi sea is to renormalize the mass of the heavy particle—the mobility is infinite.

In Sec. IV we use a weak-coupling perturbative approach to calculate the temperature dependence of the mobility in the $T\rightarrow 0$ limit. The dominant scattering process involves four fermions, absorbing and then reemitting a pair, one right and one left moving. This process changes the momentum and energy of the heavy particle, and is shown to lead to a low-temperature mobility that diverges as $\mu(T)\sim T^{-4}$.

II. THE MODEL

The Hamiltonian that describes the motion of a heavy particle coupled to a 1D interacting Fermi gas can be written as $H=H_0+H_{\rm LL}+H_{\rm int}$. Here H_0 describes the free particle of mass M:

$$H_0=\frac{P^2}{2M}, \tag{2.1}$$

with momentum P and position X. $H_{\rm LL}$ is the Hamiltonian for N interacting fermions, which in first quantized notation is

$$H_{\rm LL}=\sum_{i=1}^{N}\frac{p_i^2}{2m}+\sum_{i,j} V(x_i-x_j), \tag{2.2}$$

where x_i and p_i denote coordinate and momentum of the ith particle. The interaction between the heavy particle and fermions is assumed to take the form

$$H_{\rm int}=\sum_{i=1}^{N} U(x_i-X). \tag{2.3}$$

For simplicity we assume that $U(x)$ is repulsive and short ranged.

It is useful also to have a second quantized formulation. We denote as $\psi(x)$ the fermionic field operator describing the interacting Fermi gas. In the absence of interactions, the ground state consists of a filled Fermi sea, with Fermi momentum k_F. As usual, we decompose the field into a sum of right and left movers:

$$\psi(x)=\psi_R(x)e^{ik_Fx}+\psi_L(x)e^{-ik_Fx}, \tag{2.4}$$

where $\psi_{R/L}$ are supposed to be slowly varying. It will also be useful to bosonize the interacting electron gas, by expressing

$$\psi_{R/L}(x)=\sqrt{k_F}e^{i\sqrt{\pi}(\phi\pm\theta)}, \tag{2.5}$$

where ϕ and θ are canonically conjugate fields satisfying $[\theta(x),\partial_{x'}\phi(x')]=i\delta(x-x')$. The appropriate Luttinger liquid Hamiltonian takes the from

$$H_{\rm LL}=\frac{v}{2}\int_x\left[\frac{1}{g}(\partial_x\theta)^2+g(\partial_x\phi)^2\right]. \tag{2.6}$$

Here g is the dimensionless conductance, which is less than one for a repulsively interacting Fermi gas, equals one for free fermions, and is greater than one with attractive interactions. The Fermi velocity v is also renormalized by interactions, and will differ from the free fermion value, k_F/m.

The right and left moving electron densities, $N_R=\psi_R^\dagger\psi_R$ and $N_L=\psi_L^\dagger\psi_L$, have simple bosonic representations,

$$N_R+N_L=(1/\sqrt{\pi})\partial_x\theta \tag{2.7}$$

and

$$N_R-N_L=(1/\sqrt{\pi})\partial_x\phi. \tag{2.8}$$

III. DESCRIPTION IN FRAME COMOVING WITH PARTICLE

To transform the equations of motion into a frame comoving with the heavy particle, one can use the unitary transformation,[4]

$$\mathscr{U}=e^{i\Sigma_{i=1}^N p_iX}. \tag{3.1}$$

This transformation has been previously used[5] in a similar context, but in the special case where $M=m$. Under this transformation, the coordinates and momenta transform as

$$x_i\to x_i+X,$$

$$p_i\to p_i,$$

$$X\to X,$$

$$P\to P-\sum_{i=1}^N p_i. \tag{3.2}$$

The transformed Hamiltonian becomes

$$H\to H=\frac{1}{2M}\left[P-\sum_{i=1}^N p_i\right]^2+H_{\rm imp}, \tag{3.3}$$

with

$$H_{\rm imp}=H_{\rm LL}+\sum_{i=1}^N U(x_i). \tag{3.4}$$

Notice that P in (3.3) is no longer the momentum of the heavy particle, but rather is the *total* momentum of the system, and is conserved ($[H,P]=0$). However, the operator X remains unchanged under the transformation and still represents the position of the heavy particle.

