Short-range interactions and scaling near integer quantum Hall transitions

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We study the influence of short-range electron-electron interactions on scaling behavior near the integer quantum Hall plateau transitions. Short-range interactions are known to be irrelevant at the renormalization group fixed point which represents the transition in the noninteracting system. We find, nevertheless, that transport properties change discontinuously when interactions are introduced. Most importantly, in the thermodynamic limit the conductivity at finite temperature is zero without interactions, but nonzero in the presence of arbitrarily weak interactions. In addition, scaling as a function of frequency ω and temperature T is determined by the scaling variable ω/T^p (where p is the exponent for the temperature dependence of the inelastic scattering rate) and not by ω/T, as it would be at a conventional quantum phase transition described by an interacting fixed point. We express the inelastic exponent p and the thermal exponent z_T in terms of the scaling dimension $-\alpha<0$ of the interaction strength and the dynamical exponent z (which has the value $z=2$), obtaining $p=1+2\alpha/z$ and $z_T=2/p$.

I. INTRODUCTION

In this paper we study the effects of short-range interactions on the nature of the transitions between quantized Hall plateaus in a disordered two-dimensional electron gas (2DEG).[1] These transitions are generally believed to be prime examples of continuous quantum phase transitions, that is to say, examples of quantum critical phenomena.[2–5] We focus here on samples with sufficiently strong disorder that fractional quantum Hall states do not intervene, so that the transitions are directly from one integer Hall plateau to another. Recently, Shahar and collaborators have presented an analysis of transport measurements that would seem to indicate an absence of a true quantum Hall liquid–insulator phase transition.[6] The full implications of this are unclear at present, but we presume that this is an indication of the difficulty of reaching the asymptotic quantum critical regime in certain classes of disordered systems and will not consider it further in this paper.

The existence of quantized Hall plateaus is intimately related to the presence of disorder. In a single-particle description, all states are localized except for those at a single critical energy near the center of each Landau level. Thus the quantum phase transition is an unusual insulator to insulator transition with no intervening metallic phase. The critical point itself is quasimetallic, exhibiting anomalous diffusion.[7] Associated with each transition between plateaus in σ_{xy} there is a peak in σ_{xx} which in principle becomes infinitely sharp at zero temperature (see however Ref. 6) and whose peak value is universal and close to[8] $0.5e^2/h$. However, as we discuss below, since we have the peculiar circumstance that the set of extended states has measure zero, the zero-temperature limit is quite singular in the absence of interactions. In the noninteracting case σ_{xx} is actually rigorously zero in the limit of large sample size at all values of the magnetic field, including the critical values, for any nonzero temperature. Moreover, it has been argued previously, using a combination of renormalization group techniques and numerical calculations,[9] that interactions of sufficiently short range are perturbatively irrelevant at the noninteracting fixed point. Hence systems with short-range interactions scale into this singular noninteracting limit. We show in this paper that although interactions are irrelevant in this sense, they generate a nonzero critical value of σ_{xx} and determine the nature of temperature and frequency scaling near the critical point. We expect that interactions have similar consequences near other delocalization transitions at which they are formally irrelevant, although behavior in a different category is possible if interactions are sufficiently strongly irrelevant. We note that irrelevant interactions which control dynamical properties at a quantum critical point have been encountered previously, in the theory of metallic spin glasses.[10]

In contrast to short-range, model interactions, true Coulomb interactions are believed to be relevant at the noninteracting fixed point.[9] Hence one expects that the true critical point is interacting. One of the persistent mysteries in this problem is the fact that the experimentally observed value of the correlation exponent ν at the interacting fixed point appears to agree rather well with that predicted by numerical simulations of the noninteracting fixed point.[9,11] That is, the correlation length exponent does not appear to change even though the value of the dynamical critical exponent z is believed to change from $z=1$ for long-range interactions to $z=2$ for the short-range case.[4] In the following, we do not consider this issue, and instead restrict our attention to short-range interactions. The Coulomb interaction can be made short range by placing a metallic screening gate (ground plane) nearby. Such a situation was successfully realized by Van Keuls *et al.*[12] although they did not study the quantum critical point, but rather the insulating phase at densities well below the $0\rightarrow 1$ plateau transition. They observed that the variable range hopping exponent changed from the Efros-Shklovskii value expected for long-range interactions to the Mott value expected for short-range interactions.

The remainder of the paper is organized as follows. We summarize the scaling description of the quantum Hall plateau transitions in the next section, and discuss in Sec. III the pathologies associated with the finite temperature scaling behavior of the conductance in the noninteracting theory. From Sec. IV onward, systems with short-range interactions are considered. We first describe dephasing in the critical regime and the emergence of a long coherence time, and determine the inelastic exponent p and the thermal exponent $z_T \neq z$ in terms of the scaling dimension of the interactions. The difficulties arising from a direct application of conventional scaling ideas are discussed. In Sec. V, finite temperature scaling is analyzed in the presence of short-range interactions. We show that, although short-range interactions are formally irrelevant, they control aspects of the critical behavior. We demonstrate that the critical conductivity is nonzero provided interactions are not too strongly irrelevant. Finally, we construct new scaling variables and examine to what extent conductance scaling can be forced into the conventional scaling framework. Finite frequency scaling at $T=0$ is discussed in Sec. VI and the general scaling in temperature and frequency in Sec. VII. Concluding remarks are presented in Sec. VIII.

