Momentum Distribution Function of a Narrow Hall Bar in the FQHE Regime

S.-R. Eric Yang[1], Sami Mitra[2], M.P.A. Fisher[3], and A.H. MacDonald[2]

[1]*Department of Physics, Korea University, Seoul 136-701, Korea*

[2]*Department of Physics, Indiana University, Bloomington, Indiana 47405*

[3]*Institute for Theoretical Physics, University of California, Santa Barbara, California 93106*

The momentum distribution function ($n(k)$) of a narrow Hall bar in the fractional quantum Hall effect regime is investigated using Luttinger liquid and microscopic many-particle wavefunction approaches. For wide Hall bars with filling factor $\nu = 1/M$, where M is an odd integer, $n(k)$ has singularities at $\pm M k_F$. We find that for narrow Hall bars additional singularities occur at smaller odd integral multiples of k_F: $n(k) \sim A_p \mid k \pm p k_F \mid^{2\Delta_p - 1}$ near $k = \pm p k_F$, where p is an odd integer $M, M-2, M-4, ..., 1$. If inter-edge interactions can be neglected, the exponent $2\Delta_p = (1/\nu + p^2\nu)/2$ is independent of the width (w) of the Hall bar but the amplitude of the singularity A_p vanishes exponentially with w for $p \neq M$.

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Incompressible states associated with the fractional quantum Hall effect have gapless edge excitations which can be described using a chiral Luttinger liquid model. At $\nu = 1/M$ the momentum distribution functions of these liquids have singular contributions proportional to $|k \pm M k_F|^{M-1}$ where k_F is the Fermi wavevector[1, 2, 3, 4]. The singularities give rise to a number of power laws for the dependence of observables on temperature or on voltage which differ from the power laws which would apply for Fermi liquids and can, at least in principle, be tested experimentally. Initial experiments[5] appear to confirm the Luttinger liquid model predictions. Experimental systems in which Luttinger liquid model predictions are expected to be tested, often involve the formation of electrostatically defined constrictions in the two-dimensional electron gas, which bring opposite sides of the Hall bar into close proximity and enhance inter-edge scattering amplitudes. In this paper we consider the momentum distribution function of such a narrow Hall bar systems. Using both Luttinger liquid model and microscopic trial wavefunction approaches, we find that additional singularities arise which are absent in the limit of large Hall bars. The presence of these additional singularities may play an important role in interpreting transport and other experiments in narrow Hall bars.

The Lagrangian of the chiral Luttinger liquid model[1, 2] for the edge excitations of a Hall bar system occupied by an incompressible state with $\nu = 1/M$ is

$$L = \frac{1}{4\pi\nu}\{\partial_x\phi_R(i\partial_\tau + v\partial_x)\phi_R + \partial_x\phi_L(-i\partial_\tau + v\partial_x)\phi_L\} \tag{1}$$

This Lagrangian describes independent excitations at the right(R) and left(L) edges with oppositely directed velocities of magnitude v. The fields ϕ_R and ϕ_L satisfy the commuation rules

$$[\phi_R(x), \phi_R(x')] = -[\phi_L(x), \phi_L(x')] = i\pi\nu sgn(x - x') \tag{2}$$

The operators for creating an electron and a fractionally charged particle with charge νe at right (R) and left (L) edges may be written as $\Psi^{\dagger}_{R,L}(x) = exp(i\phi_{R,L}(x)/\nu)$ and $exp(i\phi_{R,L}(x))$, respectively. The singularites in $n(k)$ at $\pm M k_F$ follow from this form for the electron creation operator. The criteria used to construct these operators are that they create the appropriate amount of charge at the edge and that they have the correct statistics. For the electron creation operator there is abundant numerical evidence[6] that this *ansatz* is correct in wide Hall bars. Although the true microscopic creation operator evidently contains additional contributions in general, these cannot be expressed in terms of the low-energy edge degrees of freedom and will will not give rise to singularities in $n(k)$. We believe that for narrow Hall bars the microscopic electron creation operator *will* have additional low-energy edge contributions. Additional Boson operators with Fermi statistics and unit charge can be created by partitioning the charge between n quasiparticles located at the right edge and l quasiparticles located at the left edge with $n + l = M$:

$$\Psi^{\dagger}_{n,l}(x) = exp(in\phi_R)exp(il\phi_L) \tag{3}$$

where n+l=M. The propagator of this object may be calculated using the standard bosonization techniques:[1, 2]

$$< \Psi_{n,l}(x)\Psi^{\dagger}_{n,l}(0) > \sim \frac{exp(i(n-l)k_F x)}{x^{2\Delta_{n,l}}} \tag{4}$$