When $M\to\infty$ the full Hamiltonian reduces to $H_{\rm imp}$, which describes a Fermi gas interacting with a static potential $U(x)$ centered at the origin. This quantum impurity problem has recently been analyzed in great detail.[3] However, when M is finite the heavy particle can move, and exchange energy with the Fermi sea. Notice that the heavy particle is coupled to the fermions via a minimal coupling, where the ''gauge'' field is the total momentum in the Fermi sea.[6–9]

The transformed Hamiltonian can be expressed directly in second quantization using the fermion field operators (2.4). These fields can then be bosonized. It is convenient to use a path integral representation, since the Lagrangian is linear in the ''gauge'' field. The Euclidean action for the free Luttinger liquid that corresponds to (2.6) can be expressed as

$$S_{\rm LL}=\frac{g}{2v}\int_{x,\tau}[v^2(\partial_x\phi)^2+(\partial_\tau\phi)^2]. \tag{3.5}$$

The total momentum of the fermions can also be easily bosonized,

$$\sum_{i=1}^N p_i\to k_F\int_x(N_R-N_L)=\frac{k_F}{\sqrt{\pi}}\int_x\partial_x\phi. \tag{3.6}$$

which enables the total action for the heavy particle plus Luttinger liquid to be written:

$$S=\frac{M}{2}\int_\tau\dot{X}^2(\tau)+\frac{ik_F}{\sqrt{\pi}}\int_{x,\tau}\dot{X}(\tau)\partial_x\phi(x,\tau)+S_{\rm imp}. \tag{3.7}$$

To analyze the dynamics in the transformed frame, it is convenient to first consider a strong-coupling limit ($U\to\infty$). In this limit, the fermions cannot pass through the heavy particle, and the Luttinger liquid is divided into two decoupled regions on either side of the particle. Perturbations away from this limit can be included by allowing for tunneling of fermions from one side to the other, with a small amplitude t. This process can be expressed in terms of the bosonic fields as[3]

$$S_T=-t\int_\tau\cos\sqrt{\pi}[\phi(0^+,\tau)-\phi(0^-,\tau)]. \tag{3.8}$$

As we shall see, in the limit $t=0$ the heavy particle's dynamics can be obtained exactly. A perturbative analysis for small t is then possible.

To this end, we follow Ref. 3 and integrate out the bosonic field $\phi(x)$, except at $x=0$—that is at the position of the heavy particle. In terms of the phase difference across the heavy particle,

$$\Phi(\tau)=\tfrac{1}{2}[\phi(0^+,\tau)-\phi(0^-,\tau)], \tag{3.9}$$

the action becomes $S=S_0+S_T$ with

$$S_0=\frac{1}{\beta}\sum_n \left(\frac{M\omega_n^2}{2}|X_n|^2+\frac{2k_F\omega_n}{\sqrt{\pi}}X_n\Phi_{-n}+g|\omega_n||\Phi_n|^2\right), \tag{3.10}$$

$$S_T=-t\int_\tau \cos[2\sqrt{\pi}\Phi(\tau)]. \tag{3.11}$$

In (3.10) the summation is over Matsubara frequencies $\omega_n=2\pi n/\beta$, with β the inverse temperature.

In the limit of zero tunneling ($t=0$), the action is quadratic. One can then integrate over the field $\Phi(\tau)$ to obtain a simple action for the dynamics of the heavy particle:

$$S_X=\frac{1}{\beta}\sum_n \left(\frac{M\omega_n^2}{2}+\frac{k_F^2|\omega_n|}{\pi g}\right)|X_n|^2. \tag{3.12}$$

This action is of the Caldeira-Leggett form, and describes a particle undergoing Brownian motion in a viscous environment with friction coefficient, $\eta=2k_F^2/\pi g$.[10,11] The particle's mobility can be obtained from the Kubo formula,[12]

$$\mu(\omega)=\frac{1}{\omega_n}P(\omega_n)|_{\omega_n\to i\omega+\epsilon}, \tag{3.13}$$

$$P(\omega_n)=\int_\tau e^{-i\omega_n\tau}\langle\dot{X}(\tau)\dot{X}(0)\rangle=\frac{\omega_n^2}{\beta}\langle|X_n|^2\rangle. \tag{3.14}$$

For the quadratic action (3.12) this gives a dc mobility,

$$\mu=\frac{\pi g}{2\hbar k_F^2}, \tag{3.15}$$

which is independent of temperature and proportional to the Luttinger liquid conductance g. In this limit ($t=0$), the particle is heavily damped by the fermions, even at zero temperature. The damping is heavy because the fermions cannot pass through the heavy particle, so motion is only possible by "pushing" the fermions out of the way.