II. PLATEAU TRANSITIONS AND SCALING THEORY

The integer quantum Hall transition (IQHT) is driven by varying the location of the chemical potential μ relative to the critical value μ_c. Throughout this paper we denote the distance from the critical point by $\delta=|\mu-\mu_c|$. Since μ_c is dependent on magnetic field B, the transition is often reached experimentally by changing B while keeping electron density fixed. In the large B limit, μ_c lies near the center of the Landau levels. A body of experimental data, reviewed for example in Ref. 4, can be summarized by the statements that (i) on either side of the transition ($\delta\neq 0$) the Hall conductivity is quantized and the dissipative conductivity has the limit $\sigma_{xx}\rightarrow 0$ at zero temperature; (ii) at the transition ($\delta=0$) the Hall conductivity is unquantized and σ_{xx} remains finite at zero temperature, so that the critical state is conducting.

Critical behavior is cut off in the presence of a finite length scale. In this event, the transition has a finite width δ^* within which the Hall conductivity deviates from the quantized values and σ_{xx} is nonzero. This width is

$$\frac{\delta^*}{\delta_0}\sim\min\left[\left(\frac{L_0}{L}\right)^{1/\nu},\left(\frac{T}{T_0}\right)^{1/z_T\nu},\left(\frac{\omega}{\omega_0}\right)^{1/z\nu}\right], \tag{2.1}$$

where L, T, and ω are the finite system size, temperature, and measurement frequency in a specific experimental situation, and δ_0, L_0, T_0, and ω_0 are microscopic scales. The various exponents appearing in Eq. (2.1) have the following meaning: ν is the exponent of the single divergent length scale, the localization length $\xi\sim\delta^{-\nu}$; z is the dynamical exponent defining the length scale introduced by a finite frequency, $L_\omega\sim\omega^{-1/z}$; and z_T is the thermal exponent governing a temperature-dependent length scale $L_\varphi\sim T^{-1/z_T}$. In the conventional dynamical scaling description of a quantum phase transition in which interactions are relevant and scale to a finite strength at the transition, z_T is expected to be the same as z. All the three regimes in Eq. (2.1) have been probed experimentally,[2,13–15] as well as the regime in which electric field strength sets the cutoff.[16] Summarizing the results in the form in which they appear in the literature, we have $\nu=2.3\pm 0.1$, $1/z_T\nu=0.42\pm 0.04$, and $1/z\nu=0.41\pm 0.04$. This suggests that $z_T=z=1$, which is consistent with the interpretation that the Coulomb interaction is relevant at the transition. More generally, z_T and z may be independent exponents at a quantum phase transition. We show in the following that this is the case at the IQHT if the interaction scales to zero at the critical point. This happens for short-range interactions and could be realized experimentally by screening out the long-ranged Coulomb interaction with nearby ground planes or gates.

We now turn to recent theoretical developments. The Hamiltonian of interest describes interacting electrons moving in a two-dimensional random potential in the presence of a magnetic field:

$$H=\sum_i\left[\frac{1}{2m}\left(\vec{p}_i+\frac{e}{c}\vec{A}\right)^2+V_{\mathrm{imp}}(\vec{r}_i)\right]+\frac{1}{2}\sum_{i\neq j}V(\vec{r}_i-\vec{r}_j), \tag{2.2}$$

where $\vec{A}$ is the external vector potential, V_{imp} is the one-body impurity potential, and V is the two-body interaction potential. We write

$$V(\vec{r}_i-\vec{r}_j)=\frac{u}{|\vec{r}_i-\vec{r}_j|^{\lambda}}, \tag{2.3}$$

where u and λ parametrize the strength and the range of the interaction.[17] The existence of the IQHT in the model is not dependent on interactions, and the noninteracting theory, obtained by setting $u=0$, provides a simplified but concrete model that has allowed extensive quantitative calculations.[18,19] A good understanding of the main features of the noninteracting critical point has emerged: the static localization length exponent has the value $\nu\approx 2.33\pm 0.03$ and the dynamical exponent is $z=d=2$. However, the relevance of the free electron model to the IQHT in real materials depends on the nature and the effects of electronic interactions.

Imagine starting with a system at the noninteracting fixed point (NIFP), and switching on the interaction. One can ask whether this interaction is a relevant or irrelevant perturbation in the renormalization group (RG) sense. Such a stability analysis of the NIFP has been performed.[9] For the unscreened Coulomb interaction, $\lambda=1$ in Eq. (2.3), u has RG scaling dimension 1 and is therefore a relevant perturbation.

The resulting flow away from the NIFP presumably leads to another, interacting fixed point (IFP) at which the effective interaction strength is finite. Critical phenomena in this case should be described by conventional dynamical scaling theory with two independent critical exponents, z and ν, and $z_T = z$. While one expects that $z=1$ on general grounds with Coulomb interactions,[20] the value of ν is unknown and may be different from the value at the NIFP. Nevertheless, a scenario whereby Coulomb interaction changes z but not ν from the noninteracting values has been conjectured.[9,21] An alternative possibility[22,23] is that there are two divergent lengths at the critical point, with different exponents.