Note that since $n + l = M$ is odd, $p = n - l$ must also be odd. By Fourier transforming $< \Psi_{n,l}(x)\Psi^{\dagger}_{n,l}(0) >$ we see that $n(k)$ has power-law singularities at $\pm p k_F$ with the exponents $2\Delta_p - 1$, where $p = M, M-2, M-4, ..., 1$ and

$$2\Delta_{n,l} = \frac{1}{2}(\frac{1}{\nu} + (n-l)^2\nu) \tag{5}$$

We see that as $p = n - l$ decreases the exponent $2\Delta_{n,l}$ also decreases. When the width of the Hall bar greatly exceeds microscopic lengths, added electrons will be clearly associated with either left or right edges and the contribution of $\Psi^{\dagger}_{n,l}$ to the microscopic electron creation operator should become small unless $n = 0$ or $l = 0$.

To help verify that these new operators do in general contribute to the microscopic electron creation operator we consider a specific convenient microscopic model of spinless fermions with strong repulsive short-range interactions in a circularly symmetric confining potential which confines the electrons to an annulus. Under appropriate circumstances the exact ground state of such a model is given by a many-fermion wavefunction of the form

$$\Phi_{M,L}[z] = \prod_{k=1}^{N} z_k^L \Phi_M[z] \tag{6}$$

where L is an integer, N is the number of electrons, and $\Phi_M[z]$ is the Laughlin wavefunction at filling factor $\nu = 1/M$. Here a symmetric gauge is used so that the single particle wavefunctions $\phi_m(z)$ are labelled by angular momenta m and are localized around circles of radius $R_m = \ell\sqrt{2(m+1)}$ where ℓ is the magnetic length. In this wavefunction all the single particle states from $m = L$ to $L + M(N-1)$ are occupied. Electrons are confined to an annulus with inner and outer radii, R_1 and R_2, equal to $\ell(2(L+1))^{1/2}$ and $\ell(2(L+M(N-1)+1))^{1/2}$ respectively. For $L \gg 1$, the width of the annulus w is $\approx \ell\frac{M(N-1)-1}{(2L)^{1/2}}$. For $L = 0$ the innner radius of the annulus shrinks to zero and the wavefunction reduces to the Laughlin wavefunction for the ground electronic state of a circular electron droplet whose outer edge can be described by a Luttinger liquid model. The wavefunction at finite L can be thought of as being generated by creating L fractionally charged quasiholes at the center of the droplet. For large L at fixed N, the difference between inner and outer radii of the electronic annulus becomes small so that the wavefunction is the ground state wavefunction for a model of a narrow Hall bar. In the limit of infinite L, the annulus becomes arbitrarily narrow and the wavefunction becomes equivalent to that for the Calogero-Sutherland[7] model of a one-dimensional electron gas. Related connections between Laughlin and Calogero-Sutherland models have been discussed previously.[4, 8] We note, for example, that the exponents characterizing the singularities in $n(k)$ predicted by the Luttinger liquid model above are identical to those of the of the Calogero-Sutherland one-dimensional model.[9]

For this circular geometry, the momentum distribution function of a Hall bar maps to the angular momentum distribution[3] of the annulus which is given by

$$< n_m >= \int d^2z d^2z' \phi_m^*(z) n(z,z') \phi_m(z'). \tag{7}$$

Here $n(z,z')$ is the one-body density matrix for the ground state of the system, which we have computed using a Monte Carlo method. We identify $< n_m >$ and $\frac{m-m_c}{N/2}$ with $n(k)$ and k/k_F, where $m_c = L + M(N-1)/2$ is the angular momentum at the center of the annulus. Then the wavevectors corresponding to the inner and outer edges are $k/k_F = -M(N-1)/N$ and $k/k_F = M(N-1)/N$.