It is worth emphasizing that the mobility in (3.15) is the *linear response* mobility. Within linear response, the applied force is taken to zero before the frequency, so that the excursions of the position X of the heavy particle remain small. In contrast, the dc nonlinear response corresponds to a uniform force and steady-state velocity. In the $t=0$ limit, we expect that the nonlinear mobility might in fact be rather different from (3.15), since in this case all of the fermions in the sea will have to move at the same steady-state velocity. In any event, the $t=0$ limit is actually unstable at low temperatures, as we shall now discuss.

Consider now perturbing about this limit, for small tunneling t. We first integrate over $X(\tau)$ to obtain an action that depends only on the bosonic field Φ:

$$S_\Phi=\frac{1}{\beta}\sum_n \left(\frac{2k_F^2}{\pi M}+g|\omega_n|\right)|\Phi_n|^2+S_T. \tag{3.16}$$

Notice that the phase mode has a mass term, due to the motion of the heavy particle. In the static limit ($M\to\infty$) this mass term vanishes, and the action reduces to that for a Luttinger liquid with impurity. Consider now a renormalization-group (RG) transformation that consists of integrating over modes $\Phi(\omega)$, for frequencies between Λ/b and Λ, and then rescaling $\omega\to\omega'=\omega/b$. Here $\Lambda\sim E_F$ is a high-frequency cutoff, and $b=e^{dl}$ is a rescaling factor. This transformation leaves the coefficient g invariant, whereas M decreases as

$$\frac{dM}{dl}=-M. \tag{3.17}$$

The RG flows for t depend on whether the mass for the phase mode is larger or smaller than the cutoff Λ. For $M\gg k_F^2/\Lambda$ the lowest-order RG flow equation is

$$\frac{dt}{dl}=\left(1-\frac{1}{g}\right)t, \tag{3.18}$$

whereas for $M\ll k_F^2/\Lambda$ one has

$$\frac{dt}{dl}=t. \tag{3.19}$$

At finite temperatures, these RG flows will be cut off at a scale $b\sim\Lambda/T$.

Since the cutoff energy scale is essentially the Fermi energy, $\Lambda\sim k_F^2/m$, the crossover between the two flows occurs when $M(l)\sim m$. If the (bare) particle mass is very large, $M\gg m$, the scaling of t will be determined by (3.18) over a large range of temperatures, between E_F and a crossover scale $T^*\sim(m/M)E_F$. In this temperature range, for a repulsively interacting Luttinger liquid ($g<1$), the tunneling rate will scale towards zero. The mobility of the heavy particle should then be roughly independent of temperature, given by (3.15). However, at temperatures below T^*, (3.19) indicates that the tunneling rate t starts increasing. As $T\to 0$ the tunneling rate becomes large, and the perturbative expansion breaks down.

Evidently, in the low-temperature limit the fermions can tunnel easily through the heavy particle. One anticipates that as $T\to 0$ the heavy particle becomes transparent, and its dynamics decouples from the fermions.

At very low temperatures when t grows large, fluctuations in the phase Φ are greatly suppressed by the S_T term in (3.16). In this limit it is a good approximation to expand the cosine in (3.11) for small argument:

$$-t\cos(2\sqrt{\pi}\Phi)\to -t+2\pi t\Phi^2. \tag{3.20}$$

This explicitly breaks the 2π phase invariance of the action. This symmetry breaking presumably occurs spontaneously at $T=0$, but would be restored at nonzero T. This approximation is thus only expected to be strictly valid *at* $T=0$. Since each 2π phase-slip process represents an event in which a fermion backscatters off the heavy particle, these events are completely suppressed at $T=0$.