We shall not consider long-range Coulomb interactions further. Instead, we focus on the case of short-range interactions having $\lambda > 2$. As mentioned above, this case is physically relevant when the IQHT is studied in the presence of ground planes or metallic gates. It has been shown that for screened Coulomb interactions with $\lambda > 2 + x_{4s}$, $x_{4s} \simeq 0.65$, the RG dimension of u is $-\alpha = -x_{4s}$, so that interactions are an irrelevant perturbation.[9] Notice that, in particular, the dipole-dipole interaction has $\lambda = 3$ and thus belongs to this class of interactions. Moreover, for $x_{4s} > \lambda - 2 > 0$, the interaction is still irrelevant with the scaling dimension $-\alpha = 2 - \lambda$.[9,24] In all these cases, the effective interaction scales to zero at the transition in the asymptotic limit. The NIFP is therefore stable against interactions. As a result, $\nu \approx 2.33$ and $z=2$. It turns out, though, that short-range interactions, although irrelevant, control the finite temperature behavior of the conductance. As we shall see, the scaling function for the conductance is discontinuous at zero interaction strength when written in terms of a natural set of scaling variables. We will show that the scaling theory thus becomes unconventional, and a third independent critical exponent, the thermal exponent z_T, emerges in the scaling arguments. The value of z_T is set by the scaling dimension α of the interaction strength: consideration of the dephasing time in the critical regime leads to $z_T = 2z/(z+2\alpha)$. Since z_T determines the transition width in the temperature scaling regime [cf. Eq. (2.1)], experiments can, in principle, determine the scaling exponent α. We find, on the other hand, that the frequency scaling of the conductance in this case is conventional, with $z=d=2$, where d is the spatial dimension of the system. We argue that quantum critical scaling behavior of this kind may be a general feature of finite temperature transport near quantum critical points, when interactions are irrelevant. The central feature is the existence of a time scale, the dephasing time $\tau_\phi \sim T^{-p}$ where $p = 1 + 2\alpha/z$, which is longer than the single characteristic time, $\hbar/T$, at a conventional quantum phase transition. The long coherence time results from the underlying free fermion description and its associated infinite number of conservation laws. As a result, for $\omega, T \neq 0$, the ω/T scaling in conventional quantum phase transitions[4] is replaced by ω/T^p scaling.[25]

III. NONINTERACTING THEORY, $u=0$

A. $T=0$

We begin by describing the finite size scaling of the zero-frequency conductance in the absence of interactions.[26] Consider a 2D square sample of size $L \times L$. At $T=0$, the dimensionless conductance should depend only on L/ξ. Measuring the conductance in units of e^2/h, we write

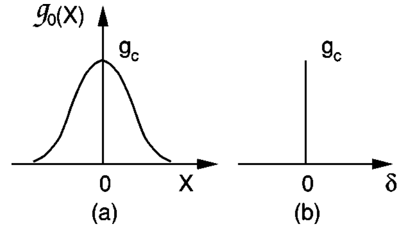

FIG. 1. (a) The conductance scaling function defined in Eq. (3.2), for the noninteracting theory. (b) The behavior of the conductance in the noninteracting theory, in the thermodynamic limit at zero temperature.

$$g(\delta, L) = \mathcal{G}_0(\delta L^{1/\nu}). \tag{3.1}$$

The scaling function $\mathcal{G}_0$ has the limiting behavior

$$\mathcal{G}_0(X) = \begin{cases} g_c, & X \to 0 \\ 0, & X \to \infty, \end{cases} \tag{3.2}$$

where g_c is a critical conductance at the transition. This quantity is expected to be universal for a given geometry and boundary conditions.[8,29–31] In phase coherent, square samples under periodic transverse boundary conditions, $g_c \simeq 0.5$. The behavior of $\mathcal{G}_0(X)$ is known from numerical work in various settings,[27,28] and in most detail for square samples from transfer matrix calculations of the two-terminal Landauer conductance:[30,31] the results of these are sketched in Fig. 1(a). It decays exponentially for large X, according to $\mathcal{G}_0(X) \sim \exp(-cX^\nu)$, where c is a constant. Hence, in the limit $L \to \infty$, g is zero for all δ except $\delta = 0$ at which it has the finite value g_c, as shown in Fig. 1(b). We will denote the conductance in the thermodynamic limit, the quantity of interest throughout the paper, by suppressing the L dependence in its argument. Thus

$$g(\delta) = \begin{cases} g_c, & \delta = 0 \\ 0, & \text{otherwise.} \end{cases} \tag{3.3}$$