It is possible to calculate $n(k)$ analytically for k near Mk_F by following Wen[1] and using a plasma analogy. The total potential energy U of the system is

$$\frac{-U}{M} = \sum_{i<j} 2M \ln|z_i - z_j| + 2L \sum_k \ln|z_k| - M \sum_k |z_k|^2/2 \tag{8}$$

where first, second, and third terms represent, respectively, the mutual Coulomb interaction energy, the external potential energy due to the charge L at the origin, and the potential energy due to the uniform positive background charge. The potential energy of a particle placed at $r >> R_2$ is a sum of the potential energies due to the fixed charges (background charges and electrons in the annulus)

$$V_{fix}(r) = \frac{Mr^2}{2\ell^2} - \frac{MR_2^2}{\ell^2} ln(r/R_2) \tag{9}$$

and the screening charge

$$Vsc(r) = Mln|r - R_2^2/r| \tag{10}$$

The electron density may be evaluated using $\rho(r) \propto e^{-(V_{fix}(r)+Vsc(r))/M}$. Equating $\rho(\mathbf{r}) = \sum_m n_m |\phi_m(\mathbf{r})|^2$ we find $n(k) \propto (k \pm Mk_F)^{M-1}$ near $k = \pm Mk_F$.

Fig.1 displays $n(k)$ vs k/k_F for M=3 at two values of $w = 1.24\ell$ and 2.52ℓ. To extract the exponent at $k = \pm k_F$ we use a scaling ansatz for the momentum distribution function of a finite sample with length R

$$n(k,R) \propto R^{-\alpha} f(|k - k_F|R) \tag{11}$$

where $R = 2\pi\ell\sqrt{2(m_c+1)}$, $|\ k - k_F\ | /k_F \ll 1$, and the scaling function $f(x) \sim x^\alpha$ for large x. The difference between $n(k)$ at two values of k adjacent to k_F is

$$\Delta n = n(k_F^+, R) - n(k_F^-, R) = g(w) R^{-\alpha} \tag{12}$$

where k_F^+/k_F and k_F^-/k_F are $1 + 1/N$ and $1 - 1/N$. Analysing our numerical data accordingly we find $\alpha = 0.6 \pm 0.1$. This estimate is consistent with the exact result 2/3. We have also investigated the width dependence $g(w)$ and find that it is approximately exponential: $g(w) \sim exp(-w/a)$ with a about 1.6ℓ. The exponent at $k = \pm 3k_F$ is numerically verified to be 2, in agreement with the analytical result. In Fig.2 we plot n(k) vs $(\frac{k}{k_F} + 3)^{2.4}$ for M=5. From the linear dependence we deduce that the relevant exponent is 2.4, in agreement with the Luttinger liquid model predictions.

In summary, we conclude on the basis of both microscopic many-particle wavefunctions and Luttinger liquid model that in a narrow Hall bar $n(k)$ has singularities at $k = \pm pk_F$, where $p = M, M-2, M-4, ..., 1$ and that the magnitude of the corresponding exponents decreases with decreasing p. For both calculations we have worked with models in which inter-edge interactions are absent so that the exponents are independent of the Hall bar width, while the amplitudes of the singularities vanish exponentially with w for $p \neq M$. These new and stronger singularities will be important[10] in the interpretation of experimental searches for Luttinger liquid behavior in narrow Hall bars.

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FIG. 1: $n(k)$ is plotted vs k/k_F for $w = 1.24\ell$ and 2.52ℓ. Data collapse is seen for each value of w. The plotting symbols are triangles, circles, squares, diamonds, triangle lefts, and triangle downs and their (N, M, L) values are (3,3,8), (4,3,21), (5,3,40), (6,3,65), (6,3,10), and (7,3,16), respectively.

FIG. 2: $n(k)$ is plotted vs $(\frac{k}{k_F}+3)^{2.4}$ for (N,M,L)= (9,5,218). The linear dependence implies that the relevant exponent is 2.4.

[1] X.G. Wen, Int. J. Mod. Phys. B **6**, 1711 (1992).

[2] C.L.Kane, M.P.A. Fisher, Phys. Rev. **B 48**, 15233 (1992); K.Moon, H. Yi, C.L.Kane, S.M. Girvin, M.P.A. Fisher, Phys. Rev. Lett., **71** 4381 (1993).

[3] Sami Mitra and A.H. MacDonald, Phys. Rev.B **48**, 2005 (1993).

[4] E.H. Rezayi and F.D.M. Haldane, Phys. Rev. B. **50**, 17199 (1994).

[5] F.P. Milliken, C.P. Umbach, and R.A. Webb, Solid State Comm. (in press); B.G. Levi, Physics Today **47**, 21 (1994).