After expanding the cosine term the full action is quadratic,

$$S_\Phi=\frac{1}{\beta}\sum_n \left(\frac{2k_F^2}{\pi M}+2\pi t+g|\omega_n|\right)|\Phi_n|^2. \tag{3.21}$$

The mobility can then be calculated using (3.14). To this end we introduce a source term, $S_s = i\int d\tau \dot{X}(\tau)J(\tau)$, which enables us to express $P(\omega_n)$ as a correlation function over the phase field:

$$P(\omega_n) = \frac{1}{M}\left(1 - \frac{2k_F^2}{\pi M}\langle|\Phi_n|^2\rangle\right). \tag{3.22}$$

This can be evaluated using (3.21) and one finds

$$P(\omega_n) = \frac{1}{M}\left(\frac{2\pi t + g|\omega_n|}{(2k_F^2/\pi M) + 2\pi t + g|\omega_n|}\right). \tag{3.23}$$

When $t \ll \omega_n \ll (m/M)E_F$, this reduces to our previous result (3.15). However, in the low-frequency limit, $\omega_n \ll t,(m/M)E_F$, it gives a diverging ac mobility:

$$\mu(\omega) = \frac{1}{i\omega M_{\rm eff}}, \tag{3.24}$$

$$M_{\rm eff} = M\left(1 + \frac{2(m/M)E_F}{\pi^2 t}\right). \tag{3.25}$$

This describes ballistic motion of the heavy particle with an effective mass $M_{\rm eff}$. This result is valid only *at* $T=0$. At nonzero but small temperatures, $T \ll (m/M)E_F$, one expects a finite mobility. As will be confirmed in Sec. IV, the dc mobility indeed diverges as $T \to 0$.

Notice that the mass renormalization becomes large when t decreases, which corresponds to an increase of the interaction strength between the heavy particle and the Fermi sea. This trend is consistent with that found by McGuire in an exact treatment of a particular Hubbard model with 1 spin down particle moving in a sea of $N-1$ spin up particles.[13] Unfortunately, a quantitative comparison with McGuire's result is not possible, since our parameter t is phenomenological, and cannot be readily related to the bare interaction potential between the heavy particle and the Fermi sea.

The above results suggest a rich temperature dependence for the mobility for $g<1$. Between the Fermi temperature and a crossover temperature, $T^* \sim (m/M)E_F$, the mobility is roughly temperature independent and given by (3.15). Below T^*, the mobility starts increasing with cooling, and diverges in the zero-temperature limit. Physically, below T^* the heavy particle becomes "transparent" to the fermions. The dynamics of the heavy particle decouples from the Fermi sea. In the next section, we employ a weak-coupling perturbative approach to calculate the functional form of $\mu(T)$ as $T \to 0$.

IV. WEAK-COUPLING PERTURBATION THEORY

Since the heavy particle tends to decouple from the Fermi sea as $T \to 0$, a weak-coupling approach should be appropriate at low temperatures. In this section we use perturbation theory in the coupling between particle and Fermi sea, to extract the temperature dependence of the mobility as $T \to 0$.

It is convenient to employ a second quantized description for the heavy particle, denoting as $c^\dagger(x)$ and $c(x)$ the creation and destruction operators. Since we are only interested in a *single* particle, $\int_x c^\dagger c = 1$. The free Hamiltonian (2.1) is

$$H_0 = \sum_k \epsilon_k c_k^\dagger c_k, \tag{4.1}$$

with dispersion $\epsilon_k = k^2/2M$. The interaction Hamiltonian (2.3) becomes

$$H_{\rm int} = U_0 \int dx c^\dagger(x) c(x) N(x), \tag{4.2}$$

where $N(x) = \psi^\dagger(x)\psi(x)$ is the fermion density. Here we have replaced the short-ranged interaction by a δ function: $U(x) \to U_0\delta(x)$. It is important to distinguish between small momentum transfer processes, and processes that scatter the fermions by $2k_F$. Using the decomposition (2.4) one can express, $N = N_0 + N_{2k_F}$, where

$$N_0(x) = \psi_R^\dagger\psi_R + \psi_L^\dagger\psi_L = N_R + N_L \tag{4.3}$$

involves small momentum transfer, and

$$N_{2k_F}(x) = \psi_R^\dagger\psi_L e^{i2k_F x} + \text{H.c.} \tag{4.4}$$

denotes the large momentum contributions. The two corresponding terms generated from the interaction Hamiltonian will be denoted $H_{\rm int,0}$ and $H_{{\rm int},2k_F}$, respectively.