B. $T \neq 0$

For noninteracting electrons, the conductivity at $T \neq 0$ is

$$\sigma_{xx}(\delta, T, L) = \int dE \left(-\frac{\partial f}{\partial E} \right) \mathcal{G}_0(EL^{1/\nu}), \tag{3.4}$$

where $\mathcal{G}_0$ is the $T=0$ conductance scaling function given in Eq. (3.1), and $f(E)$ is the Fermi-Dirac distribution function

$$f(E) = \frac{1}{e^{\beta(E-\delta)} + 1}. \tag{3.5}$$

Equation (3.4) is a convolution of the derivative of the Fermi function (which has width $k_B T$) with the $T=0$ conductance scaling function (which has width $L^{-1/\nu}$), as illustrated in Fig. 2. In the limit $L \to \infty$, Eqs. (3.3) and (3.4) imply that

$$\sigma_{xx}(\delta, T) = 0 \tag{3.6}$$

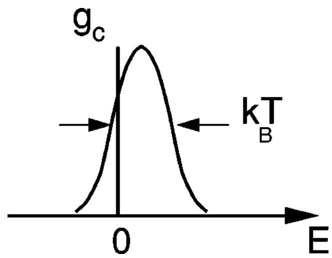

FIG. 2. The convolution of $(-\partial f/\partial E)$ with $\mathcal{G}_0$ in the thermodynamic limit leads to a vanishing conductivity at finite temperature in the noninteracting theory.

for any δ if $T\neq 0$: within the noninteracting theory, the conductivity vanishes for all values of the Fermi energy at finite temperature. This strange result follows from the fact that the set of conducting states is of measure zero for this transition.

IV. SHORT-RANGED INTERACTION, $u\neq 0$

For the conductivity to be nonzero at finite temperatures near the transition, interactions are necessary, and we now examine the effect of short-range interactions. Since u is an irrelevant coupling in the RG sense, the transitions at $T=0$ are described by the noninteracting fixed point. In general at such a fixed point, provided the density of states is finite, $z=d$ in d dimensions, and so for the IQHT $z=2$. Under a RG length scale transformation b, u transforms according to $u'=b^{-\alpha}u$, and energy scales ϵ transform as $\epsilon'=b^z\epsilon$.

A. Naive scaling at $\delta=0$

The finite temperature conductivity at criticality is expected to have the scaling form

$$\sigma_{xx}(T,u)=b^{2-d}\mathcal{G}'(b^zT,b^{-\alpha}u). \tag{4.1}$$

Choosing the scale factor $b=T^{-1/z}$, we obtain a new scaling function

$$\sigma_{xx}(T,u)=\mathcal{G}(uT^{\alpha/z}). \tag{4.2}$$

Equation (3.6) implies, setting $u=0$, that $\mathcal{G}(X=0)=0$.

If u were a conventional irrelevant scaling variable $\mathcal{G}$ would have a power series expansion and one could write

$$\sigma_{xx}(T,u)=\mathcal{G}(0)+\sum_{l=1}^{\infty}(uT^{\alpha/z})^l\mathcal{G}_l(0). \tag{4.3}$$

Since $\mathcal{G}(0)=0$, Eq. (4.3) implies that $\sigma_{xx}(T\rightarrow 0,u)=0$. This result would, paradoxically, exclude the existence of a conducting critical state. In fact, as we show in the following sections, $\mathcal{G}(X)$ is a discontinuous function of its argument X at $X=0$ so that

$$\sigma_{xx}(T\neq 0,\ u=0)=\mathcal{G}(X=0)=0, \tag{4.4}$$

$$\sigma_{xx}(T\rightarrow 0,\ u\neq 0)=\mathcal{G}(X\rightarrow 0)=g_c. \tag{4.5}$$

This discontinuous behavior is shown schematically in Fig. 3.

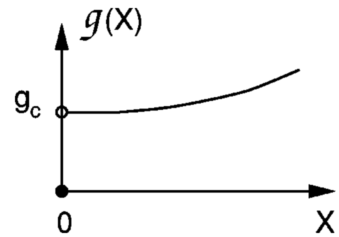

FIG. 3. Discontinuity of the scaling function $\mathcal{G}(X)$, Eq. (4.2), at $X=0$.

B. Dephasing in the critical regime by interactions

For $T\neq 0$, interactions, relevant or irrelevant in the RG sense, will cause transitions between single-particle states.[32] This leads to a finite quasiparticle dephasing rate[33] $\tau_\varphi=T^{-p}$. At a quantum phase transition, the exponent p that enters the dephasing rate should not be taken from those for simple disordered metals in the large conductance regime, for it is the decay time of the critical eigenstates that matters. This should be determined by the underlying critical phenomena. A natural scaling form for the dephasing rate is

$$\frac{h}{\tau_\varphi}=TY'(b^zT,b^{-\alpha}u), \tag{4.6}$$

where the prefactor T is determined by the engineering dimension of $1/\tau_\varphi$. Setting $b=T^{-1/z}$, we have

$$\frac{h}{\tau_\varphi}=TY(uT^{\alpha/z}). \tag{4.7}$$

As u is an irrelevant coupling (perturbation) which scales toward zero under renormalization group scale transformations, the unperturbed state (noninteracting fixed point) is therefore analytically connected to the perturbed state in the presence of u. Thus, a perturbative expansion in u is justified. To lowest order, $1/\tau_\varphi\sim u^2$ from a Fermi's golden rule estimate of the inelastic scattering rate. Thus, the expected leading scaling behavior is

$$\frac{1}{\tau_\varphi}\sim u_{\mathrm{eff}}^2T\sim u^2T^{1+2\alpha/z}, \tag{4.8}$$

or

$$\tau_\varphi\sim T^{-p},\ \ p=1+\frac{2\alpha}{z}. \tag{4.9}$$

For the case of a quantum Hall transition in the presence of a screening gate, we have $z=2$ and $\alpha\simeq 0.65$, and we obtain $p\simeq 1.65$.