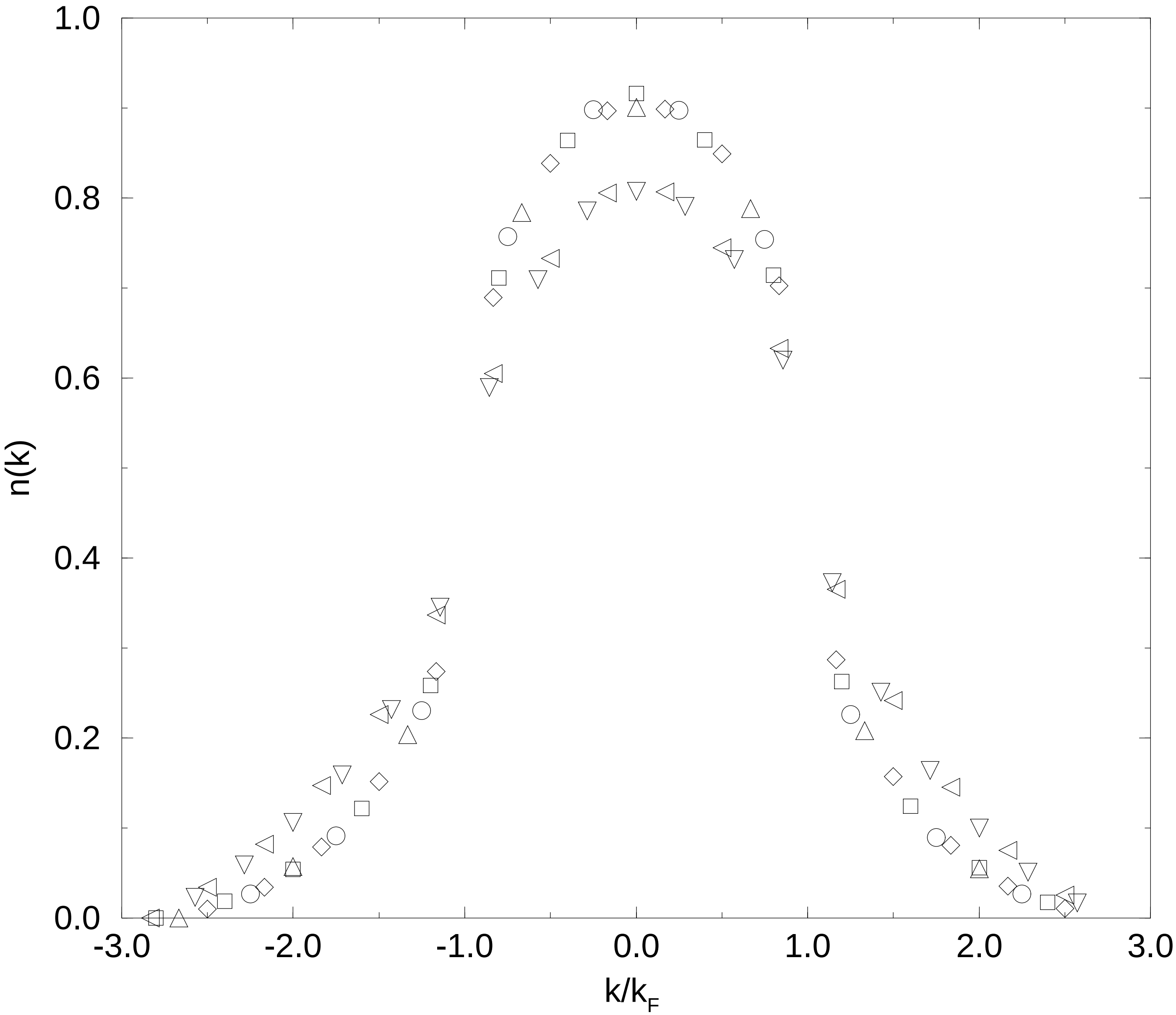
[6] J.J. Palacios and A.H. MacDonald, Phys. Rev. Lett. to appear, (1995) and work cited therein.

[7] F. Calogero, J. Math. Phys. **10**, 2191 (1969); B. Sutherland, J. Math. Phys. **12**, 246 (1971).

[8] Michael Stone and Matthew P.A. Fisher, Int. J. of Mod. Phys. B **8**, 2539 (1994).