Consider first computing the scattering rate for the heavy particle using Fermi's golden rule, where the perturbing Hamiltonian is $H_{\rm int,0}$. Since the fermion density at small momentum transfer is simply $N_0 = (1/\sqrt{\pi})\partial_x\theta$, the interaction Hamiltonian $H_{\rm int,0}$ takes the form of an "electron-phonon" interaction. It is thus useful to introduce "phonon" creation and destruction operators, which create and destroy the harmonic Luttinger liquid excitations. To this end, we expand the boson field as

$$\theta(x) = \frac{1}{\sqrt{L}}\sum_k \theta_k e^{ikx} \tag{4.5}$$

and introduce boson operators:

$$b_k = (1/\sqrt{2g|k|})(|k|\theta_k + ig\Pi_k), \tag{4.6}$$

where Π_k denote Fourier modes of the conjugate momentum, $\Pi(x) = \partial_x\phi(x)$. The operators b_k satisfy canonical Bose commutation relations. The Luttinger liquid Hamiltonian can be expressed as

$$H_{\rm LL} = \sum_k \omega_k b_k^\dagger b_k, \tag{4.7}$$

with dispersion $\omega_k = v|k|$. Finally, the small momentum interaction takes the form:

$$H_{\rm int,0} = \frac{\sqrt{qvU_0}}{\sqrt{2\pi L}}\sum_{k,q}(iq/\sqrt{\omega_q})c_{k+q}^\dagger c_k(b_q + b_{-q}^\dagger). \tag{4.8}$$

The rate to scatter the heavy particle from an initial state with momentum k to a final state $k' = k+q$, with absorption or emission of a single phonon, can now be readily obtained using Fermi's golden rule. After summing over all possible phonon modes, assuming they are in equilibrium at temperature T, the rate is found to be

$$\Gamma_{k\to k+q}=\frac{qU_0^2}{2vL}\sum_{\pm}\omega_q(2n_q+1\pm 1)\,\delta(\epsilon_{k+q}-\epsilon_k\pm\omega_q). \tag{4.9}$$

Here $n_q=[\exp(\beta\omega_q)-1]^{-1}$ is the Bose distribution function.

These processes are severely restricted by energy and momentum conservation. For example, for zero initial momentum, $k=0$, the above δ functions vanish unless $\epsilon_q=\omega_q$ or $q=2Mv_F$. But at this momentum, the heavy particle has energy $\epsilon_{q=2Mk_F}=(4M/m)E_F$. These processes will thus freeze out exponentially fast for temperatures below this energy scale. If these were the only processes present, the mobility would diverge exponentially in the $T\to 0$ limit. But other processes will dominate at low temperatures, as we now discuss.

Consider next the $2k_F$ scattering contribution,

$$H_{\mathrm{int},2k_F}=U_0\int_x c^\dagger(x)c(x)[\psi_R^\dagger\psi_L e^{i2k_Fx}+\mathrm{H.c.}]. \tag{4.10}$$

Unfortunately, to leading order this interaction does not contribute to the low-temperature scattering rate. To see this, consider the scattering process that transfers $2k_F$ momentum but zero energy to the heavy particle. Energy and momentum conservation require $\epsilon_k=\epsilon_{k'}$ and $k-k'=2k_F$, where k and k' are the initial and final particle momenta. Together, these imply $k=-k'=k_F$, which corresponds to a large particle energy, $\epsilon_{k_F}=(m/M)E_F$. At temperatures below this energy scale, this process will freeze out.

However, higher-order processes that are generated by $H_{\mathrm{int},2k_F}$ will contribute to the low-temperature scattering. Specifically, consider the interaction term,

$$H_{\mathrm{eff}}=\tilde{\lambda}\int_x c^\dagger(x)c(x)\psi_R^\dagger\psi_R\psi_L^\dagger\psi_L, \tag{4.11}$$

which will be generated by $H_{\mathrm{int},2k_F}$ at second order. The coupling constant is $\tilde{\lambda}=U_0^2/\epsilon_{2k_F}$, where the denominator ϵ_{2k_F} is the energy of the heavy particle in the "intermediate state." This interaction term can be readily bosonized using (2.7) and (2.8), giving

$$H_{\mathrm{eff}}=\lambda\int_x c^\dagger(x)c(x)[(\partial_x\theta)^2-(\partial_x\phi)^2], \tag{4.12}$$

with $\lambda=\tilde{\lambda}/4\pi$.