C. Dephasing length and thermal exponent z_T

For a conventional quantum phase transition (with finite interaction strength at the fixed point), there is one length scale $(\xi\sim\delta^{-\nu})$ and one time scale $(\Omega^{-1}\sim\xi^z\sim\delta^{-z\nu})$ away from criticality. There are no finite correlation length or time scales at criticality.[4] In such a critical system at finite temperature T, one expects to have one characteristic time $\hbar/T$, the significance of which is particularly clear in imaginary time, where it sets a finite size in the time direction, as shown in Fig. 4. However, in the present case, we have ob-

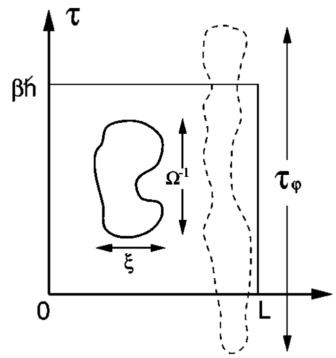

FIG. 4. Schematics of the time and length scales close to a quantum phase transition. The correlation volume (in space and time) is indicated by the full line for an interacting fixed point. The corresponding volume is indicated by a dashed line in the case where interactions are irrelevant and a (real) coherence time, $\tau_\varphi \gg \hbar/T$, emerges.

tained an additional (real) time τ_φ, which is much larger than $\hbar/T$ as $T\to 0$, provided $p>1(\alpha>0)$, which is the case if interactions are irrelevant. For further discussion of quantum critical transport in the incoherent long time limit see Ref. 5.

We now turn to the dephasing length L_φ associated with τ_φ. The irrelevance of the interaction at the NIFP allows us to view the system in terms of weakly interacting diffusive quasiparticles. The dephasing length that cuts off the phase coherent dc transport is thus

$$L_\varphi=\sqrt{D\tau_\varphi}\sim T^{-p/2}, \tag{4.10}$$

where D is the diffusion constant at the noninteracting critical point, obtained from the wave vector q and frequency ω dependent coefficient $D(q,\omega)$ in the limit first $q\to 0$ and then $\omega\to 0$. Thus, anomalous diffusion[7] present in the opposite limit will not enter our discussion. We show below that, even though u is irrelevant in the RG sense, the important length scale introduced by temperature is L_φ, so that

$$L_\varphi\sim T^{-1/z_T}, \tag{4.11}$$

$$z_T=\frac{2}{p}=\frac{2z}{z+2\alpha}. \tag{4.12}$$

This length enters the scaling of the transition width in Eq. (2.1). For the IQHT in the presence of short-range interactions, we thus obtain $z_T\simeq 1.21$.

V. TEMPERATURE SCALING OF CONDUCTIVITY NEAR CRITICALITY

To calculate the conductivity in the presence of a finite dephasing length, we follow the standard procedure and divide the system into $L_\varphi\times L_\varphi$ phase coherent blocks. Transport within each block can be described by phase coherent single-electron transport using the underlying noninteracting theory. The disorder-averaged conductivity that we are interested in can be obtained by averaging over the phase coherent blocks. The outcome of this exercise is that the system size L in Eq. (3.4) should be replaced by L_φ, which leads to

$$\sigma_{xx}(\delta,T,u)=\int dE\left(-\frac{\partial f}{\partial E}\right)\mathcal{G}_0(EL_\varphi{}^{1/\nu}), \tag{5.1}$$

where $\mathcal{G}_0$ is a scaling function. Although the precise phase coherent geometry appropriate for this averaging procedure is unclear, this scaling function is expected to have the same *qualitative* behavior as $\mathcal{G}_0$ in Eq. (3.4). Note that this discussion omits contributions to transport from variable range hopping, which will in fact dominate when $\mathcal{G}_0$ is very small.

Let $x=\beta(E-\delta)$. We then have

$$\sigma_{xx}(\delta,T,u)=-\int dx\,\frac{\partial f(x)}{\partial x}\mathcal{G}_0(xk_{\mathrm{B}}TL_\varphi{}^{1/\nu}+\delta L_\varphi{}^{1/\nu}), \tag{5.2}$$

where $f(x)=1/(e^x+1)$.

A. At criticality: $\delta=0$, $T\to 0$

We first study the behavior of the critical conductivity at low temperatures. At $\delta=0$, the second term in the argument of $\mathcal{G}_0$ in Eq. (5.2) vanishes, leading to

$$\sigma_{xx}(\delta=0,T,u)=-\int dx\,\frac{\partial f(x)}{\partial x}\mathcal{G}_0[x(T/T_0)^{1-p/2\nu}], \tag{5.3}$$

where $T_0\sim(u^2/D)^{1/(2\nu-p)}$ is a constant determined by the bare interaction strength and the diffusion constant. To understand the behavior of σ_{xx} that results from Eq. (5.3), one should compare the width of the thermal window, determined by $-(\partial f/\partial x)$, with the width of the window over which electrons are mobile, determined by the scaling function $\mathcal{G}_0(X)$ [see Fig. 1(a)]. There are two different low-T behaviors for σ_{xx}, depending on the value of $p/2\nu$.