[9] N. Kawakami and S.-K. Yang, Phys. Rev. Lett. **67**, 2493 (1991).

[10] S.-R. Eric Yang, Sami Mitra, Matthew P.A. Fisher, and A.H. MacDonald, in preparation.



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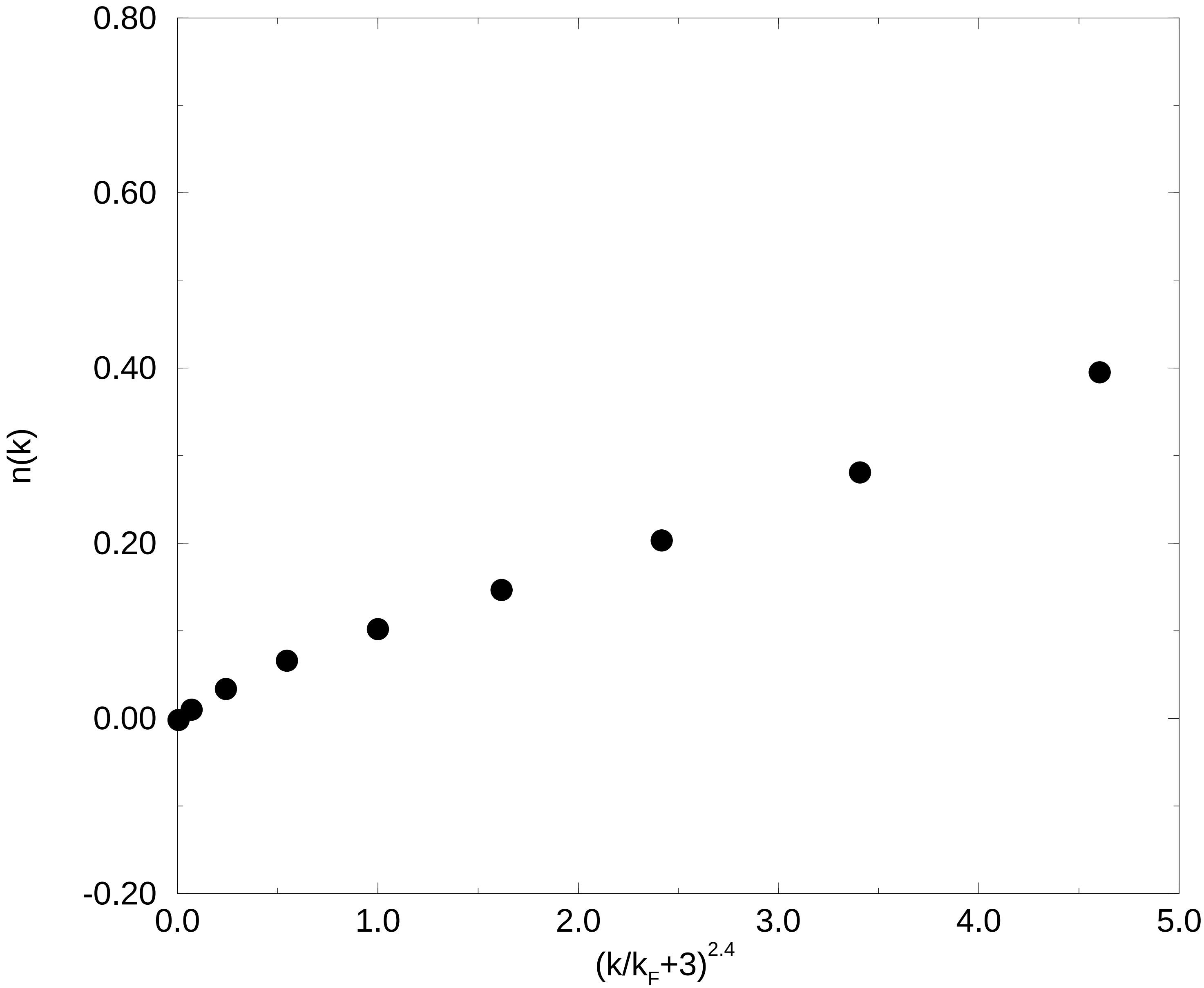
2.0
3.0
k/k_F



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-0.20
n(k)
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1.0
2.0
3.0
4.0
5.0
$(k/k_F+3)^{2.4}$