The scattering rate from the process H_{eff} can be computed using Fermi's golden rule giving

$$\Gamma_{k\to k+q}=a_g[\omega_q^2-(\Delta\epsilon)^2]n_{|\omega_q+\Delta\epsilon|/2}[n_{|\omega_q-\Delta\epsilon|/2}+1], \tag{4.13}$$

with $a_g=(\lambda^2/8Lg^2v^3)(1+g^4)$ and $\Delta\epsilon=\epsilon_{k+q}-\epsilon_k$. As required, Γ satisfies a detailed balance condition, $f_0(k)\Gamma_{k\to p}=f_0(p)\Gamma_{p\to k}$, where $f_0(k)=\mathrm{const}\times e^{-\beta\epsilon_k}$ is the equilibrium momentum distribution function for the heavy particle at temperature T. Notice that this rate has appreciable weight at small energy and momentum transfer, vanishing as a power rather than exponentially. This leads to a power-law dependence of the mobility $\mu(T)$ on temperature, as we now demonstrate.

The mobility can be obtained by solving a Boltzmann equation for the momentum distribution function $f(p,t)$ in the presence of an applied electric field E:

$$\partial_t f(p,t)+E\partial_p f(p,t)=I(p,t). \tag{4.14}$$

As usual, the "collision integral" is expressed in terms of the scattering rates, (4.13), into and out of the state p:

$$I(p)=\sum_k [f(k,t)\Gamma_{k\to p}-f(p,t)\Gamma_{p\to k}]. \tag{4.15}$$

We seek a solution of the form $f(k)=f_0(k)G(k)$, and determine $G(k)$. The collision term can be reexpressed as

$$I(p)=\sum_q \Gamma_{p+q\to p}f_0(p+q)[G(p+q)-G(p)]. \tag{4.16}$$

Due to the Bose factors in (4.13), the scattering rate Γ is a sharply peaked function of the momentum transfer q with width $q\sim T/v$. At low temperatures it is then legitimate to expand both $f_0(p+q)$ and $G(p+q)$ for small q. Moreover, in the low-temperature limit, $\Delta\epsilon$ in (4.13) can be set to zero, and the scattering rate simplifies:

$$\Gamma_{k\to k+q}\to\Gamma_q\equiv a_g\omega_q^2 n_{\omega_q/2}[n_{\omega_q/2}+1]. \tag{4.17}$$

This requires

$$\frac{\Delta\epsilon}{T}\sim(v_Fq/T)(k/Mv_F)\sim\sqrt{T/Mv_F^2}\ll 1, \tag{4.18}$$

where we have used the fact that $v_Fq/T\sim 1$ and $k\sim\sqrt{MT}$. In this low-temperature regime, the collision integral can be written as

$$I(p)=A\partial_p f_0\partial_p G+\tfrac{1}{2}Af_0\partial_p^2 G, \tag{4.19}$$

where we have defined

$$A=\sum_q q^2\Gamma_q=\mathrm{const}\times T^5, \tag{4.20}$$

and used the fact that $\Sigma_q q\Gamma_q=0$.

With this form for the collision integral, the steady-state Boltzmann equation reduces to a differential equation for $G(p)$:

$$A\partial_p G-EG=\frac{M}{2\beta p}(A\partial_p^2 G-2E\partial_p G). \tag{4.21}$$

We now specialize to the linear response limit, for small electric fields. To linear order in E, the terms on the right side can be dropped, and the equation readily integrated to give $G(p)=\mathrm{const}\times e^{Ep/A}$. The momentum distribution function $f=f_0G$ is then given by

$$f(p)=f_0\left(p-\frac{EM}{\beta A}\right). \tag{4.22}$$

The linear response mobility readily follows,

$$\mu = \frac{\langle v \rangle}{E} = \frac{1}{ME} \int_p p f(p) = \frac{1}{\beta A}. \qquad (4.23)$$

Since $A \sim T^5$, we deduce a mobility which diverges as $\mu \sim T^{-4}$. This result agrees with a strong-coupling analysis[6] based on the Brownian motion of solitons and calculations of the diffusion coefficient in real space.[14]

V. CONCLUSION

In this paper we have analyzed the dynamics of a heavy particle moving in a 1D repulsively interacting Luttinger liquid. The behavior of the particle's mobility depends on whether the temperature is larger or smaller than a crossover scale, $T^* \sim (m/M) E_F$. Above T^* the mobility is roughly independent of temperature and proportional to the conductance g of the Luttinger liquid. Below T^* the mobility grows upon cooling, and diverges in the zero-temperature limit as $\mu(T) \sim T^{-4}$. *At* zero temperature, the heavy particle moves ballistically, with a renormalized mass.

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