1. $p<2\nu$: the case of IQHT

For $p<2\nu$, the argument of the scaling function in Eq. (5.3) approaches zero as $T\to 0$. Thus, using Eq. (3.2), we have

$$\sigma_{xx}(\delta=0,\ T\to 0,u)\simeq\mathcal{G}_0(X\to 0)=g_c. \tag{5.4}$$

In this case, the low-T conductance is finite [cf. Eq. (4.5)] (despite the fact that the set of conducting states is of measure zero) and has a value comparable to the critical phase coherent conductance in the noninteracting theory. Hence interactions control the low-temperature behavior, even though they are irrelevant in the RG sense. The quantum Hall transition with short-range interactions produced by a screening gate falls into this category since $p\simeq 1.65$ and $\nu\simeq 2.33$ so that $p/2\nu\simeq 0.35$.

2. $p>2\nu$

For sufficiently irrelevant interactions (large α), the condition $p>2\nu$ may be satisfied. In this case, the argument of $\mathcal{G}_0$ in Eq. (5.3) diverges as $T\to 0$ for fixed x. Taking $\mathcal{G}_0(X)$ from Eq. (3.2),

$$\sigma_{xx}(\delta=0,T,u)\simeq\int dx\,\mathcal{G}_0[x(T_0/T)^{p/2\nu-1}]$$
$$\sim T^{p/2\nu-1}. \tag{5.5}$$

Thus the critical conductivity vanishes as $T\to 0$ according to a universal power law. Note that the power law exponent *cannot* be obtained using naive scaling with irrelevant couplings by following the approach discussed in Sec. III A. Again, this vanishes because the set of conducting states is of measure zero. The difference between the results for the two cases $p<2\nu$ and $p>2\nu$ will be further elucidated below.

B. Transition width: $\delta\neq 0$, $T\neq 0$

Hereafter, we specialize to $p<2\nu$ (case 1 above) which is appropriate for the quantum Hall transition with short-range interactions. For $\delta\neq 0$ and small T, the first term in the argument of the $\mathcal{G}$ in Eq. (5.2) can be ignored, leading to

$$\sigma_{xx}(\delta,T)\simeq\mathcal{G}_0(\delta L_\varphi{}^{1/\nu}). \tag{5.6}$$

Making use of $L_\varphi\sim T^{-p/2}=T^{-1/z_T}$ from Eqs. (4.12) and (4.10), this can be rewritten as

$$\sigma_{xx}(\delta,T)=\mathcal{G}_0\left(\frac{c\,\delta}{T^{1/z_T\nu}}\right). \tag{5.7}$$

The transition width is determined by the value of δ at which the scaling variable in Eq. (5.7) is of order 1. We obtain

$$\delta^*\sim T^{1/z_T\nu}. \tag{5.8}$$

We can view δ^* as the width of the energy window of states whose localization length exceeds the phase coherence length. If the width of this window exceeds the energy window defined by the Fermi function through the temperature (i.e., if $z_T\nu>1$ or equivalently $p<2\nu$), then the conductivity will scale to a finite value as discussed above. Conversely, if the energy window of states is narrower than the temperature, the conductivity becomes sensitive to the fact that the set of conducting states is of measure zero.

At large argument, the scaling function in Eq. (5.6) falls off exponentially with L_φ/ξ, being controlled by the crossover to the noninteracting localized phase. But the interaction u, although irrelevant at the critical fixed point, will give rise to conduction by variable range hopping in the localized phase. Because u is dangerously irrelevant in this sense, variable range hopping will not be part of the universal crossover scaling function in Eq. (5.6), but will only set in when L_φ exceeds the hopping length R_{hop}. Naive scaling suggests that the ratio of this longer crossover length to ξ will diverge as a power in ξ.

From Eq. (5.8) we deduce the temperature scaling exponent κ for the case of short-range interactions,

$$\kappa=\frac{1}{z_T\nu}\simeq 0.36. \tag{5.9}$$

Interestingly, because the value of z_T happens to be close to 1—the expected value with long-range Coulomb interactions—the value of κ is quite close to the corresponding value $\kappa\simeq 0.42$ as well, provided that ν is indeed the same in both cases. This suggests that temperature scaling of the transition width will not be dramatically altered by the presence of a screening gate and careful measurements will need to be made to see the change in the exponent. An important feature of Eq. (5.6) is that it implies that the correct thermal scaling variable is

$$\frac{L_\varphi}{\xi}\leftrightarrow\frac{1}{T\xi^{z_T}}, \tag{5.10}$$

and the thermal scaling function has the form

$$\sigma_{xx}(\delta,T)=\mathcal{G}_0([T\xi^{z_T}]^{-1/z_T\nu}). \tag{5.11}$$

These results suggest that by choosing appropriate scaling variables, the conductivity can be expressed in terms of a scaling function that is free of singularities in the limit of small scaling arguments. This will allow a description of transport within the conventional scaling framework, despite the fact that the scaling function $\mathcal{G}(X)$ of Eq. (4.2) is discontinuous.

C. Conventional scaling framework

The basic scaling form at the noninteracting fixed point reads

$$\sigma_{xx}(\delta,T,u)=\mathcal{G}'(b^{1/\nu}\delta,b^zT,b^{-\alpha}u). \tag{5.12}$$

At scale $b=\xi$, one writes

$$\sigma_{xx}(\delta,T,u)=\mathcal{G}(T\xi^z,u\xi^{-\alpha}), \tag{5.13}$$

where, as we have shown earlier, the scaling function has a discontinuity when its second argument approaches zero. In view of Eqs. (5.10) and (5.11), it is convenient to change the scaling variables according to

$$(T\xi^z,u\xi^{-\alpha})\to(L_\varphi/\xi,u\xi^{-\alpha}). \tag{5.14}$$

This is possible because

$$\frac{L_\varphi}{\xi}=\frac{1}{(T\xi^z)^{p/2}(u\xi^{-\alpha})}. \tag{5.15}$$

Hence, we can write as an alternative to Eq. (5.13)

$$\sigma_{xx}(\delta,T,u)=\mathcal{G}_{\mathrm{reg}}(L_\varphi/\xi,u\xi^{-\alpha}), \tag{5.16}$$

in which $\mathcal{G}_{\mathrm{reg}}$ is a regular scaling function when its second argument is taken to zero. Specifically,

$$\mathcal{G}_{\mathrm{reg}}(L_\varphi/\xi,0)=\mathcal{G}_{\mathrm{reg}}(\delta^\nu/T^{1/z_T},0)=\mathcal{G}_0(\delta/T^{1/z_T\nu}), \tag{5.17}$$

where use has been made of Eq. (5.6) in the last step and the behavior of $\mathcal{G}_0(X)$ is shown in Fig. 1(a). It is perhaps important to note that the change of variables in Eq. (5.14) has not removed the singularity associated with the scaling function in Eq. (5.13). Instead, it simply makes the singularity inaccessible in Eq. (5.16), since $u\to 0$ implies $L_\varphi\to\infty$.

VI. FREQUENCY SCALING AT $T=0$

A. Noninteracting case, $u=0$

For studying the frequency scaling, we start by returning to the noninteracting theory.[25] Scaling implies

$$\sigma_{xx}(\delta,\omega)=\mathcal{G}_0'(b^{1/\nu}\delta,b^z\omega). \tag{6.1}$$

Putting $b=\xi$ leads to

$$\sigma_{xx}(\delta,\omega)=\mathcal{G}_0(\omega\xi^z). \tag{6.2}$$

The behavior of the scaling function in Eq. (6.2) is expected from the Mott formula to be

$$\mathcal{G}_0(X)=\begin{cases} X^2\ln^{d-1}X, & X\to 0\\ \text{const}, & X\to\infty, \end{cases} \tag{6.3}$$

and has been studied numerically.[35] Thus the natural frequency scaling variable is $\omega\xi^z$, in contrast to the temperature scaling variable $T\xi^{z_T}$, which appears in Eqs. (5.11) and (5.12).

B. Short-range interactions, $u\neq 0$

Including u as in Eq. (5.13), we write

$$\sigma_{xx}(\delta,\omega)=\mathcal{G}(\omega\xi^z,u\xi^{-\alpha}). \tag{6.4}$$

This function has a nonsingular limit, i.e., $\mathcal{G}(X,Y\to 0)=\mathcal{G}_0(X)$. Thus we conclude that frequency scaling is conventional, so long as $p<2\nu$. Anticipating that this is the case for the IQHT with short-range interactions, further subtleties that occur in the opposite limit ($p>2\nu$) will not be discussed here. The transition width for $\omega\neq 0$ but $T=0$ is determined by setting $\omega\xi^z(\delta^*)=1$, giving

$$\delta^*(T=0,\omega)\sim\omega^{1/z\nu}. \tag{6.5}$$

This should be contrasted with $\delta^*(T,\omega=0)\sim T^{1/z_T\nu}$ where $z_T=2/p$, Eq. (5.8).

C. Irrelevance of frequency dephasing

A finite frequency can also lead to dephasing through interactions. For $u=0$, the only length scale introduced by a finite frequency is

$$L_\omega=\sqrt{D/\omega}. \tag{6.6}$$

However, when $u\neq 0$, there is a frequency-induced dephasing time $\tau_\varphi(\omega)$ which can be accounted for by including ω in the discussion of Sec. III B. Following Eqs. (4.6)–(4.9), one obtain,

$$\frac{1}{\tau_\varphi(\omega)}\sim u^2\omega^p \tag{6.7}$$

at $T=0$. This leads to another frequency-dependent length scale in the diffusive regime, $L_\omega^u=\sqrt{D\tau_\varphi(\omega)}$. Thus

$$L_\omega^u\sim\sqrt{D/u^2}\,\omega^{-1/z_T}. \tag{6.8}$$

The ratio of the two lengths is

$$\frac{L_\omega^u}{L_\omega}\sim\omega^{-(z-z_T)/zz_T}. \tag{6.9}$$

Provided interactions are irrelevant, so that $\alpha>0$ and $z_T<2$ from Eq. (4.12), this ratio diverges in the limit $\omega\to 0$. The fact that $L_\omega^u\gg L_\omega$ ensures that frequency dephasing results only in corrections to scaling of the conductivity, and is irrelevant in the asymptotic limit.

VII. GENERAL TEMPERATURE AND FREQUENCY SCALING

In this section, we discuss the general scaling behavior of the conductivity as a function of both frequency and temperature. We start with the basic scaling form at the NIFP,

$$\sigma_{xx}(\delta,T,\omega,u)=\mathcal{G}(T\xi^z,\omega\xi^z,u\xi^{-\alpha}). \tag{7.1}$$

We convert to new scaling variables as in Eq. (5.15). Then

$$\sigma_{xx}(\delta,T,\omega,u)=\mathcal{G}_{\rm reg}(L_\varphi/\xi,\omega\xi^z,u\xi^{-\alpha}), \tag{7.2}$$

where $\mathcal{G}_{\rm reg}(X,Y,Z)$ is continuous in Z at $Z=0$. Let

$$\mathcal{G}_{\rm reg}(L_\varphi/\xi,\omega\xi^z,0)=\mathcal{G}_0(L_\varphi/\xi,\omega\xi^z). \tag{7.3}$$

Thus for $\xi\gg 1$ we have

$$\sigma_{xx}(\delta,T,\omega,u)=\mathcal{A}(T\xi^{z_T},\omega\xi^z). \tag{7.4}$$

Now consider the approach to the critical point at $\delta=0$. As $\xi\to\infty$, one argument of $\mathcal{G}_0$ diverges and the other approaches zero, but the scaling variable

$$(L_\varphi/\xi)^z\omega\xi^z=\frac{\omega}{T^p} \tag{7.5}$$

remains finite for $\omega,T\neq 0$. (We have used $z_T=2/p$ and $z=2$.) Thus at the critical point

$$\sigma_{xx}(\delta=0,T,\omega,u)=\mathcal{A}(\omega\tau_\varphi)=\mathcal{A}\left(\frac{\omega}{T^p}\right). \tag{7.6}$$

We see that $\omega/T^p\sim\omega/T^{1.65}$ is the scaling variable at criticality, in contrast to the conventional situation in which the interaction u scales to a finite value at the fixed point and the scaling variable is ω/T.

VIII. SUMMARY

We have shown that, in the presence of short-range Coulomb interactions, the integer quantum Hall transition is a quantum phase transition of an unconventional kind. We find that the interactions, though irrelevant, are responsible for the existence of a finite critical conductivity. In addition, the conventional ω/T scaling at criticality is replaced by ω/T^p scaling, where p is a critical exponent controlling the inelastic dephasing time. As a result, there exist two independent dynamical scaling exponents $z_T\neq z$ for temperature and frequency, respectively. The dynamic exponents determine the physical length scales associated with T and ω: $(L_\varphi,L_\omega)\sim(T^{-1/z_T},T^{-1/z})$. These unconventional results follow from the fact that, though short-range interactions are irrelevant at the critical point, the physical behavior is discontinuous in the interaction strength in the noninteracting limit. Associated with this is the existence of a coherence time much longer than the conventional quantum coherence time $\hbar/T$, as interactions scale to zero and the system scales toward the noninteracting fixed point. We have shown that the scaling exponent z_T (or p) is completely determined by the scaling dimension of the leading irrelevant interaction. The physics discussed here may in fact be quite general for quantum critical transport phenomena such as the conventional Anderson-Mott metal-insulator transitions, whenever the interactions scale to zero at the fixed point.

For the IQHT with short-range interactions, we have the set of critical exponents

$$\nu \simeq 2.3,\ z_T \simeq 1.2,\ z=2, \tag{8.1}$$

which describe the scaling with sample size, temperature, and frequency according to Eq. (2.1).

This behavior can be checked experimentally, for example, by looking for a change in the temperature scaling of the transition width whose exponent will change from $\kappa \simeq 0.42$ to $\kappa \simeq 0.36$, or by looking at the frequency/temperature scaling described in Eq. (7.6) where a larger change in exponent is expected. The experimental requirement is that the long-range Coulomb interaction between electrons at large distances be screened, so that they interact via a residual, short-range interacting potential.

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[33] Note that one should distinguish between the electron-electron inelastic scattering time $\tau_{\rm ee}$, which is a single-particle (energy) relaxation time, and the two-particle or phase coherence time τ_φ defined in the weak localization theory (Ref. 34). The difference between the two in a disordered metal depends on dimensionality, although on general grounds $\tau_\varphi \leqslant \tau_{\rm ee}$. It turns out within perturbative weak localization theory (Ref. 34) that $\tau_\varphi^{-1} \sim \tau_{\rm ee}{}^{-1} \sim T^{3/2}$ in $3D$, $\tau_\varphi^{-1} \sim T^{2/3}$, and $\tau_{\rm ee}{}^{-1} \sim T^{1/2}$ in $1D$. In the marginal case of $d=2$, $\tau_\varphi \sim \tau_{\rm ee} \sim T^{-1}$ up to possible $\log T$ corrections. Here τ_φ can also be formally defined in terms of the temperature dependence of the conductivity (although it is not weak localization). The result for the temperature dependence here is different from that in the weak localization case and contains the exponent α because the disorder eigenstates are those for a critical point, not a metallic phase. We have assumed that in perturbation theory for an irrelevant interaction, both $\tau_{\rm ee}$ and τ_φ scale with $|u_{\rm eff}|^2$.

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